2023 International Graduate School on Control
Course M16

Control of Soft and Articulated Elastic Robots

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Robots with Flexible Joints: Modeling and Control

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Outline

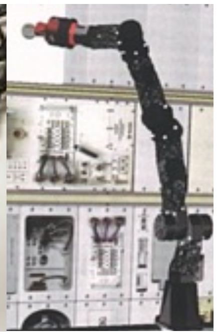
- cases of articulated soft robots
  - manipulators with flexible transmissions, variable stiffness actuation (VSA), serial elastic actuation (SEA), …
  - application: safe physical Human-Robot Interaction (pHRI)
- dynamic modeling of flexible joint manipulators
  - with few comments on their structural properties and extensions
- classical control tasks and their solution
  - a closer look into the linear case: single elastic joint (with no gravity)
  - regulation with partial/full state feedback and gravity compensation
  - inverse dynamics and feedback linearization for trajectory tracking
- model-based design based on feedback equivalence
  - exact gravity cancellation
  - damping injection on the link side of the flexible transmission
- conclusions and basic references

Classes of articulated soft robots

Robots with elastic joints

- design of lightweight robots with stiff links for end-effector accuracy
- compliant elements absorb impact energy
  - soft coverage of links (safe bags)
  - elastic transmissions (HD, tendons, cable-driven, ...)


- elastic joints decouple instantaneously the larger inertia of the driving motors from smaller inertia of the links (involved in contacts/collisions!)
- relatively soft joints need more sensing (e.g., joint torque) and better control to compensate for static deflections and dynamic vibrations

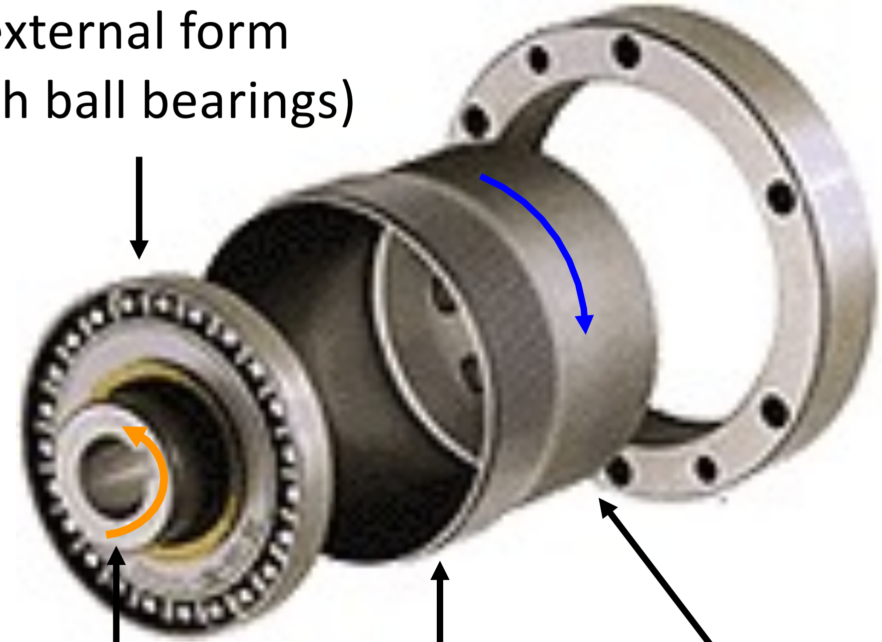

➡ torque-controlled robots (DLR LWR-III, KUKA LWR-IV & iiwa, Franka, ...)

Harmonic Drive

Compact, in-line, high reduction (up to 1:160), power efficient transmission

Wave Generator
of slightly elliptic
external form
(with ball bearings)

Circular Spline

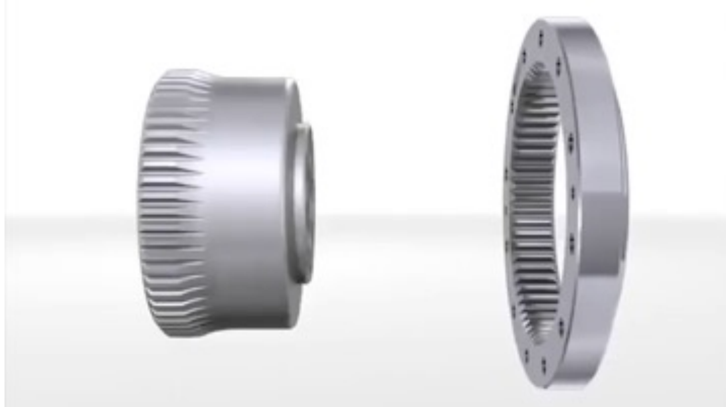


FlexSpline
(two contact points)

input from motor

output to load

video



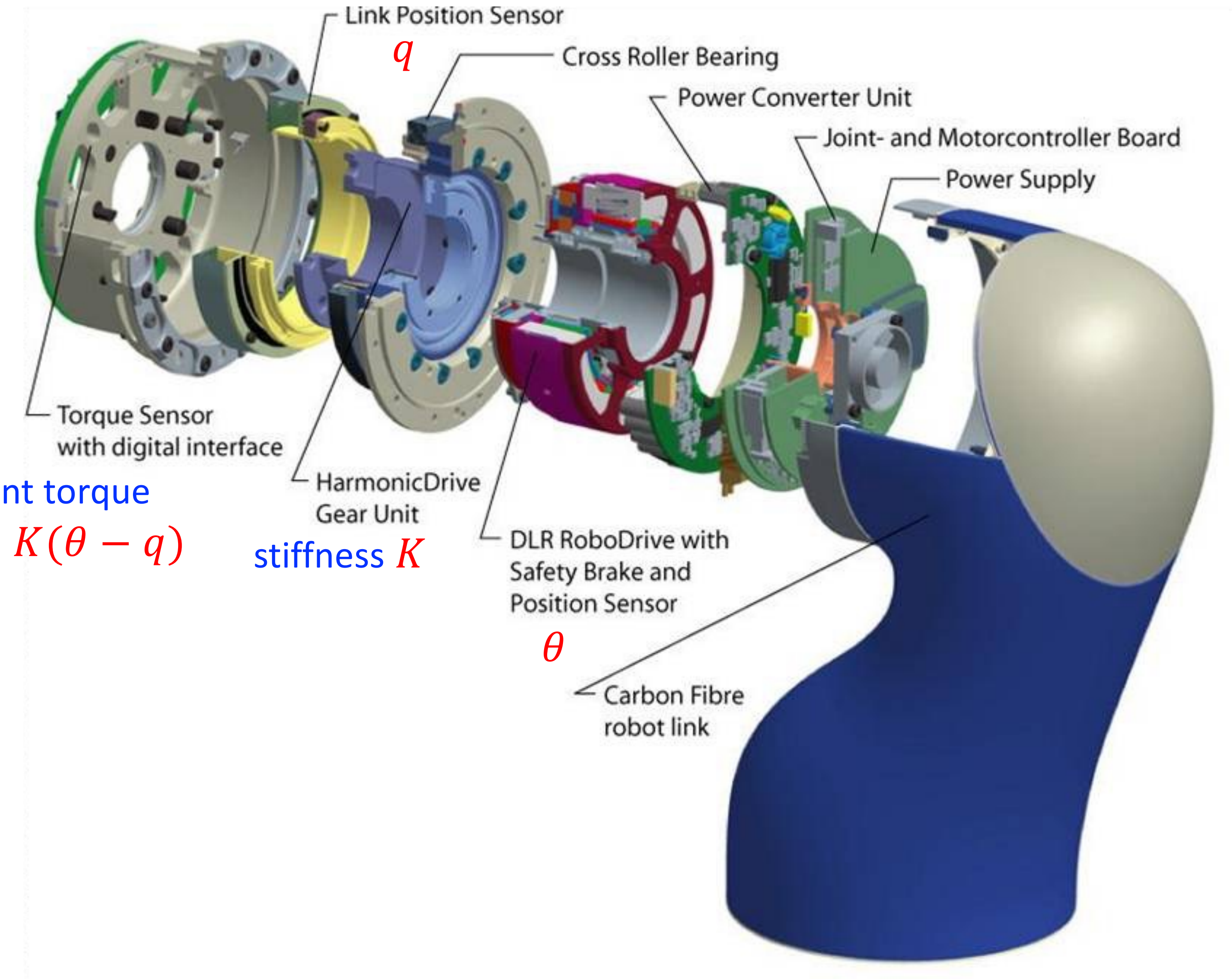
inner #teeth CS = outer #teeth FS + 2

reduction ratio

n = #teeth FS / (#teeth CS - #teeth FS)
= #teeth FS / 2

Sensors in an elastic joint

Exploded view of a joint of the DLR-III robot



joint torque

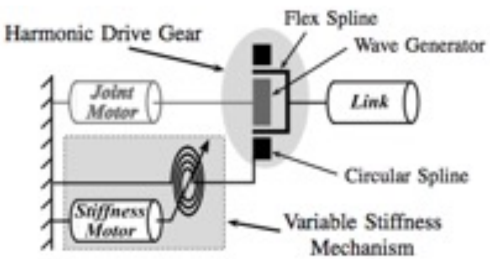
$$\tau_J = K(\theta - q)$$

stiffness K

Classes of soft robots

Robots with Variable Stiffness Actuation (VSA)

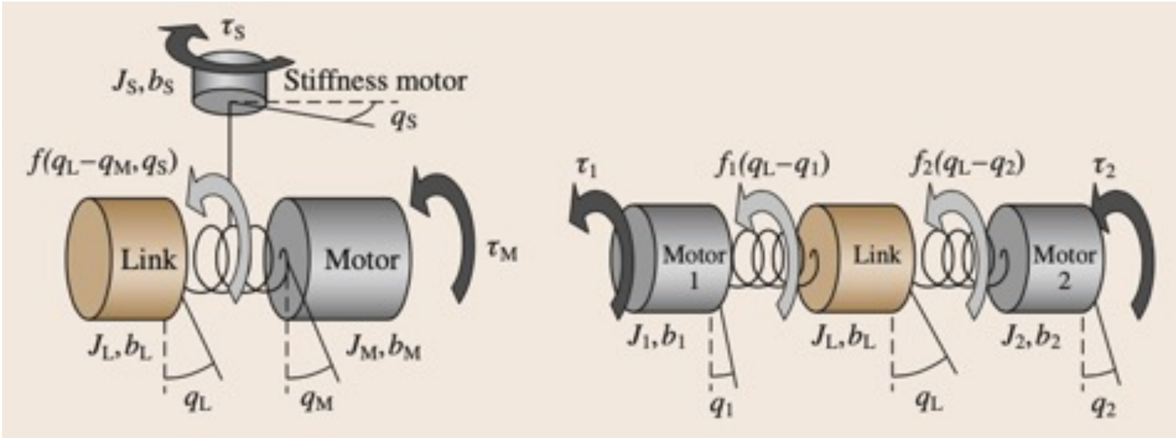
- uncertain/dynamic interaction with the environment requires to adjust the compliant behavior of the robot and/or to control contact forces
  - passive joint elasticity & active impedance control used in parallel
- nonlinear flexible joints with variable (controlled) stiffness work at best:
  - can be made stiff when moving slow (performance), soft when fast (safety)
  - enlarge the set of achievable task-oriented compliance matrices
  - feature also: **robustness**, **optimal energy** use, **explosive motion** tasks, ...



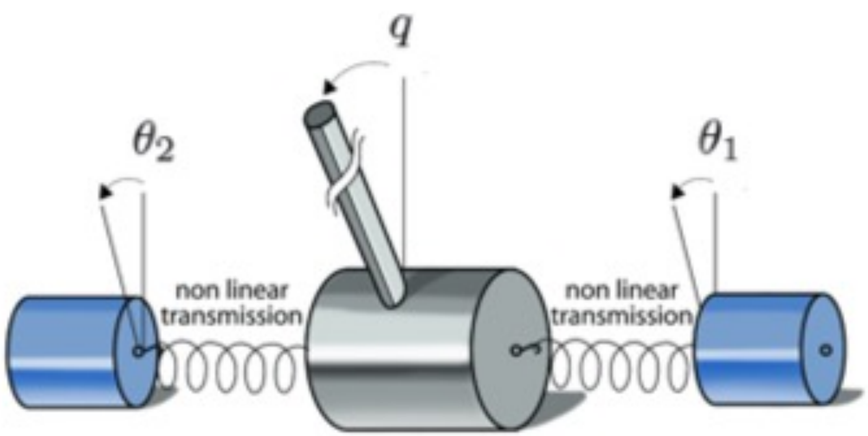
Serial and Antagonistic VSA

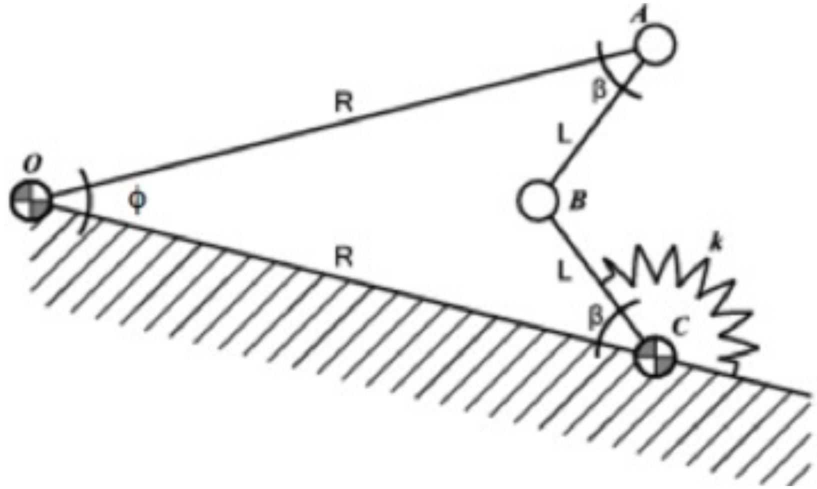
With antagonistic VSA-II by University of Pisa

serial VSA



antagonistic VSA





- bi-directional, symmetric arrangement of two motors in antagonistic mode
- nonlinear flexible transmission: four-bar linkage + linear spring

A matter of terminology (or of purpose?)

Different sources of softness/flexibility, though similar robotic systems

- elastic joints vs. SEA (Serial Elastic Actuators)
  - based on the same physical phenomenon: compliance in actuation
  - compliance added on purpose in SEA, mostly a disturbance in elastic joints
  - different range of stiffness: 5-10K Nm/rad down to 0.2-1K Nm/rad in SEA
- joint deformation is often considered in the linear domain
  - modeled as a concentrated torsional spring with constant stiffness at the joint
  - nonlinear flexible joints share similar control properties
  - nonlinear stiffness characteristics are needed instead in VSA
  - a (serial or antagonistic) VSA working at constant stiffness is an elastic joint
- flexible joint robots are classified as underactuated mechanical systems
  - have less commands than generalized coordinates
  - non-collocation of command inputs and of dynamic behaviors to be controlled
  - however, they are controllable in the first approximation (the *easier* case!)

