Synchronous Games in the Situation Calculus

(Extended Abstract)

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ABSTRACT
We develop a situation calculus-based account of multi-player synchronous games. These are represented as action theories called situation calculus synchronous game structures (SCSGSs) that involve a single action tick whose effects depend on the combination of moves chosen by the players. Properties of games, e.g., winning conditions, playability, weak and strong winnability, etc. can be expressed in a first-order variant of alternating-time mu-calculus. Computationally effective verification can be performed. SCSGSs can be viewed as a variant of the Game Description Language (GDL) where states are represented by first-order theories.

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1. INTRODUCTION
Many types of problems can be viewed as games, e.g., contingent planning, service orchestration, etc. Logics for reasoning about game settings has been an active area, with Alternating-Time Temporal Logic (ATL) [1] a popular choice, and model checking techniques have been used to verify properties of games specified in ATL and to synthesize agent strategies. However, such logics are usually propositional and limited to finite domains. Moreover, the game settings are usually specified using low-level automata-like languages. One exception is the Game Description Language (GDL) [6] developed for the general game playing competition, which is based on logic programming. But, GDL is intended to represent games with finite domains and its semantics based on “negation as failure” is somewhat complex.

[4] proposes an expressive first-order (FO) logical framework for specifying and reasoning about game-like problems based on the situation calculus (SC)[7]. Game settings are specified as a special kind of SC action theory. Legal moves can be specified declaratively, or procedurally in a variant of the SC programming language Golog [7]. Complex temporal properties of games can be expressed in a FO variant of alternating-time µ-calculus (µATL). Methods for verification and synthesis based on fixpoint approximation and regression are also proposed. But [4] assumes only one agent may act in any state, i.e., it deals with turn-taking games.

Inspired by [4], we develop a SC-based specification and verification framework that deals with multi-players synchronous games, and is similar in spirit to GDL. Games are represented as action theories of a special form, called situation calculus synchronous game structures (SCSGSs), in which we have a single action tick whose effects depend on the combination of moves selected by the players. As in [4] a FO-order variant of µATL [2] is used to specify and verify properties of the game, including winning conditions, playability, weak and strong winnability, etc.

2. SYNCHRONOUS GAME STRUCTURES
Here, we concentrate on games where there are n players/agents each of whom chooses a move at every time step. All such moves are executed synchronously and determine the next state of the game. At each time step, the state of the game is fully observable by all agents, as are all past moves. These assumptions are built into GDL [6]. To represent such multi-player synchronous games, we define a special class of basic action theories [7], called situation calculus synchronous game structures (SCSGSs), defined as follows.

Agents A SCSGS involves a finite set of n agents, and we introduce a subset Agents of Objects which includes these finitely many agents A1,...,An, each denoted by a constant, and for which unique names and domain closure hold.

Moves. We also introduce a second subset Moves of Objects, representing the possible moves of the agents. These come in finitely many types, represented by function symbols Mi(x), which are parameterised by objects x. Given the parameterisation, each agent may have an infinite number of possible moves at each time step. We have unique names and domain closure axioms for these functions.

Actions. In SCSGSs, there is only one action type, tick(m1, ..., mn), representing the execution of a joint move by all the agents at a given time step. tick has n parameters, one per agent, which are of sort Moves and corresponds to the simultaneous choice of the move to perform by the n agents.

Legal moves. SCSGSs include a characterization of the legal moves available to each agent in a given situation. This
is specified using a special predicate $LegalM$ defined as:

$$LegalM(Ag_i, M_i(x), s) \equiv \Phi_{Ag_i,M_i}(x, s)$$

meaning that agent $Ag_i$ can legally do move $M_i(x)$ in situation $s$ if $\Phi_{Ag_i,M_i}(x, s)$ holds. Technically, $LegalM$ is an abbreviation for $\Phi_{Ag_i,M_i}(x, s)$, which is a uniform formula (i.e., a formula that only refers to a single situation $s$).

**Precondition axioms.** The precondition axiom for the action $\textit{tick}$ is fixed and specified in terms of $LegalM$ as follows:

$$\text{Poss}(\textit{tick}(m_1,\ldots,m_n), s) \equiv \bigwedge_{i=1,\ldots,n} LegalM(Ag_i, m_i, s)$$

Thus action $\textit{tick}(m_1,\ldots,m_n)$ denoting the joint move of all agents can be performed iff each selected move $m_i$ is a legal move for agent $Ag_i$ in situation $s$. Since we only have one action type $\textit{tick}$, this is the only precondition axiom in $D_{\text{poss}}$.

**Successor-state axioms.** Then, we have successor-state axioms $D_{\text{ss}},$ specifying the effects and frame conditions of the joint moves $\textit{tick}(m_1,\ldots,m_n)$ on the fluents. Such axioms, as usual in basic action theories, are domain specific, and characterize the game under consideration. Within such axioms, the agent moves that occur as arguments of $\textit{tick}$ determine how fluents change as a result of the joint move.

**Initial situation description.** Finally, the initial state of the game is axiomatized in the initial situation description $D_0$ as usual, in a domain specific way.