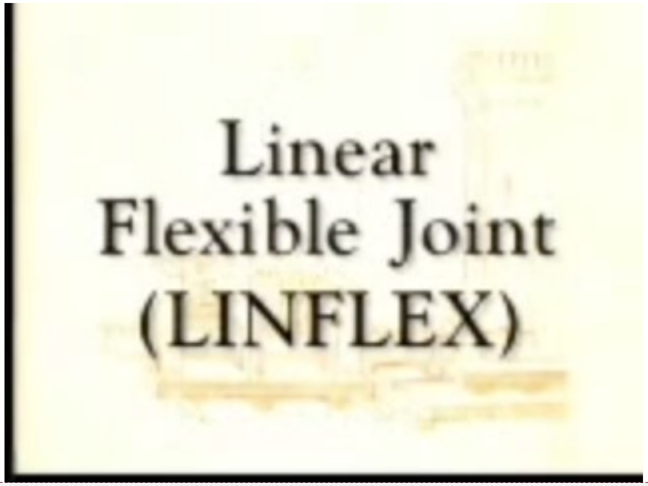
Control drawbacks due to joint elasticity

Neglecting softness may generate vibrations and trajectory oscillations

- anywhere: conventional/massive industrial manipulators, lightweight (loaded) research-oriented robots, educational devices, …

video

video

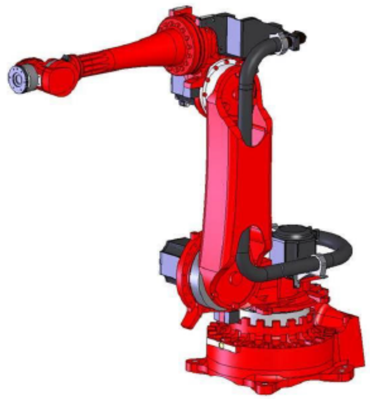


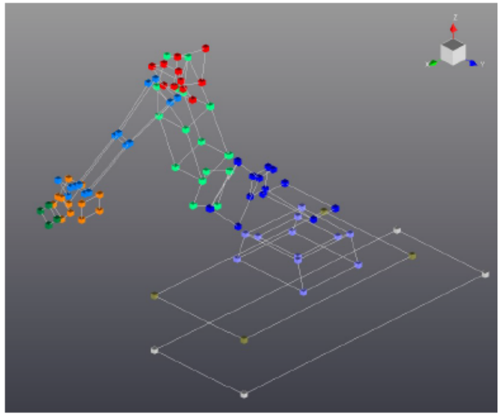
video

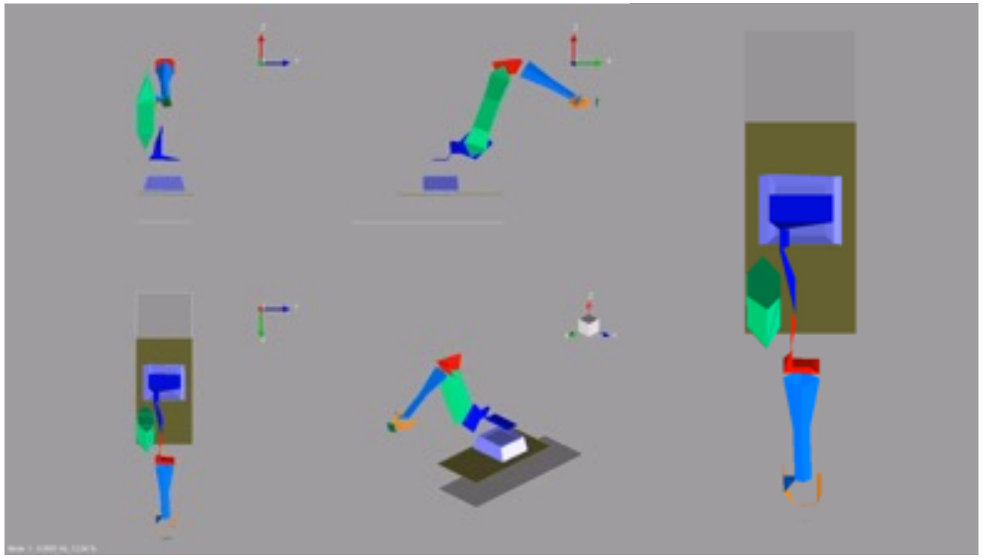
Modal analysis of an industrial robot

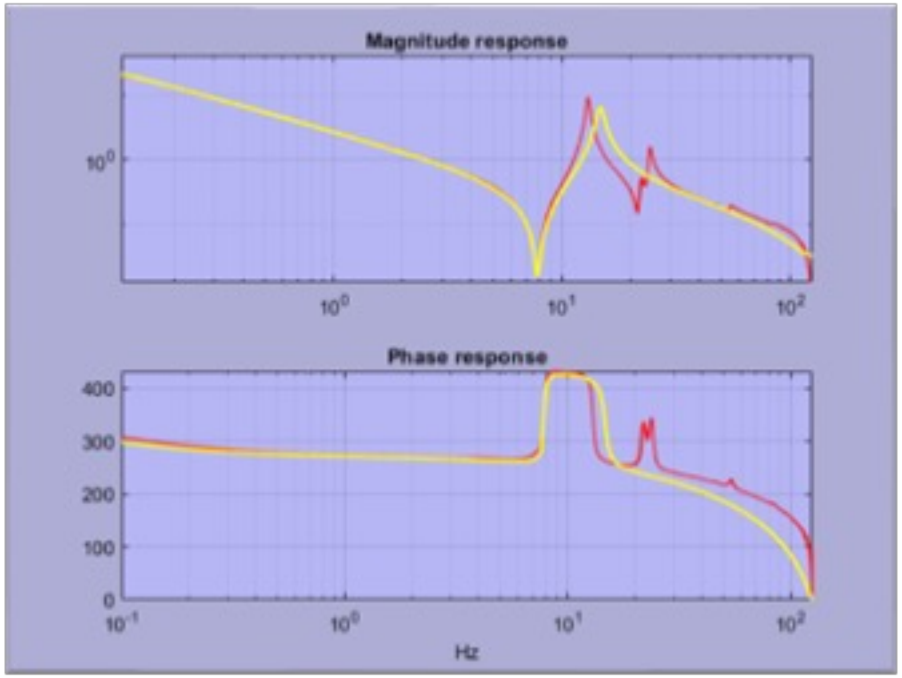
Assumed to be fully rigid

- lowest mode is a torsional vibration around the base vertical joint axis with $f_1 = 6.9$ Hz (but slightly changing with robot configuration and payload)









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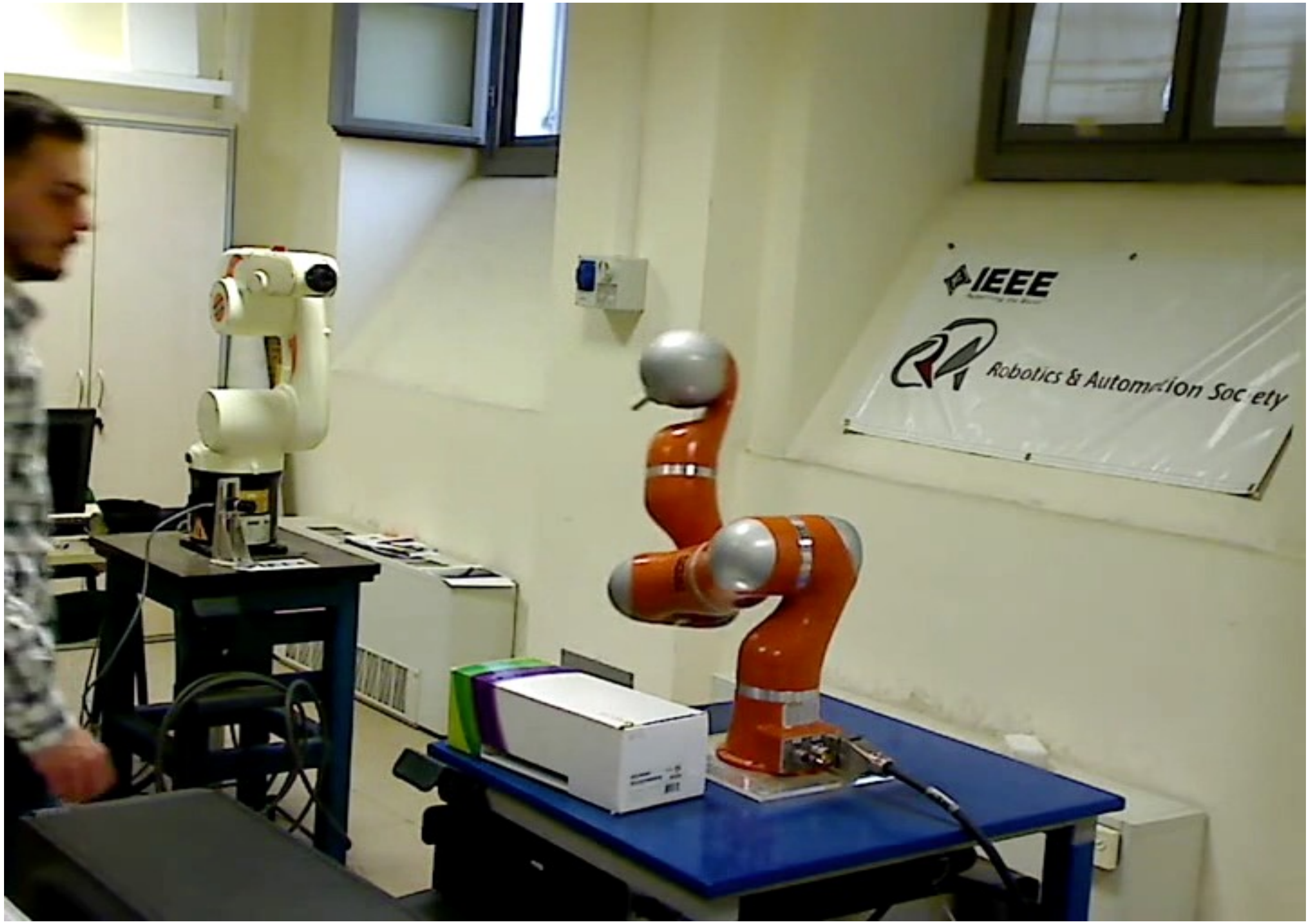
Exploiting joint elasticity in pHRI

Detection and selective reaction in torque control mode, based on residuals

- collision detection & reaction for safety (model-based + joint torque sensing)



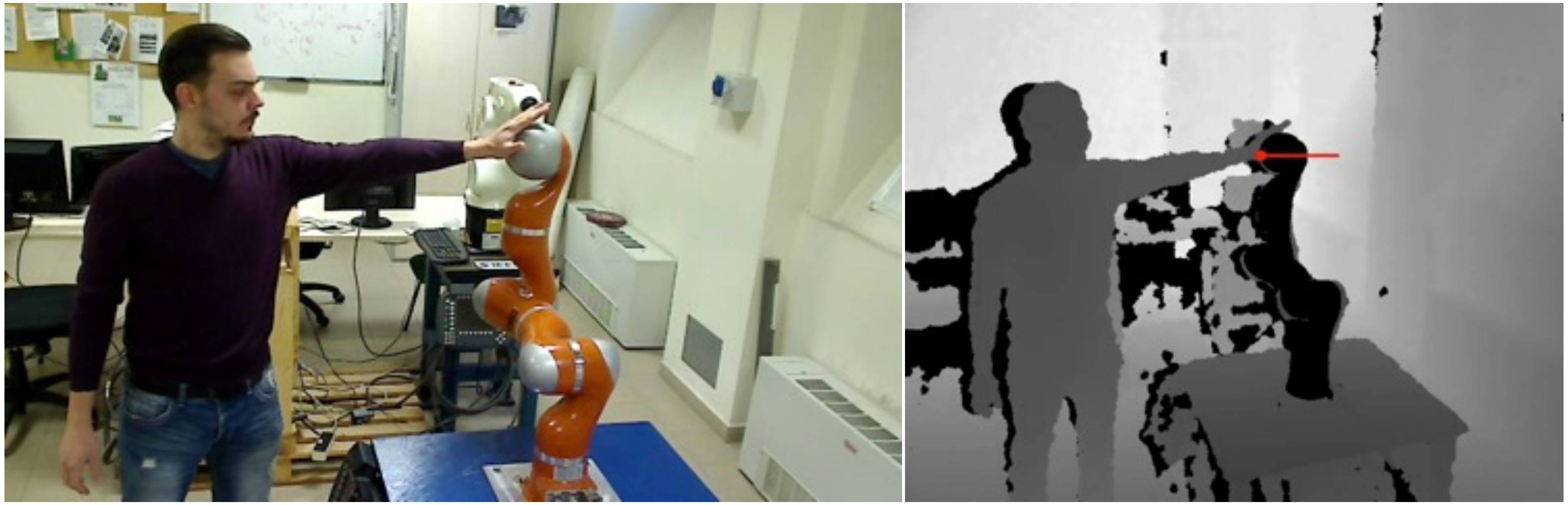
video

[De Luca, Mattone, 2005; Haddadin *et al*, 2017]

Exploiting joint elasticity in pHRI

Human-robot collaboration in torque control mode

- contact force estimation & control (virtual force sensor, anywhere/anytime)



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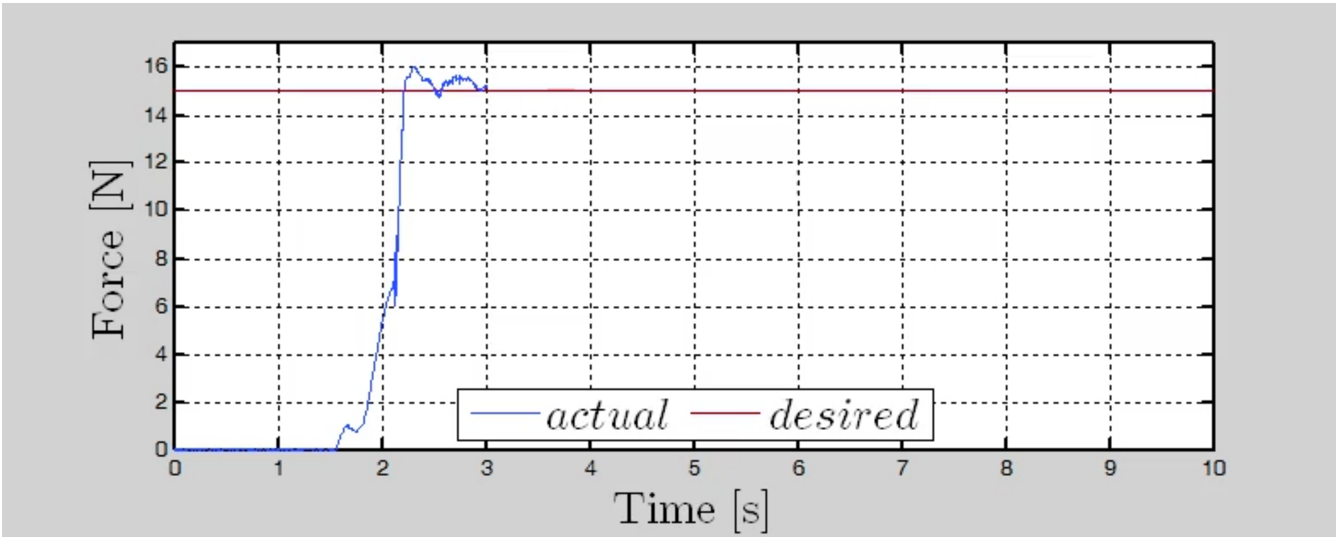


[Magrini *et al*, 2015]

Dynamic modeling

Lagrangian formulation (so-called reduced model of [Spong, 1987])

- open chain manipulator with N joints driven by electrical actuators, with elastic transmission to N rigid links
- use N motor variables θ (as reflected through the gear ratios $\dot{\theta}_{mi} = n_{ri}\dot{\theta}_i$) and N link variables q
- assumptions

  A1) small deflection at joints

  A2) axis-balanced motors

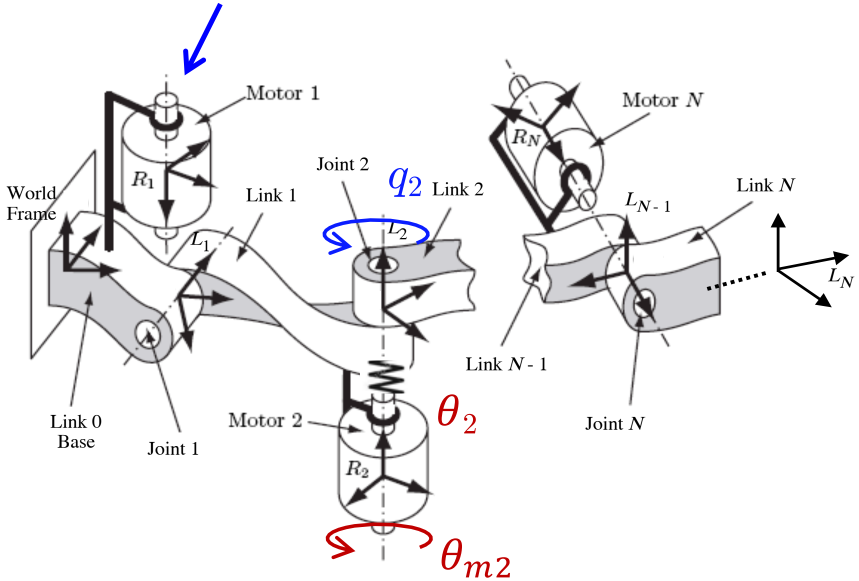
  A3) each motor mounted on the robot in a position preceding the driven link

  A4) no inertial couplings between motors and links

  ⇕

  angular kinetic energy of each motor is due only to its own spinning
- no dissipative effects here (can be added later)

center of mass of rotors on rotation axes



Dynamic modeling

Derivation

- kinetic energy and potential energy due to gravity of the links (including on each link the mass of the carried actuator, under assumption A3 (and A2)

$$T_l = \frac{1}{2}\dot{q}^T M(q)\dot{q} \qquad U_g = U_g(q)$$

- angular kinetic energy of the motors, under assumption A4 (and A2)

$$T_{mi} = \frac{1}{2} I_{mi}\dot{\theta}_{mi}^2 = \frac{1}{2} I_{mi} n_{ri}^2 \dot{\theta}_i^2 = \frac{1}{2} B_i \dot{\theta}_i^2 \qquad T_m = \sum_{i=1}^{N} T_{mi} = \frac{1}{2}\dot{\theta}^T B \dot{\theta}$$

- potential energy due to joint elasticity (under assumption A1)

$$U_{ei} = \frac{1}{2} K_i (q_i - \theta_i)^2 \qquad U_e = \sum_{i=1}^{N} U_{ei} = \frac{1}{2}(q-\theta)^T K (q-\theta)$$

both > 0, diagonal matrices

- robot Lagrangian and E-L equations

$$L = T - U = (T_l + T_m) - (U_g + U_e) = L(q, \theta, \dot{q}, \dot{\theta})$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)^T - \left(\frac{\partial L}{\partial q}\right)^T = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)^T - \left(\frac{\partial L}{\partial \theta}\right)^T = \tau$$

Dynamic model

Robots with elastic joints

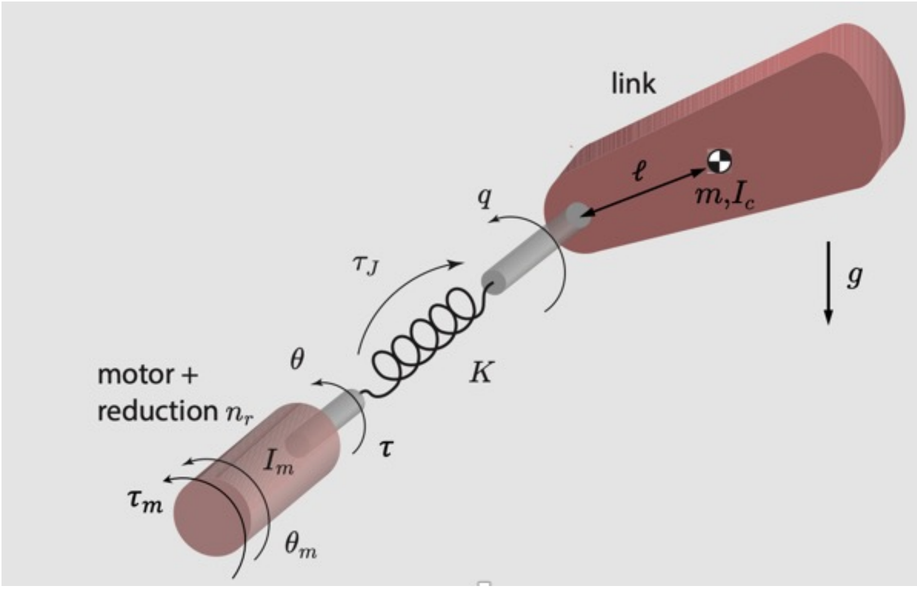
A4) ⇒ 2N × 2N inertia matrix Is block diagonal

A2) ⇒ inertia matrix and gravity vector are not dependent from θ

$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

link equation

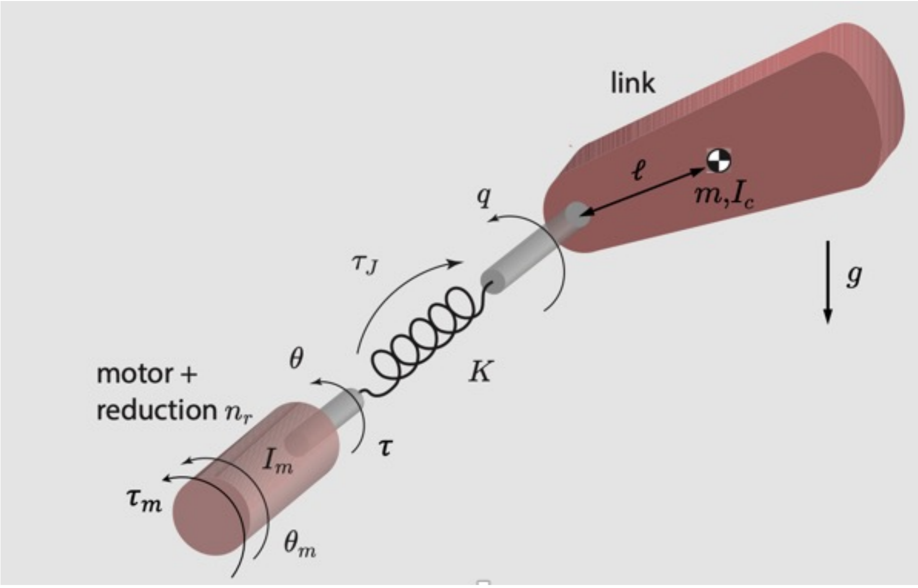
motor equation



work out this 1 DOF example …. [5 min]

Single elastic joint

Adding also viscous friction on motor and link sides



$$T_m = \frac{1}{2} I_m \dot{\theta}_m^2 = \frac{1}{2} I_m n_r^2 \dot{\theta}^2 = \frac{1}{2} B \dot{\theta}^2$$

$$T_l = \frac{1}{2}(I_c + m\ell^2)\dot{q}^2 = \frac{1}{2} M \dot{q}^2$$

$$U_g = mg\ell \sin q + U_0$$

$$U_e = \frac{1}{2} K (q - \theta)^2$$

link equation

$$M\ddot{q} + K(q - \theta) + mg\ell \cos q = -D_q \dot{q}$$

$$B\ddot{\theta} + K(\theta - q) = n_r\left(\tau_m - D_{m\theta}\dot{\theta}_m\right) = \tau - D_\theta \dot{\theta}$$

on the rhs non-conservative torques performing work on q and θ

motor equation

$$\tau = n_r \tau_m$$

$$D_\theta = D_{\theta m} n_r^2$$

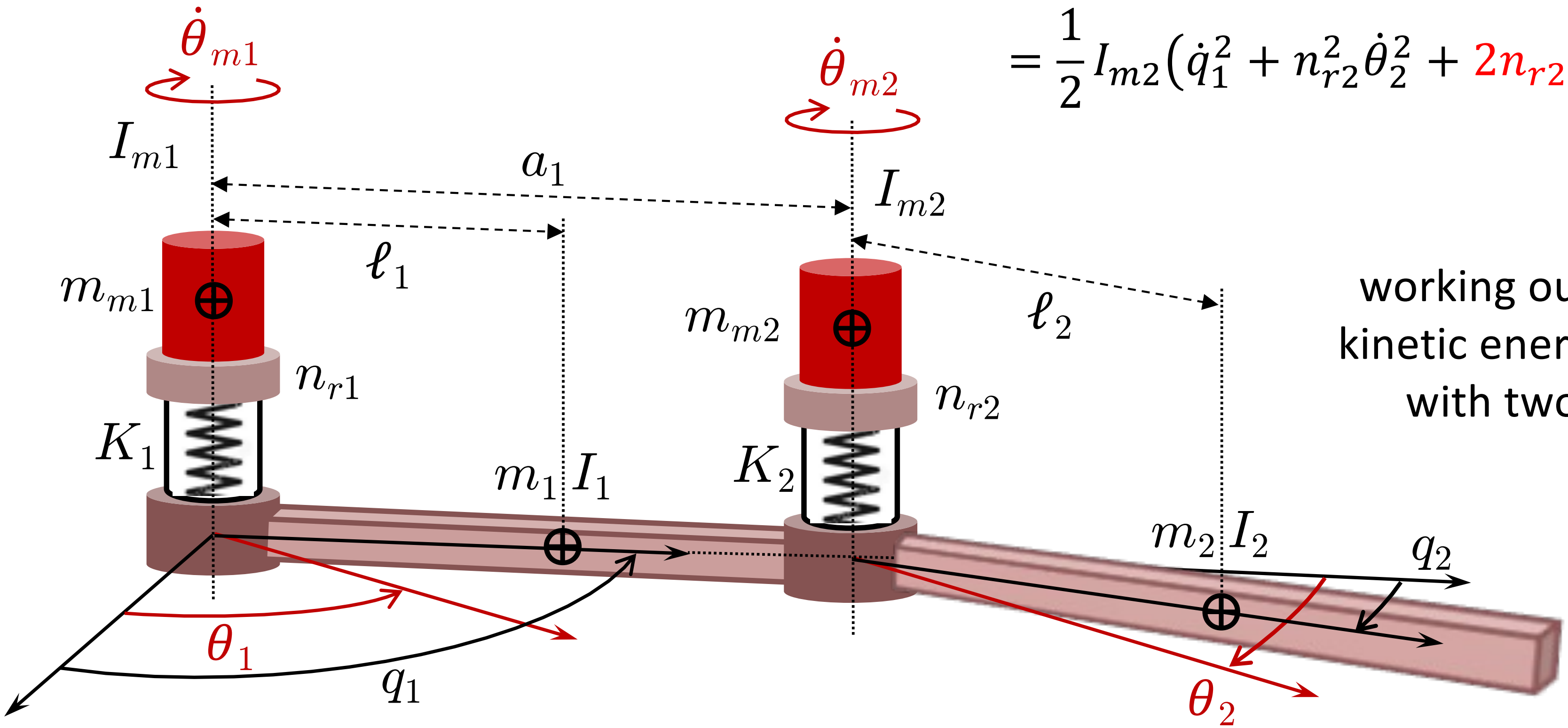
Dynamic modeling

A more complete model without the Spong assumption A4

the angular kinetic energy of the two motors is …

$$T_{m1} = \frac{1}{2} I_{m1} n_{r1}^2 \dot\theta_1^2 = \frac{1}{2} B_1 \dot\theta_1^2$$

$$\boldsymbol{T_{m2}} = \frac{1}{2} I_{m2} \left(\dot q_1 + \dot\theta_{m2}\right)^2 = \frac{1}{2} I_{m2} \left(\dot q_1 + n_{r2}\dot\theta_2\right)^2 = \frac{1}{2} I_{m2}\left(\dot q_1^2 + n_{r2}^2 \dot\theta_2^2 + 2 n_{r2} \dot q_1 \dot\theta_2\right)$$



working out the complete kinetic energy of this robot with two elastic joints …

$$\Rightarrow \begin{pmatrix} M(q) & S \\ S^T & B \end{pmatrix} \qquad S = \begin{pmatrix} 0 & I_{m2} n_{r2} \\ 0 & 0 \end{pmatrix}$$

strictly upper-triangular

$M(q)$ contains also m_{m2} and I_{m2}

(B contains $I_{m1} n_{r1}^2$ and $I_{m2} n_{r2}^2$)

Model properties

Robots with elastic joints

- for $K \to \infty$ (rigid joints), $\theta \to q$ and $K(q-\theta) \to \infty$ (a finite value) and the equivalent rigid model is recovered (adding up link and motor equations)

$$(M(q)+B)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

- the nonlinear dynamic model is linear in a set of dynamic coefficients $\tilde{a} = (a, a_K, a_B)$ (i.e., including K and B)

$$\begin{aligned} Y(q,\dot{q},\ddot{q})\, a + \mathrm{diag}(q-\theta)\, a_K &= 0 \\ \mathrm{diag}(\ddot{\theta})\, a_B - \mathrm{diag}(q-\theta)\, a_K &= \tau \end{aligned} \quad \Rightarrow \quad \tilde{Y}(q,\dot{q},\ddot{q},\theta,\ddot{\theta})\tilde{a} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$\tilde{Y} = \begin{pmatrix} Y(q,\dot{q},\ddot{q}) & \mathrm{diag}(q-\theta) & 0 \\ 0 & -\mathrm{diag}(q-\theta) & \mathrm{diag}(\ddot{\theta}) \end{pmatrix}$$