**EXAMPLE 1.** Consider an iterated version of the game Rock-Paper-Scissors. We can specify legal moves as follows:

$$LegalM(ag, play(k), s) \equiv (ag = Ag_{R} \lor (ag = Ag_{P}) \land (k = Rock \lor k = Paper \lor k = Scissors))$$

We have a fluent $\textit{WinCount}$ keeping track of how many rounds each agent has won, with the successor-state axiom:

$$\textit{WinCount}(ag, i, do(a, s)) \equiv \phi \lor \textit{WinCount}(ag, i, s) \land \neg \phi$$

where $\phi \equiv \exists k, l, j.a = \textit{tick}(\textit{play}(k), \textit{play}(l)) \land \textit{Beats}(k, l) \land \textit{WinCount}(ag, j, do(a, s)) \land i = j + 1 \land i \leq R$

We can say an agent wins as soon as she has won $R$ rounds. Note that we get an infinite-domain version of the game if we allow move $\textit{play}(k)$ to take any integer $k$ as argument, and define $\textit{Beats}(k, l)$, e.g., as $(k - l) \text{ mod } 3 = 1$.

In the full paper [5], we show how one can also specify legal agent moves procedurally in variant of the SC programming language Golog [7]. Our SCSGSs amount to a variant of GDL where states are represented by FO theories. In [5], we show this by giving a translation of GDL specifications into SCSGSs and showing its soundness and completeness.

### 3. VERIFICATION AND GOAL LANGUAGE

To express properties about SCSGSs, we introduce a logic, inspired by ($\mu$) ATL [1], based on a FO variant of the $\mu$-calculus [2] but that works on games, by suitably considering coalitions acting towards the realization of a temporarily extended goal. The key building block in this logic is the so-called $force$-next operator, $\langle \langle G \rangle \rangle \circ \Psi$, meaning that coalition $G$ can ensure that $\Psi$ holds in the next situation. Informally, $\langle \langle G \rangle \rangle \circ \Psi$ holds in a situation if there exist moves for the agents in coalition $G$ such that for all moves by agents not in $G$, $\Psi$ holds in the situation resulting from such a joint move. The syntax of our logic $\mu\mathcal{L}$ is as follows:

$$\Psi \leftarrow \varphi \mid Z \mid \Psi_1 \land \Psi_2 \mid \exists x. \Psi \mid \langle \langle G \rangle \rangle \circ \Psi \mid \mu Z. \Psi$$

Here, $\varphi$ is a (possibly open) situation-suppressed SC uniform formula, $Z$ is a predicate variable of a given arity, and $\mu$ is the least fixpoint construct from the $\mu$-calculus. We also use the usual FOL abbreviations, $[G] \circ \Psi \equiv \neg [\langle \langle G \rangle \rangle \circ \neg \Psi]$, and $\nu Z. \Psi(Z) \equiv \neg \mu Z. \neg [\Psi(Z)]$, i.e., the greatest fixpoint of $\Psi(Z)$. The modal $\mu$-calculus [2] is one of the most expressive temporal logic, generalizing of CTL* for instance.

**EXAMPLE 2.** Several key properties of games [6] can be expressed: a) $Playability$, i.e., at every step which is not terminal there exists a legal joint move: $\nu Z. \text{Terminal} \lor \langle \langle \text{ALL} \rangle \rangle \circ Z$. b) $Termination$, i.e., there is a way of playing the game that eventually leads to termination: $\mu Z. \text{Terminal} \lor \langle \langle \text{ALL} \rangle \rangle \circ Z$. c) $Weak Winnability (by agent $Ag$)$, i.e., there is a way for agent $Ag$ to win if the others cooperate:

$$\mu Z. \text{Terminal} \land \text{Goal}(Ag, v) \land \langle \langle \lambda_{Ag'} \neq Ag \text{Goal}(Ag', v') \land v' \leq v \rangle \rangle \lor \langle \langle \text{ALL} \rangle \rangle \circ Z$$

d) $Strong Winnability (by agent $Ag$)$, i.e., there is a way for agent $Ag$ to win no matter what the others do:

$$\mu Z. \text{Terminal} \land \text{Goal}(Ag, v) \land \langle \langle \lambda_{Ag'} \neq Ag \text{Goal}(Ag', v') \land v' \leq v \rangle \rangle \lor \langle \langle \text{ALL} \rangle \rangle \circ Z$$

SC reasoning can be used for verifying properties of games, to analyze them and develop better players. These include sound but incomplete techniques that apply to the general setting [4], which is undecidable, and sound and complete techniques for the decidable “bounded fluent extension” setting [3]. See the full paper for details.

### 4. CONCLUSION

We have defined a logical framework, SCSGSs, for representing synchronous games-like systems and verifying temporal properties over them. GDL games can be represented as SCSGSs. Our framework is truly first-order and can be used to specify games/systems that involve infinite domains and an infinite set of states. Furthermore, when the SCSGS is “bounded”, verification of large classes of temporal formulas is decidable. Effective reasoners for our framework can be implemented. A prototype verifier for SC game structures that uses the iterated fixpoint approximation technique [4] has been developed. In future work, we will generalize our account to partially observable game settings.

REFERENCES