- as in the rigid case, there exists a bound on the norm of the gradient of the gravity vector $g(q)$

$$\left\| \frac{\partial g}{\partial q} \right\| \le \alpha \quad \forall q \qquad \Rightarrow \qquad \|g(q_1) - g(q_2)\| \le \alpha \|q_1 - q_2\| \quad \forall q_1, q_2$$

Control problems

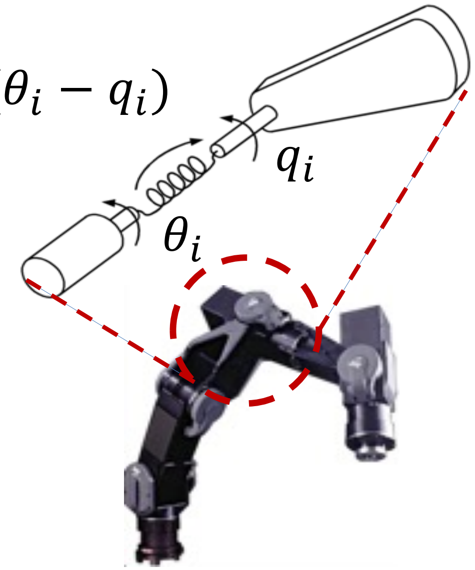
Robots with elastic joints

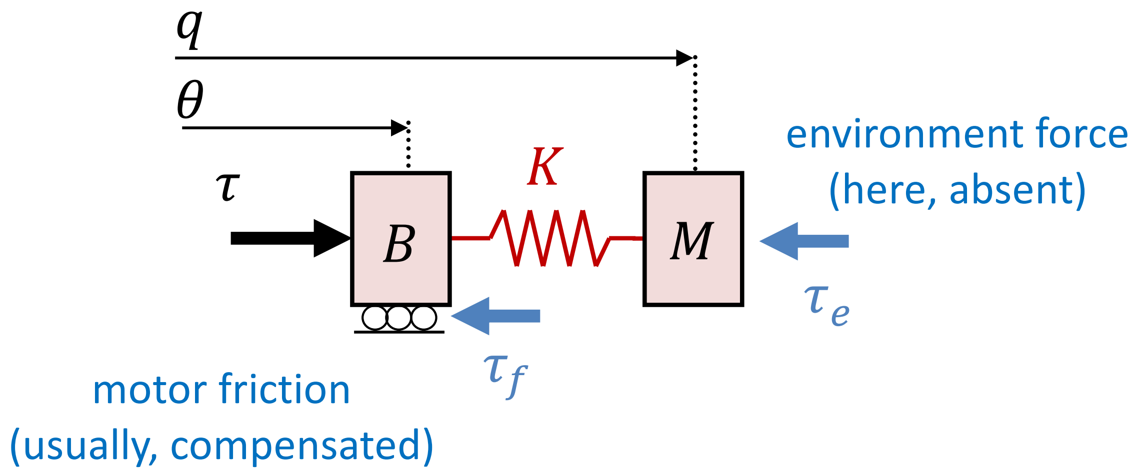
- regulation to an equilibrium configuration $(q, \theta, \dot{q}, \dot{\theta}) = (q_d, \theta_d, 0, 0)$
  - direct kinematics of elastic joint robots is a function of link variables only: $r = \mathrm{kin}(q)$
  - only a desired link position q_d is given, θ_d is to be determined
  - q_d may come from the inverse kinematics of a desired Cartesian pose/position r_d
  - using partial or full state feedback
- asymptotic tracking of a (sufficiently) smooth link trajectory $q_d(t)$
  - the corresponding motor trajectory $\theta_d(t)$ is to be determined
  - mostly using full, but also partial state feedback
- model matching by feedback
  - less conventional problem, based on equivalence under feedback transformations

Single elastic joint

Transfer functions of interest

$$\tau_{Ji} = K_i(\theta_i - q_i)$$





we often look rather at the torque-to-velocity mappings … (eliminating one integrator)

$$P_{\text{motor}}(s) = \frac{\theta(s)}{\tau(s)} = \frac{Ms^2 + K}{MBs^2 + (M+B)K}\,\frac{1}{s^2}$$

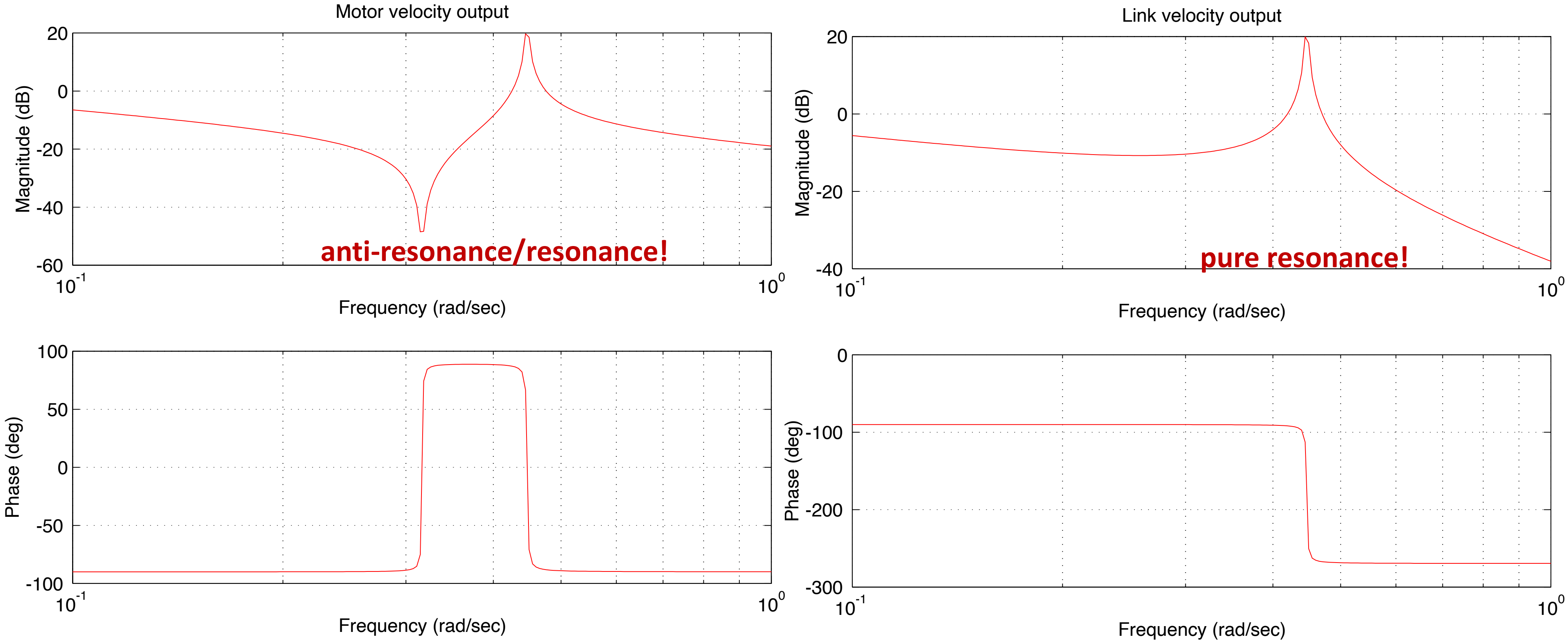
- system with two zeros and relative degree = 2
- passive (zero *precedes* pole on imaginary axis)
- stabilization can be achieved via output θ feedback

$$P_{\text{link}}(s) = \frac{q(s)}{\tau(s)} = \frac{K}{MBs^2 + (M+B)K}\,\frac{1}{s^2}$$

- NO zeros!!
- maximum relative degree = 4

Single elastic joint

Transfer functions of interest (some small damping added on motor/link side)



- single anti-resonance/resonance behavior on motor output
- pure resonance on link output (weak or no zeros)

Regulation of a single elastic joint

Feedback schemes with reduced measurements

- link PD feedback $\tau = u_{qd} - \left(K_{P,q} q + K_{D,q} \dot{q}\right)$

$$W_{qq}(s) = \frac{q(s)}{u_{qd}(s)} = \frac{K}{MBs^4 + (M+B)Ks^2 + KK_{D,q}s + KK_{P,q}}$$

- always unstable for any value of the gains (s^3 term is missing …)
- inclusion of dissipative terms would lead to a very small interval of stability

- motor PD feedback $\tau = u_{\theta d} - \left(K_{P,\theta} \theta + K_{D,\theta} \dot{\theta}\right)$

$$W_{\theta\theta}(s) = \frac{q(s)}{u_{\theta d}(s)} = \frac{K}{MBs^4 + MK_{D,\theta}s^3 + \left((M+B)K + MK_{P,\theta}\right)s^2 + KK_{D,\theta}s + KK_{P,\theta}}$$

- asymptotically stable for any $K_{P,\theta} > 0$, $K_{D,\theta} > 0$ (Routh criterion …)
- as in a rigid joint!

Regulation of a single elastic joint

Feedback schemes with reduced measurements (mixed cases)

- link position and motor velocity feedback $\tau = u_{qd} - \left(K_{P,q} q + K_{D,\theta} \dot{\theta}\right)$

$$W_{q\theta}(s) = \frac{q(s)}{u_{qd}(s)} = \frac{K}{MBs^4 + MK_{D,\theta}s^3 + (M+B)Ks^2 + KK_{D,\theta}s + KK_{P,q}}$$

- asymptotically stable for $0 < K_{P,q} < K,\ K_{D,\theta} > 0$
- limited proportional gain, not overriding the spring stiffness

- motor position and link velocity feedback $\tau = u_{\theta d} - \left(K_{P,\theta} \theta + K_{D,q} \dot{q}\right)$

$$W_{\theta q}(s) = \frac{q(s)}{u_{\theta d}(s)} = \frac{K}{MBs^4 + \left((M+B)K + MK_{P,\theta}\right)s^2 + KK_{D,q}s + KK_{P,\theta}}$$

- always unstable for any value of the gains

- caution must be used in dealing with different partial state measurements
- in the nonlinear/MIMO case (regulation under gravity) we consider only the best of these feedback schemes: motor PD feedback

Regulation with motor PD + feedforward

Partial state feedback solution

very similar to flexible link case!

- consider the control law

$$\tau = K_P(\theta_d - \theta) - K_D\dot{\theta} + g(q_d)$$

with symmetric (diagonal) $K_P > 0$, $K_D > 0$, and with the motor reference position at steady state corresponding to q_d given by

$$\theta_d = q_d + K^{-1} g(q_d)$$

Theorem [Tomei, 1991]

If

$$\left\| \frac{\partial g}{\partial q} \right\| \leq \alpha \quad \text{and} \quad \lambda_{min}(K_E) = \lambda_{min}\begin{pmatrix} K & -K \\ -K & K + K_P \end{pmatrix} > \alpha > 0$$

then the desired closed-loop equilibrium state $(q_d, \theta_d, 0, 0)$ is globally asymptotically stable ◀

Regulation with motor PD + feedforward

Lyapunov-based proof in detail

- all closed-loop equilibria ($\dot{q} = \dot{\theta} = \ddot{q} = \ddot{\theta}$) satisfy

$$K(q-\theta) + g(q) = 0$$
$$K(\theta - q) - K_P(\theta_d - \theta) - g(q_d) = 0$$

- adding/subtracting $K(\theta_d - q_d) - g(q_d)$ ($= 0$, by definition of θ_d) yields

$$K(q - q_d) \; - K(\theta - \theta_d) + g(q) - g(q_d) = 0$$
$$-K(q - q_d) + (K + K_P)(\theta - \theta_d) = 0$$

- or in matrix form

$$\begin{pmatrix} K & -K \\ -K & K + K_P \end{pmatrix} \begin{pmatrix} q - q_d \\ \theta - \theta_d \end{pmatrix} = K_E \begin{pmatrix} q - q_d \\ \theta - \theta_d \end{pmatrix} = \begin{pmatrix} g(q_d) - g(q) \\ 0 \end{pmatrix}$$

Regulation with motor PD + feedforward

Lyapunov-based proof in detail

- using the assumptions of the Theorem, for all $(q,\theta) \neq (q_d,\theta_d)$ we have

$$\left\| K_E \begin{pmatrix} q - q_d \\ \theta - \theta_d \end{pmatrix} \right\| \geq \lambda_{min}(K_E) \left\| \begin{pmatrix} q - q_d \\ \theta - \theta_d \end{pmatrix} \right\|$$
$$> \alpha \left\| \begin{pmatrix} q - q_d \\ \theta - \theta_d \end{pmatrix} \right\| \geq \alpha \|q - q_d\|$$
$$\geq \|g(q_d) - g(q)\| = \left\| \begin{pmatrix} g(q_d) - g(q) \\ 0 \end{pmatrix} \right\|$$

and hence (q_d,θ_d) is the unique equilibrium configuration

- define the position-dependent (potential-like) function

$$P(q,\theta) = \frac{1}{2}(q-\theta)^T K (q-\theta) + \frac{1}{2}(\theta_d - \theta)^T K_P (\theta_d - \theta) + U_g(q) - \theta^T g(q_d)$$

- the gradient $\nabla P(q,\theta) = 0$ only at (q_d,θ_d) (using the same argument above) **+** the Hessian $\nabla^2 P(q,\theta) > 0 \Rightarrow (q_d,\theta_d)$ is an absolute minimum of $P(q,\theta)$

Regulation with motor PD + feedforward

Lyapunov-based proof in detail

- the following is thus a Lyapunov candidate

$$V(q,\theta,\dot{q},\dot{\theta}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\dot{\theta}^T B\dot{\theta} + P(q,\theta) - P(q_d,\theta_d) \geq 0$$

$$\ldots = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\dot{\theta}^T B\dot{\theta} + \frac{1}{2}(q-\theta)^T K(q-\theta) + \frac{1}{2}(\theta_d-\theta)^T K_P(\theta_d-\theta)$$
$$+ U_g(q) - U_g(q_d) - (\theta-\theta_d)^T g(q_d) - \frac{1}{2}(q_d-\theta_d)^T K(q_d-\theta_d)$$

- its time derivative evaluated along the closed-loop system trajectories is

$$\dot{V} = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{\theta}^T B\ddot{\theta} + (\dot{q}-\dot{\theta})^T K(q-\theta) - \dot{\theta}^T K_P(\theta_d-\theta) + \frac{\partial U_g(q)}{\partial q}\dot{q} - \dot{\theta}^T g(q_d)$$
$$= \dot{q}^T \left(-\cancel{C(q,\dot{q})\dot{q}} - \cancel{g(q)} - \cancel{K(q-\theta)} + \cancel{\frac{1}{2}\dot{M}(q)\dot{q}} + \cancel{K(q-\theta)} + \cancel{\left(\frac{\partial U_g(q)}{\partial q}\right)^T} \right)$$
$$+ \dot{\theta}^T \left(\tau - \cancel{K(\theta-q)} - \cancel{K(q-\theta)} - K_P(\theta_d-\theta) - g(q_d) \right)$$
$$= \dot{\theta}^T \left(\cancel{K_P(\theta_d-\theta)} - K_D\dot{\theta} + \cancel{g(q_d)} - \cancel{K_P(\theta_d-\theta)} - \cancel{g(q_d)} \right) = -\dot{\theta}^T K_D \dot{\theta} \leq 0$$

where the skew-symmetry of $\dot{M} - 2C$ has been used

Regulation with motor PD + feedforward

Lyapunov-based proof in detail

- since $\dot{V} = 0 \Leftrightarrow \dot{\theta} = 0$, the proof is completed using LaSalle
- substituting $\ddot{\theta} = 0$ in the closed-loop equations yields

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + Kq = K\theta = \text{constant} \qquad (*)$$

$$Kq = K\theta - K_P(\theta_d - \theta) - g(q_d) = \text{constant} \qquad (**)$$

- from $(**)$ it follows that $\dot{q} = \ddot{q} = 0$, which in turn simplifies $(*)$ to

$$g(q) + Kq - K\theta = 0 \qquad (***)$$

- from the first part of the proof $q = q_d, \theta = \theta_d$ is the unique solution to $(**)$-$(***)$ and thus the configuration (q_d, θ_d) is the only one contained in the largest invariant set of states such that $\dot{V} = 0$

$\Rightarrow$ global asymptotic stability of the desired equilibrium state $(q_d, \theta_d, 0,0)$ ■

Comments

… on this regulation control law in the joint elasticity case

- if joint stiffness K is large enough (always true in non-pathological cases), the assumption of the Theorem $\lambda_{min}(K_E) > \alpha$ can always be satisfied by increasing $\lambda_{min}(K_P)$
- in the presence of model uncertainties, the control law

$$\tau = K_P\left(\hat{\theta}_d - \theta\right) - K_D\dot{\theta} + \hat{g}(q_d) \qquad \hat{\theta}_d = q_d + \hat{K}^{-1}\hat{g}(q_d)$$

provides asymptotic stability for a different equilibrium $(\bar{q}, \bar{\theta})$ (still unique, and possibly close to the desired one, if K_P is sufficiently large)

- a motor PD + on-line gravity compensation scheme

$$\tau = K_P(\theta_d - \theta) - K_D\dot{\theta} + g(\bar{\theta}) \qquad \bar{\theta} = \theta + \hat{K}^{-1}\hat{g}(q_d)$$

biased motor position

can be proven to achieve global asymptotic stability (with expected better transients), by using a modified Lyapunov candidate

$$P(q,\theta) = \frac{1}{2}(q-\theta)^T K(q-\theta) + \frac{1}{2}(\theta_d - \theta)^T K_P(\theta_d - \theta) + U_g(q) - U_g(\bar{\theta})$$

Regulation with motor PD+ …

Comparative numerical results with constant or on-line gravity compensation

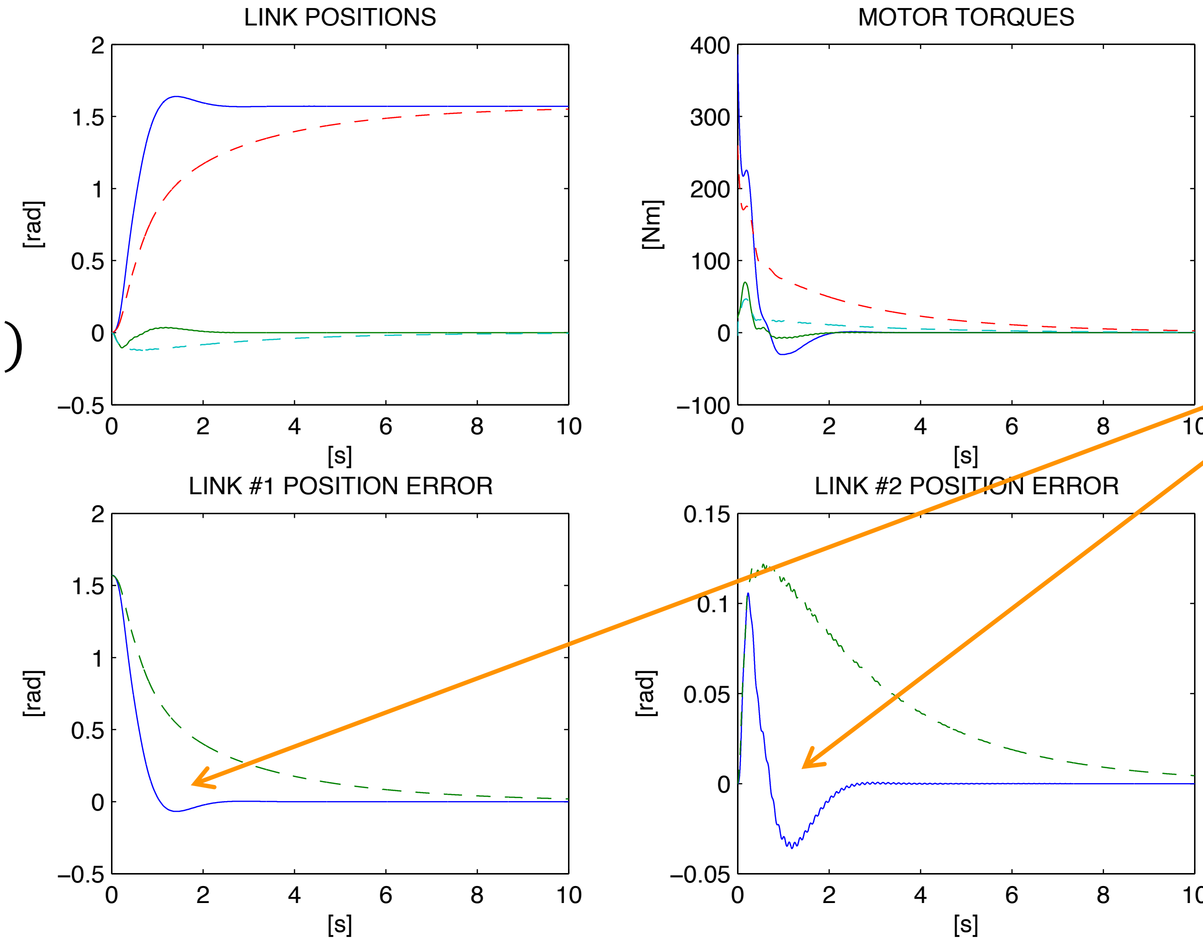
- a planar robot with two elastic joints robot **under** gravity (in the vertical plane), with $K_1 = K_2 = 1000$ [Nm/rad] and $\alpha \cong 133$
- at rest from the horizontal $q(0) = (0°, 0°)$ to the upward vertical $q_d = (90°, 0°)$

$$\theta(0) = \begin{pmatrix} 7.3° \\ 1.4° \end{pmatrix}$$

$$K_P = \mathrm{diag}(180{,}180)$$

K_E satisfies the assumption

$$K_D = \mathrm{diag}(80{,}80)$$



faster transients!

— = online (solid)

- - = constant (dashed)

Regulation with motor PD+ …

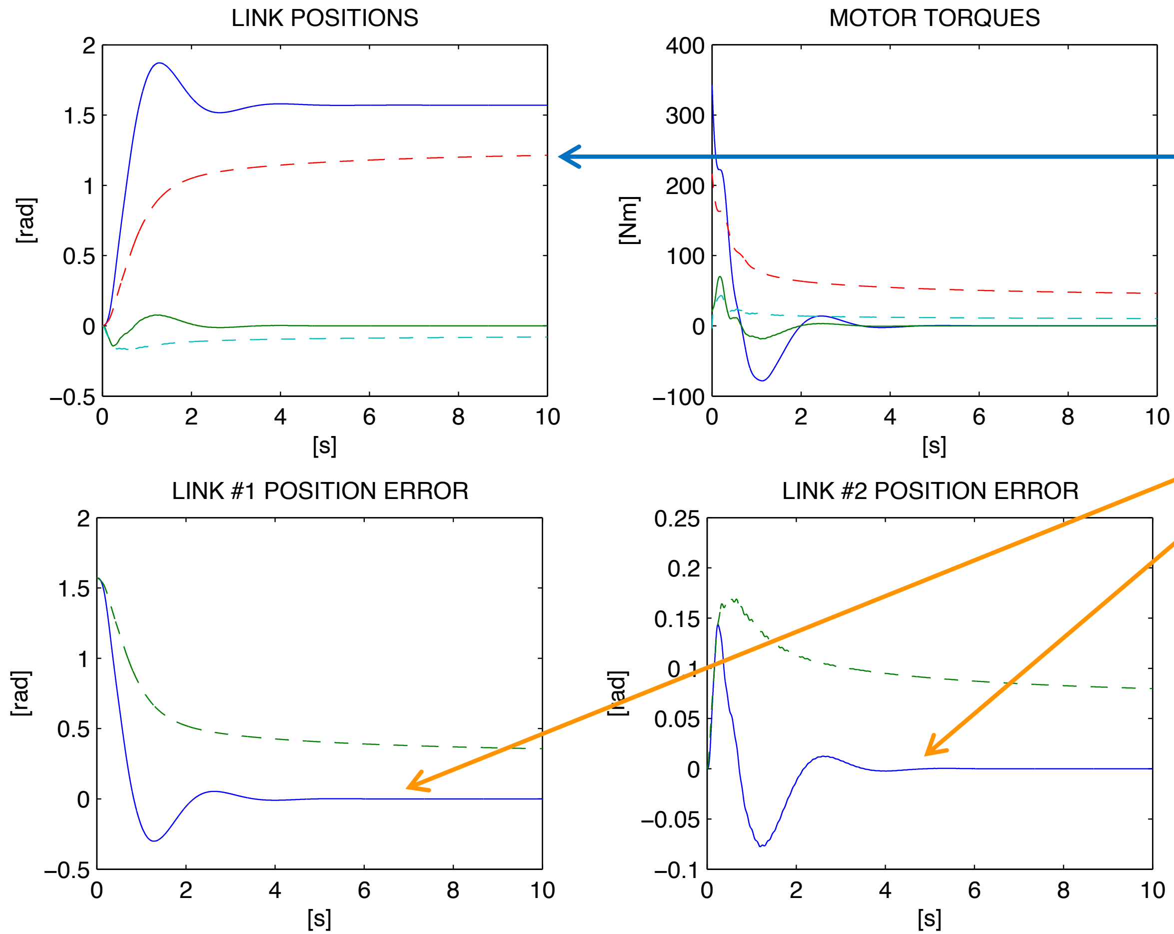
Comparative numerical results with constant or on-line gravity compensation

- a planar robot with two elastic joints robot **under** gravity (in the vertical plane), with $K_1 = K_2 = 1000$ [Nm/rad] and $\alpha \cong 133$
- at rest from the horizontal $q(0) = (0°, 0°)$ to the upward vertical $q_d = (90°, 0°)$

$$K_P = \text{diag}(150,150)$$

K_E violates the assumption (because of K_P)

$$K_D = \text{diag}(50,50)$$



does **not** converge

.. still **works!**

— = online (solid)

- - = constant (dashed)

Summary of control laws for regulation

Using a minimal PD+ action on the motor side

for a desired constant link position q_d

- evaluate the associated desired motor position θ_d at steady state
- collocated (partial state) feedback on motor variables preserves passivity
- a sufficiently stiff K_P gain should be used to dominate gravity
- focus on term for (link side) **gravity compensation** based on motor measurements

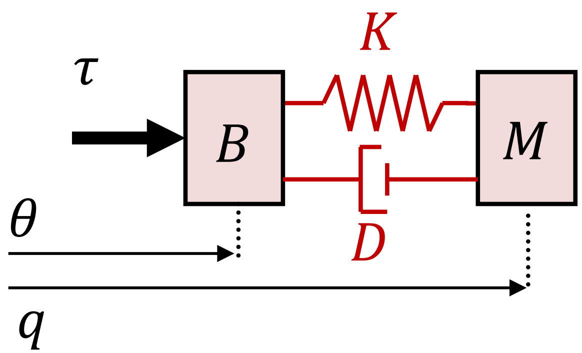
$$\theta_d = q_d + K^{-1} g(q_d) \qquad \tau = \tau_g + K_P(\theta_d - \theta) - K_D \dot{\theta} \qquad K_D > 0$$

τ_g	gain criteria for stability	
$g(q_d)$	$\lambda_{min}\begin{bmatrix} K & -K \\ -K & K+K_P \end{bmatrix} > \alpha$	[Tomei, 1991]
$g(\theta - K^{-1} g(q_d))$	$\lambda_{min}\begin{bmatrix} K & -K \\ -K & K+K_P \end{bmatrix} > \alpha$	[De Luca, Siciliano, Zollo, 2004]
$g(\bar{q}(\theta))$, $\bar{q}(\theta)$: $g(\bar{q}) = K(\theta - \bar{q})$	$K_P > 0$, $\lambda_{min}(K) > \alpha$	[Ott, Albu-Schäffer, *et al* 2004]
$g(q) + BK^{-1}\ddot{g}(q)$	$K_P > 0$, $K > 0$	[De Luca, Flacco, 2010]

↖ **exact gravity cancellation**
(with **full state** feedback)
more on this soon…

Visco-elasticity at the joints

Introduces a structural change …



on Spong model

$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) + D(\dot{q}-\dot{\theta}) \\ K(\theta-q) + D(\dot{\theta}-\dot{q}) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

coupling type	consequence for the model
stiffness	basic static coupling, maximum relative degree (= 4) of output q
damping	reduced relative degree (= 3), only I/O linearization# by static feedback
inertia*	reduced relative degree, I/O linearization needs dynamic feedback

with asymptotically stable zero dynamics

* so-called complete dynamic model

Inverse dynamics

Feedforward action for following a desired trajectory in nominal conditions

given a desired smooth link trajectory $q_d(t) \in C^4$

- compute symbolically the desired motor acceleration and, therefore, also the desired link jerk and snap (i.e., up to the fourth time derivative of the desired motion)

$$\begin{pmatrix} M(q) & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

$$\tau_{m,d} = B\ddot{\theta}_d + K(\theta_d - q_d)$$

$$= BK^{-1}\left(M(q_d)\, q_d^{[4]} + 2\dot{M}(q_d)\, q_d^{[3]} + \ddot{M}(q_d)\ddot{q}_d + \frac{d^2}{dt^2}\left(C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)\right)\right)$$

$$+(M(q_d) + B)\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)$$

- the inverse dynamics can be efficiently computed using a modified Newton-Euler algorithm (with link recursions up to the fourth differential order) running in $O(N)$
- the feedforward command can be used in combination with a PD feedback control on the motor position/velocity error, so to obtain a local but simple trajectory tracking control law

Feedback linearization

For accurate trajectory tracking tasks

- the link position q is a linearizing (a.k.a. **flat**) output

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} \iff q^{(4)} = u$$

- differentiating twice the link equation and using the motor acceleration yields

$$\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dot{M}q^{[3]} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q))\right)$$

- an exactly linear and I/O decoupled closed-loop system is obtained
  - to be stabilized with standard techniques for linear dynamics (pole placement, LQ, …)
- requires higher derivatives of q —— $q, \dot{q}, \ddot{q}, q^{(3)}$
- … but these can be computed from the model using the state measurements
- requires higher derivatives of the dynamic components —— $\ddot{M}, \ddot{C}, \ddot{g}$
- … a $O(N^3)$ Newton-Euler recursive numerical algorithm is available for this

Feedback linearization

Based on the rigid model only vs. when modeling also joint elasticity

$$\tau = (M(q) + B)(\ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P\,(q_d - q)) + C(q,\dot{q})\dot{q} + g(q)$$

$$\tau \;= BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dot{M}q^{[3]} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q))\right)$$

$$u = \left(q_d^{[4]} + K_J(\dddot{q}_d - \dddot{q}) + K_A(\ddot{q}_d - \ddot{q}) + K_D(\dot{q}_d - \dot{q}) + K_P\,(q_d - q)\right)$$

video

video

rigid computed torque

[Spong, 1987]

elastic joint feedback linearization

Feedback linearization

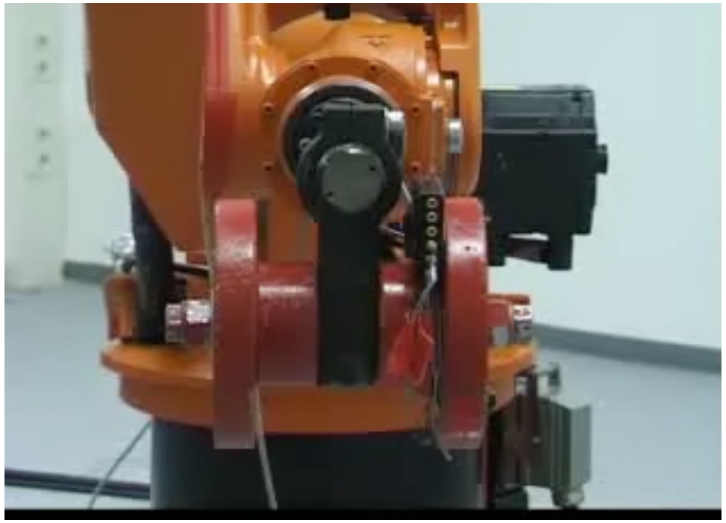
Benefits on an industrial KUKA KR-15/2 robot (235 kg) with joint elasticity

video

conventional industrial robot control

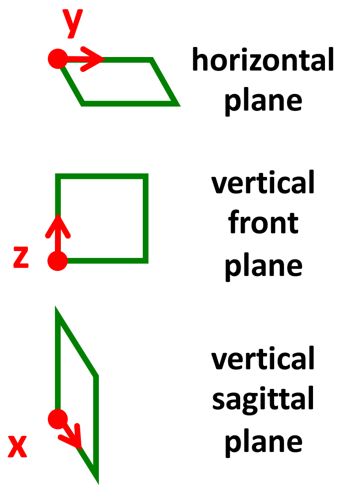
video

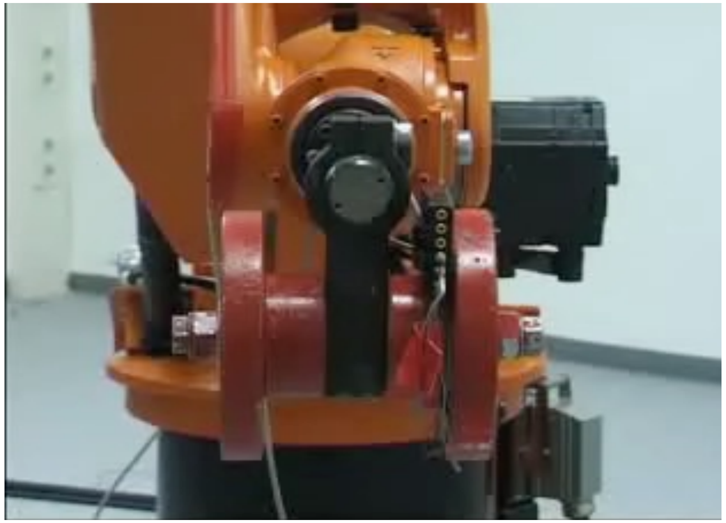
feedback linearization + high-damping



video

three squares in:





trajectory tracking with model-based control

[Thümmel, PhD@TUM 2007]

Torque control

A different set of state measurements can be used directly for tracking control

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$\tau_J = K(\theta - q)$$ measurable by a joint torque sensor

$$\boxed{BK^{-1}\ddot{\tau}_J + \tau_J = \tau - B\ddot{q}}$$ rewriting the motor dynamics

$$\tau = BK^{-1}\ddot{\tau}_{J,d} + \tau_{J,d} + K_T(\tau_{J,d} - \tau_J) + K_S(\dot{\tau}_{J,d} - \dot{\tau}_J)$$

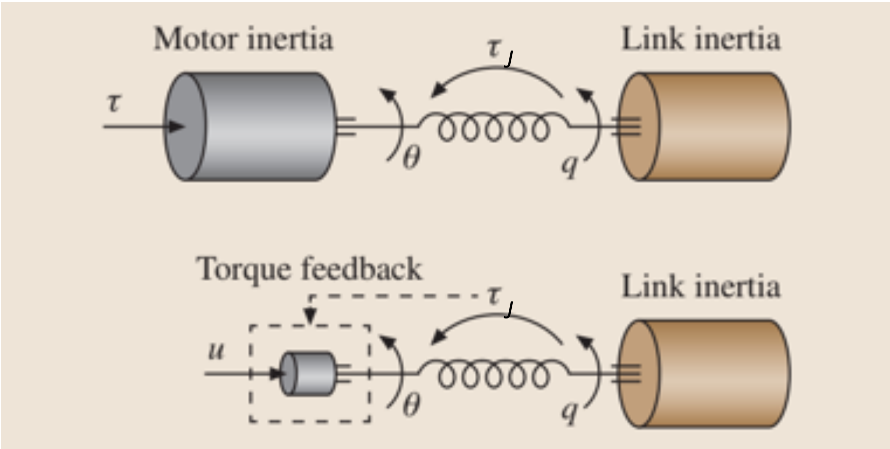
- useful for designing a motor side disturbance observer, e.g., to realize friction compensation
- basis for many cascaded controller designs, starting from a given rigid body control law $\tau = \tau(q, \dot{q}, \mathrm{t})$ taken as $\tau_{J,d}(\mathrm{t})$ in the above formulas
- higher derivatives are still required (either $\ddot{q}$ or $\ddot{\tau}_J$)

Torque feedback

An inner loop that largely reduces motor inertia (and friction)

a pure proportional torque feedback (+ a derivative term for the visco-elastic case)

$$\tau = BB_d^{-1}u + \underbrace{\left(I - BB_d^{-1}\right)}_{-K_T}\tau_J \;\; + \underbrace{\left(I - BB_d^{-1}\right)DK^{-1}}_{-K_S}\dot{\tau}_J$$



physical interpretation:

scaling down motor inertia and friction!

[Ott *et al*, 2008]

original motor dynamics

$$B\ddot{\theta} + K(\theta - q) = \tau$$

visco-elastic case

$$B\ddot{\theta} + \tau_J + DK^{-1}\dot{\tau}_J = \tau$$

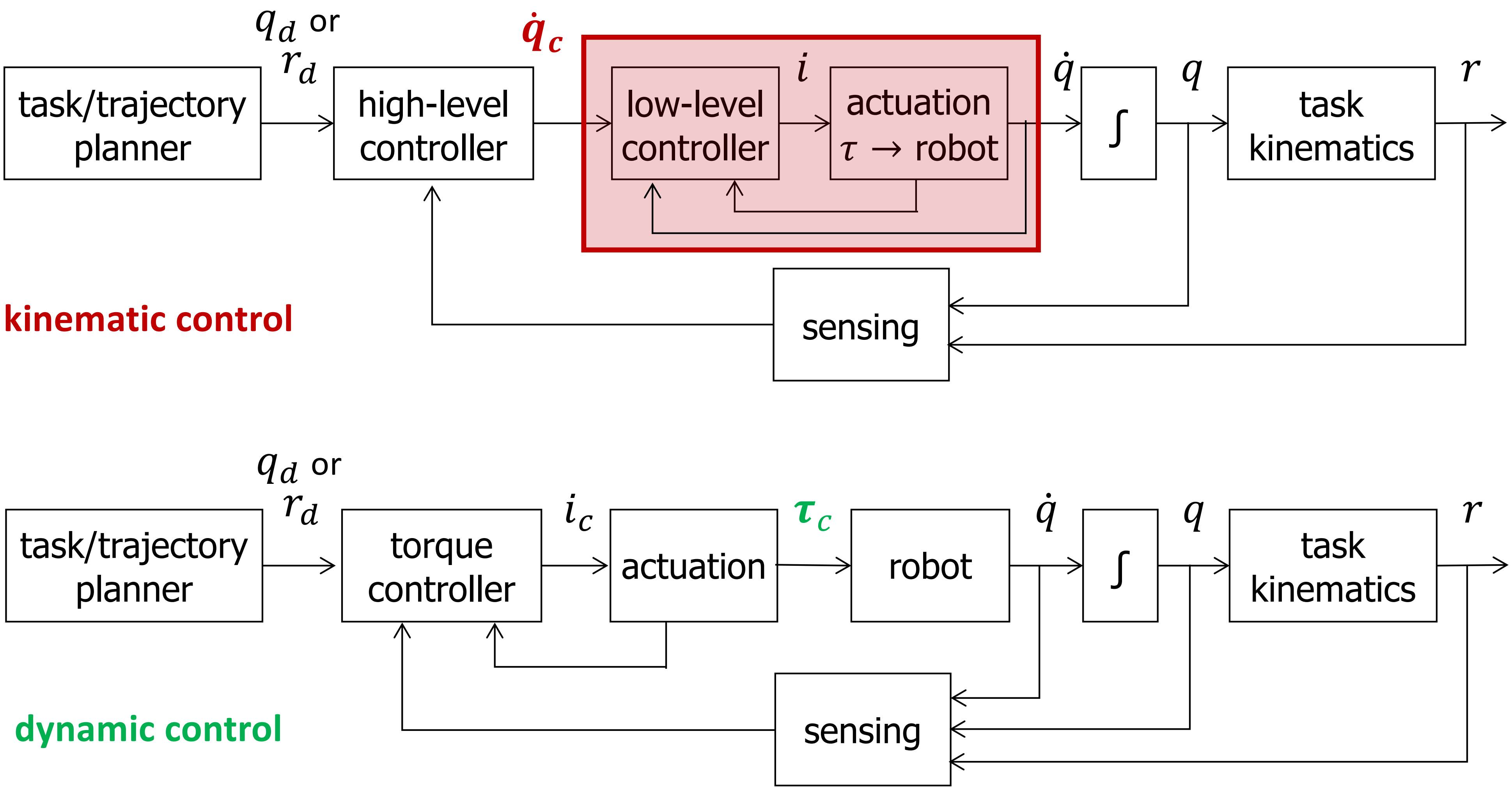
after the torque feedback

$$B_d\ddot{\theta} + K(\theta - q) = u$$

$$B_d\ddot{\theta} + \tau_J + DK^{-1}\dot{\tau}_J = u$$

Position- vs torque-controlled robots

Joint elasticity and joint torque sensing allows better dynamic control



Full-state feedback

Combining torque feedback with a motor PD regulation law

[Albu-Schäffer *et al*, 2007]

inertia scaling via torque feedback $\tau = (I + K_T)u - K_T\,\tau_J - K_S\dot{\tau}_J$

regulation via motor PD, e.g., with $u = g(\bar{q}(\theta)) + K_\theta(\theta_d - \theta) - D_\theta\dot{\theta}$

⇒ joint level control structure of the DLR (and KUKA) lightweight robots

dynamics feedforward and desired torque command

setpoint control

$$\tau = \tau_{J,d} - K_T\left(\tau_J - \tau_{J,d}\right) - K_S\dot{\tau}_J - K_P(\theta_d - \theta) - K_D\dot{\theta} + \tau_f + \tau_{dob}$$ (+ integral actions)

motor inertia scaling

vibration damping

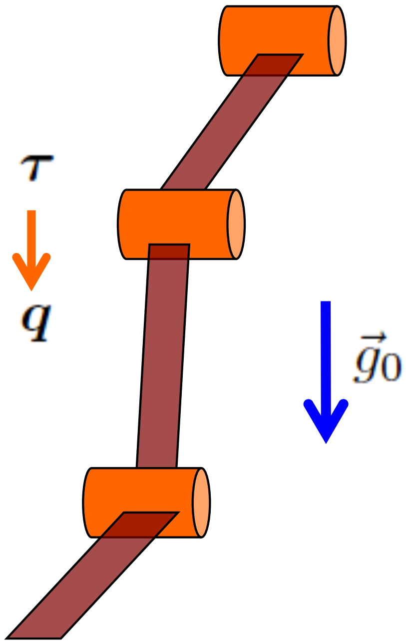
friction compensation and/or disturbance observer

torque control	position control	impedance control
$K_P = 0$	$K_P > 0$	$K_P = K_T K_\theta$
$K_D = 0$	$K_D > 0$	$K_D = K_T D_\theta$
$K_T > 0$	$K_T > 0$	$K_T = (BB_d^{-1} - I)$
$K_S > 0$	$K_S > 0$	$K_S = \left(BB_d^{-1} - I\right)DK^{-1}$
$\tau_{J,d} = \tau_d$	$\tau_{J,d} = g(q)$	$\tau_{J,d} = g(\bar{q}(\theta))$

Exact gravity cancellation

A slightly different view

- for rigid robots this is trivial, due to full actuation and collocation



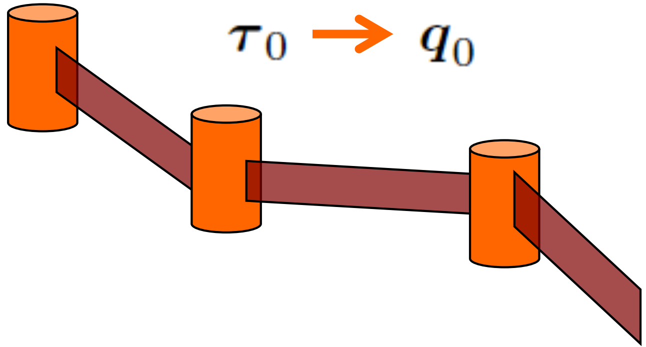
$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}$$

$$\boldsymbol{\tau} = \boldsymbol{\tau}_g + \boldsymbol{\tau}_0$$

$$\Rightarrow$$

$$\boldsymbol{\tau}_g = \boldsymbol{g}(\boldsymbol{q})$$

$$\boldsymbol{q} \equiv \boldsymbol{q}_0$$

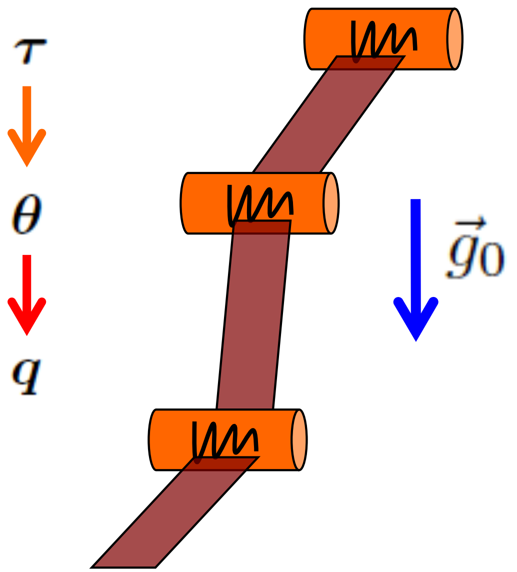


$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{\tau}_0$$

Exact gravity cancellation

… exploiting the concept of feedback equivalence between nonlinear systems

- for elastic joint robots, non-collocation of input torque and gravity term

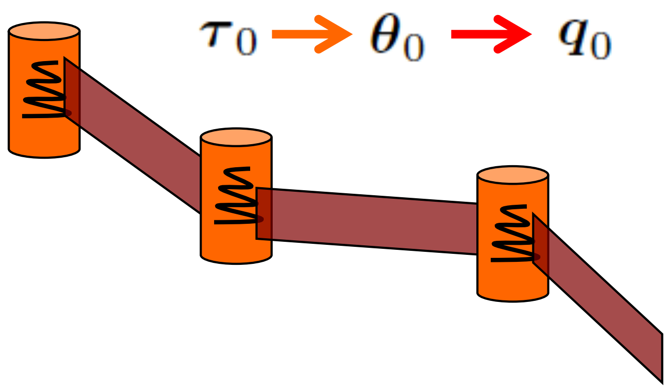


??

$$\tau = \tau_g + \tau_0$$

$$q \equiv q_0$$

??

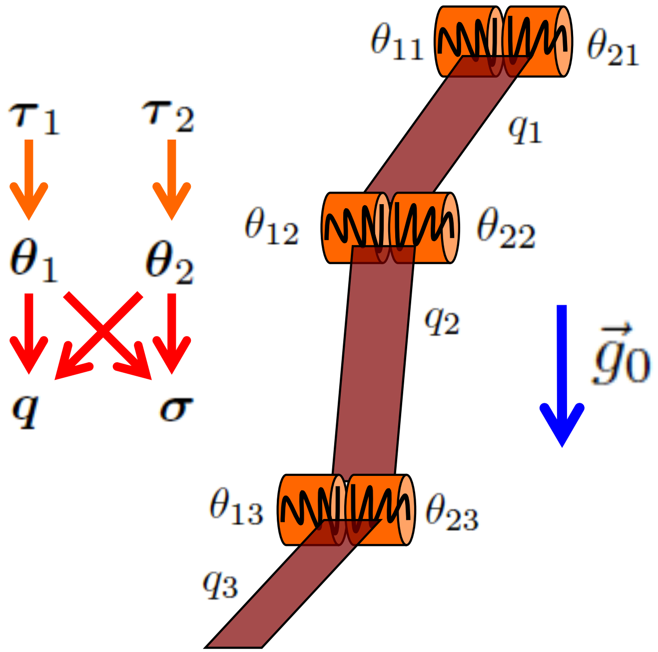


$$
\begin{aligned}
M(q)\ddot{q} + c(q,\dot{q}) + g(q) + K(q-\theta) &= 0 \\
B\ddot{\theta} + K(\theta - q) &= \tau
\end{aligned}
$$

Exact gravity cancellation

… can be generalized also to VSA robots

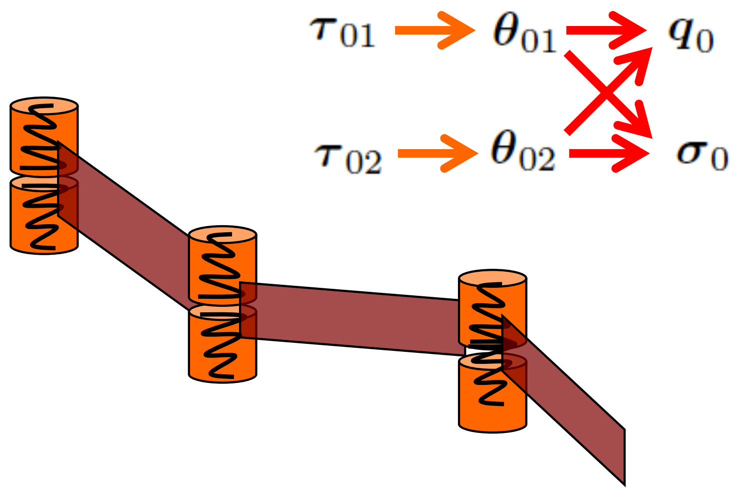
- same problem formulation holds also for VSA robots (here, in antagonistic configuration), with the additional consideration of the internal stiffness state



$\boldsymbol{\tau}_1 \quad \boldsymbol{\tau}_2$??

$\boldsymbol{q} \equiv \boldsymbol{q}_0$??

$\boldsymbol{\sigma} \equiv \boldsymbol{\sigma}_0$??



$$\phi_i = q - \theta_i \qquad i = 1,2$$

$$\begin{aligned} M(q)\ddot{q} + c(q,\dot{q}) + g(q) + \tau_{e1}(\phi_1) + \tau_{e2}(\phi_2) &= 0 \\ B_1\ddot{\theta}_1 - \tau_{e1}(\phi_1) &= \tau_1 \\ B_2\ddot{\theta}_2 - \tau_{e2}(\phi_2) &= \tau_2 \end{aligned}$$

Feedback equivalence

Use the system property of being feedback linearizable (without forcing it!)

$$u = \tilde{\alpha}(x) + \tilde{\beta}(x)u_0$$
$$x_0 = T_0^{-1}(T(x)) = \tilde{T}(x)$$

$$\dot{x} = f(x) + G(x)u$$

gravity-loaded system

$$\dot{x}_0 = f_0(x_0) + G_0(x_0)u_0$$

gravity-free system

feedback transformations
static **state feedback**
+ **change of coordinates**
both invertible

$$u = \alpha(x) + \beta(x)v$$
$$z = T(x)$$

$$u_0 = \alpha_0(x_0) + \beta_0(x_0)v$$
$$z = T_0(x_0)$$

$$\dot{z} = Az + Bv$$

linear, controllable system

z ≈ linearizing outputs

Exact gravity cancellation

Elastic joint robots (including link/motor damping) [De Luca, Flacco, 2010]

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + D_q\dot{q} + K(q-\theta) = 0$$

$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

$$q(t) \equiv q_0(t) \quad \forall t \geq 0 \qquad \tau = \tau_g + \tau_0$$

$$\Rightarrow \quad \boxed{\tau_g = g(q) + D_\theta K^{-1}\dot{g}(q) + BK^{-1}\ddot{g}(q)}$$

$$\dot{g}(q) = \frac{\partial g(q)}{\partial q}\,\dot{q}$$

$$\ddot{g}(q) = \frac{\partial g(q)}{\partial q} M^{-1}(q)\big(K(\theta - q) - c(q,\dot{q}) - g(q) - D_q\dot{q}\big) + \sum_{i=1}^{n} \frac{\partial^2 g(q)}{\partial q\,\partial q_i}\,\dot{q}\,\dot{q}_i$$

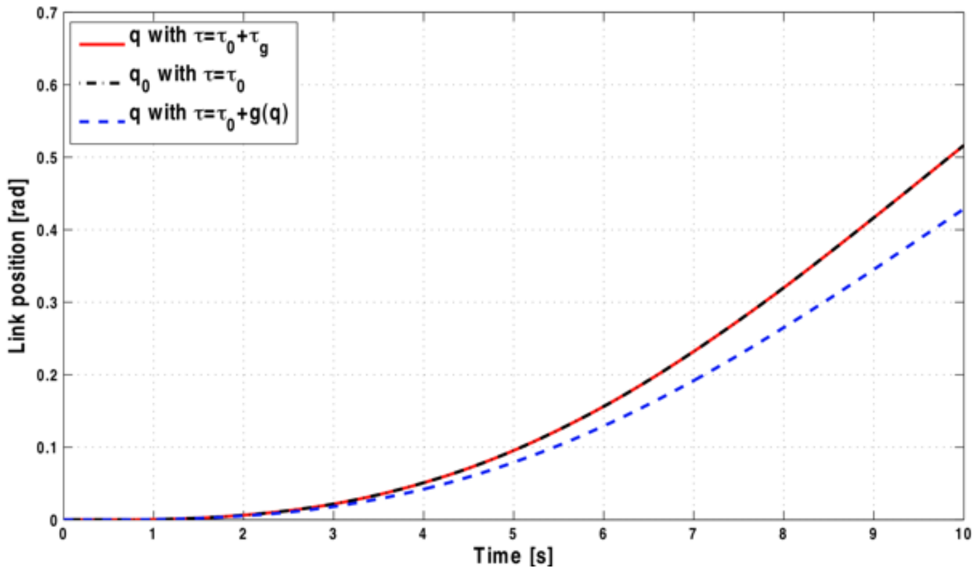
requires (in principle) full state feedback

Numerical results

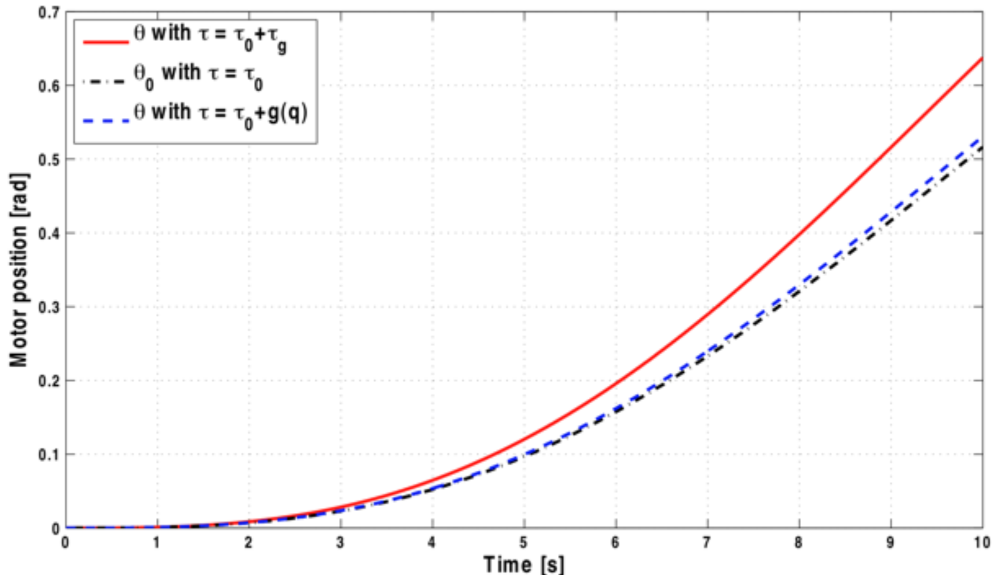
Exact gravity cancellation for a 1-DOF elastic joint

$$\tau_g = m d g_0 \left\{ \left(1 - \frac{B}{K}\,\dot{q}^2\right)\sin q - \frac{B}{M}\frac{m d g_0}{K}\,\sin q \cos q + \frac{M D_\theta - B D_q}{K M}\,\dot{q}\cos q + \frac{B}{M}(\theta - q)\cos q \right\}$$

$$\boxed{\tau_0 = \sin 0.1\pi t} \qquad g(q) = m d g_0 \sin q$$



exact reproduction of same link behavior with and without gravity



different motor behavior with and without gravity

$$\boldsymbol{\theta} = \boldsymbol{\theta}_0 + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q})$$

A global PD-type regulator

Exact gravity cancellation combined with PD law on modified motor variables

$$\boldsymbol{\tau} = \boldsymbol{\tau}_g + \boldsymbol{\tau}_0$$

$$\boldsymbol{\tau}_g = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}_\theta \boldsymbol{K}^{-1}\dot{\boldsymbol{g}}(\boldsymbol{q}) + \boldsymbol{B}\boldsymbol{K}^{-1}\ddot{\boldsymbol{g}}(\boldsymbol{q})$$

$$\boxed{\boldsymbol{\tau}_0 =} \boldsymbol{K}_P(\boldsymbol{\theta}_{d0} - \boldsymbol{\theta}_0) - \boldsymbol{K}_D\dot{\boldsymbol{\theta}}_0$$

$$= \boxed{\boldsymbol{K}_P(\boldsymbol{q}_d - \boldsymbol{\theta} + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q})) - \boldsymbol{K}_D(\dot{\boldsymbol{\theta}} - \boldsymbol{K}^{-1}\dot{\boldsymbol{g}}(\boldsymbol{q}))}$$

Global asymptotic stability can be shown using a Lyapunov analysis under “minimal” sufficient conditions (also without viscous friction)

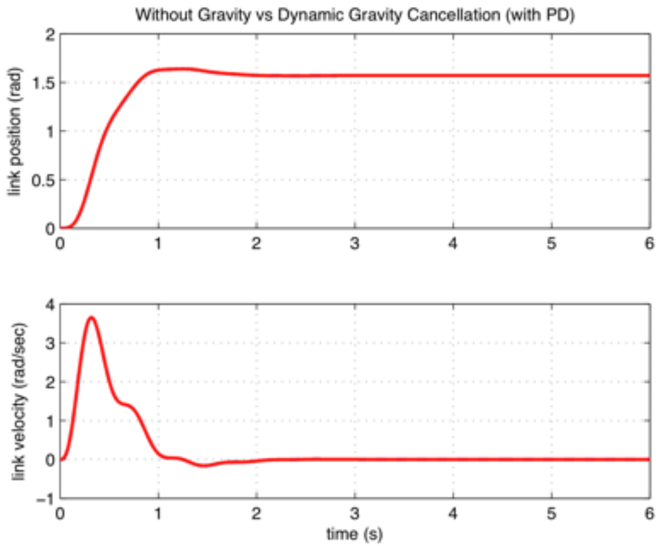
$\boxed{\boldsymbol{K}_P > 0}$ $\boxed{\boldsymbol{K} > 0}$ i.e., **no** strictly positive lower bounds are needed any longer and $\boldsymbol{K}_D > 0$

[De Luca, Flacco, 2011]

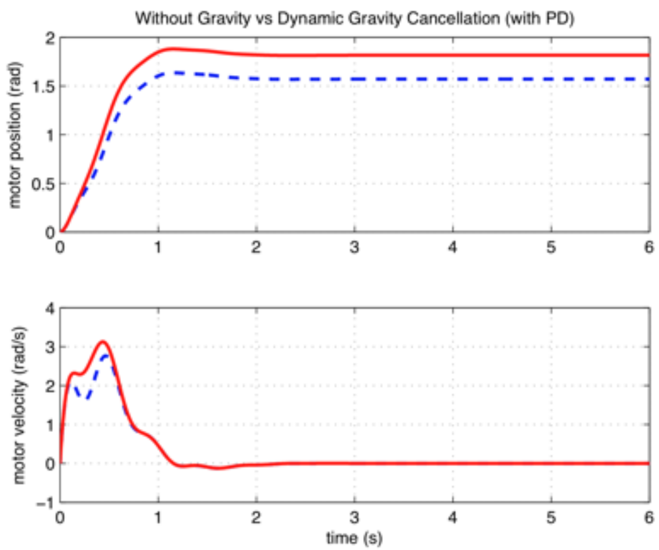
Numerical results

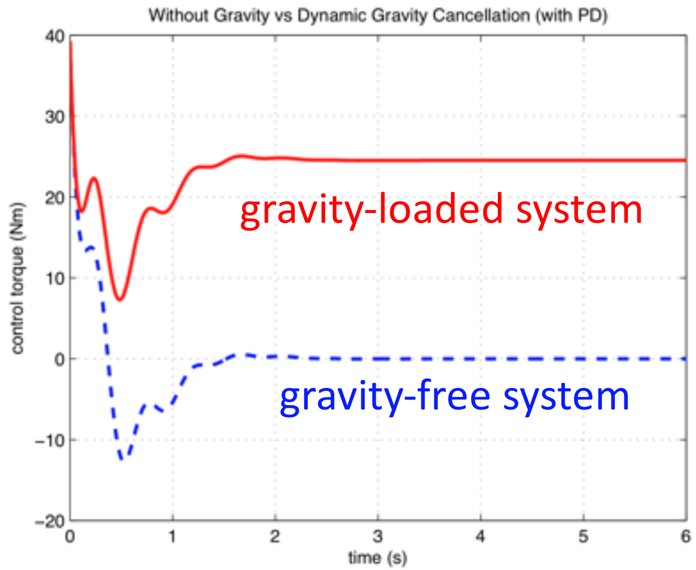
Regulation of a 1-DOF arm with elastic joint under gravity

identical link behavior



different motor behavior



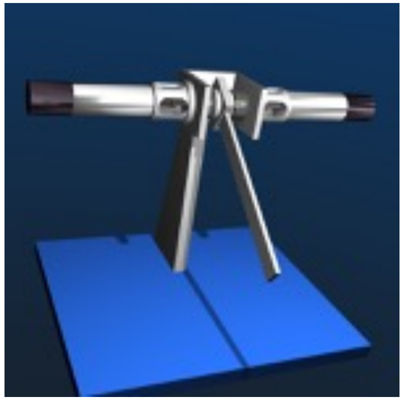


total **control torque**

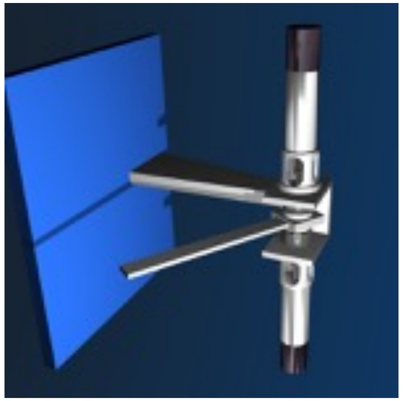
gravity-loaded system under PD
+ gravity cancellation
vs.
gravity-free system under PD
(with same gains)

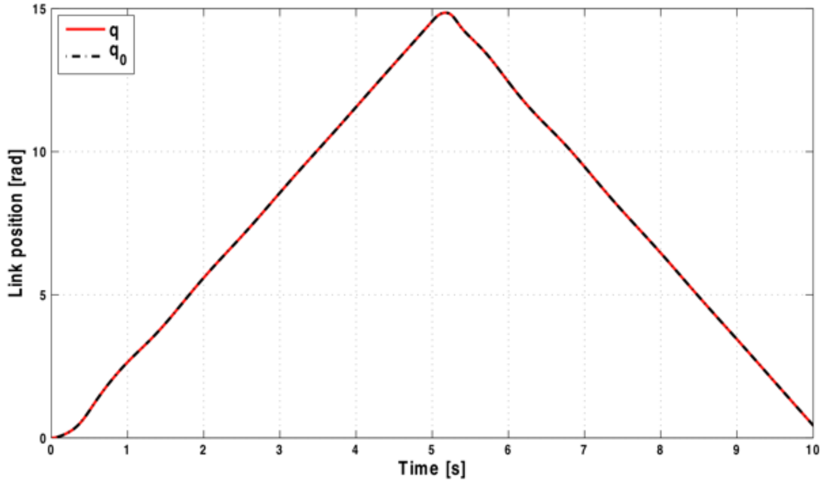
Numerical results

Exact gravity cancellation for VSA-II of the University of Pisa



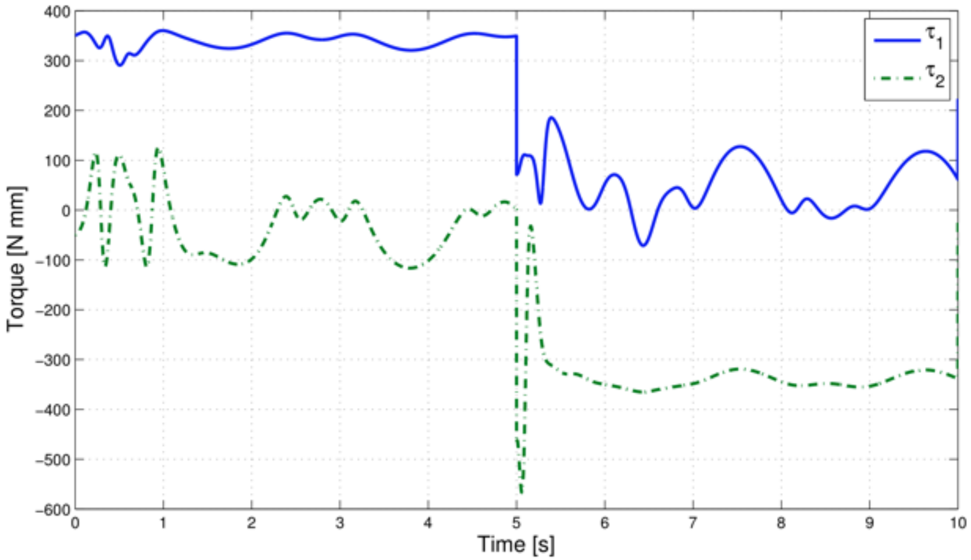
via feedback



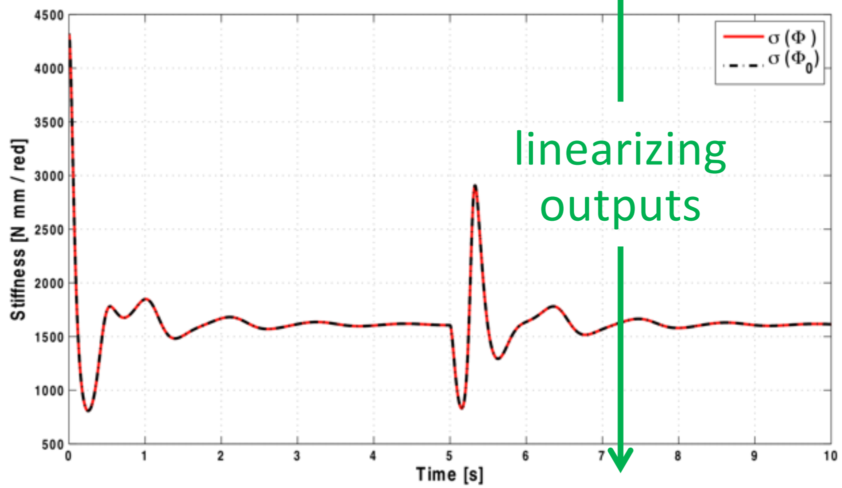


exact reproduction of link behavior

linearizing outputs



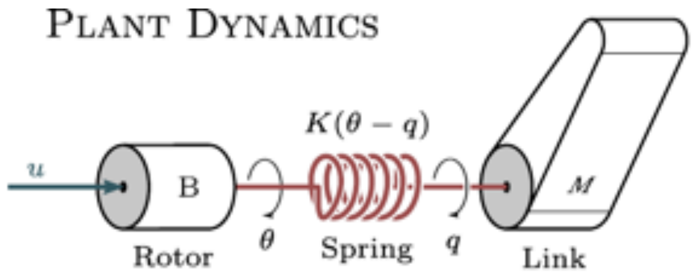
applied torques for gravity cancellation



exact reproduction of stiffness behavior

Damping injection on the link side

Method for the VSA-driven bimanual humanoid torso David



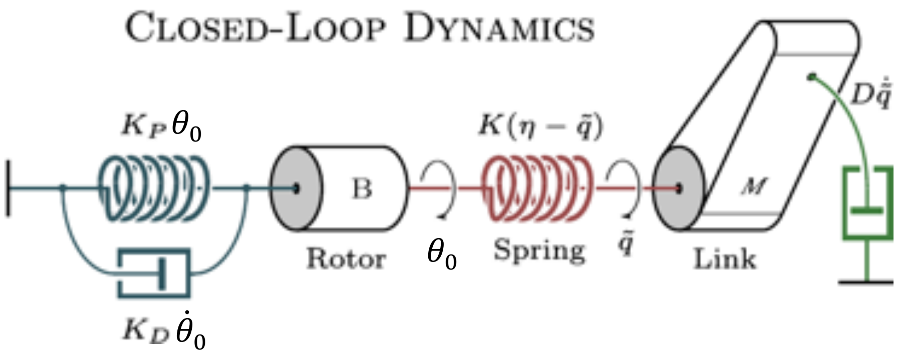
$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta - q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$\boldsymbol{K(q - \theta) = K(q - \theta_0) + D\dot{q}}$$

state transformation

$$\boldsymbol{\tau = \tau_0 - D\dot{q} - BK^{-1}D\ddot{q}}$$

feedback control



$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\boldsymbol{\theta}}_{\mathbf{0}} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\boldsymbol{\theta}_{\mathbf{0}}) \\ K(\boldsymbol{\theta}_{\mathbf{0}} - q) \end{pmatrix} = \begin{pmatrix} -\boldsymbol{D\dot{q}} \\ \tau_0 \end{pmatrix}$$

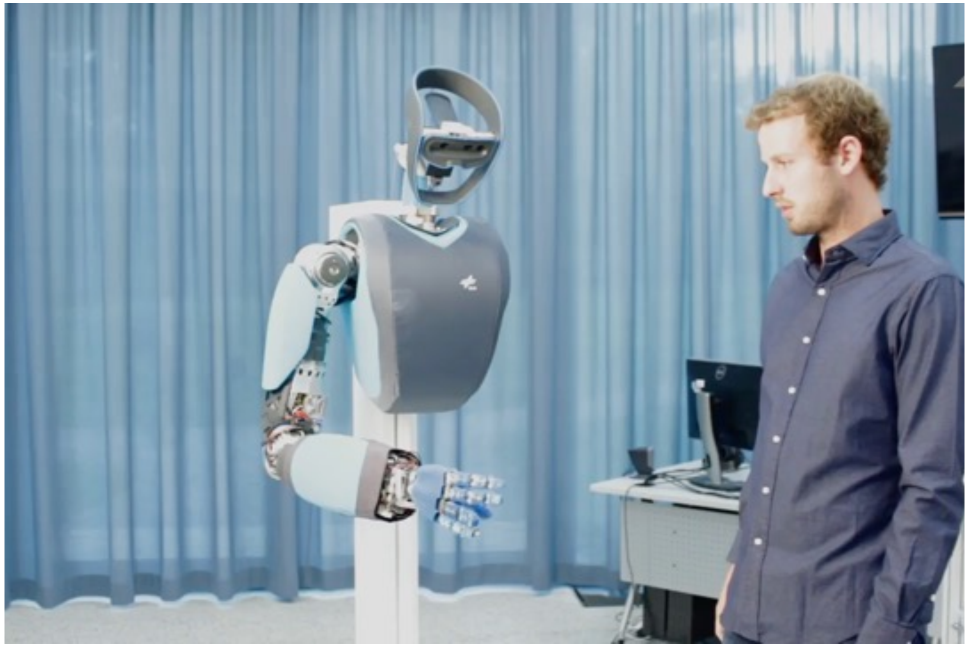
- same principle of **feedback equivalence** (including state transformation)
- **ESP** = Elastic Structure Preserving control by DLR **[Keppler *et al*, 2018]**
- generalizations to **trajectory tracking**, to nonlinear joint flexibility, and to visco-elastic joints

Damping injection on the link side

Method for **VSA-driven** bimanual humanoid torso **David** at DLR

video

video



[Keppler *et al*, 2018]

Conclusions

Control of flexible link/joint robots vs. continuum soft robots in 2023+

- mature field revamped by a new "explosion" of interest
  - simpler control laws for compliant and soft robots are very welcome
  - sensing requirements could be a bottleneck
  - combine (learned) feedforward and feedback to achieve robustness
    - learning on repetitive tasks (ILC) already available for flexible manipulators
  - optimal control (min time, min energy, max force, …) still "open for fun"
- revisiting model-based control design
  - do not fight against the natural dynamics of the system
    - unwise to stiffen what was designed/intended to be soft on purpose!
  - don't give up too much of desirable performance (use feedback equivalence)
  - keep in mind under-actuation and control limitations (e.g., instabilities in the system inversion of tip trajectories for flexible link robots, I/O synergies, …)
- ideas assessed for joint and link elasticity may migrate to many application domains and other classes of soft-bodied robots (and vice versa)
  - locomotion, shared manipulation, physical interaction in complex tasks …

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