A review on the control of flexible joint manipulators

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Summary

- **Motivations and definitions**
  - elastic/flexible joint, serial elastic actuation (SEA), variable stiffness actuation (VSA)
  - concentrated, collocated and distributed flexibility

- **Dynamic modeling of elastic joint manipulators**
  - control properties
  - differences with flexibility in the links

- **Regulation tasks**
  - partial state vs. full state feedback
  - PD+ control laws, with different gravity compensation/cancellation techniques

- **Trajectory tracking tasks**
  - inverse dynamics (feedforward)
  - feedback linearization
  - torque control

- **Latest approach**
  - least modification of elastic dynamics: exact gravity cancellation, link damping, ESP ...
Classes of soft robots

Robots with **elastic joints**

- **lightweight but stiff link** design reduces robot inertia and preserves kinematic accuracy at end-effector level
- **compliant elements** can absorb impact energy
  - soft coverage of links (safe bags)
  - elastic transmissions/joints (HD, cable-driven, ...)
- **elastic joints** decouple instantaneously the *larger* inertia of the driving motors from the *smaller* inertia of the links (where collisions occur!)
  - robots with *relatively soft* joints need more *sensing* and better *control* laws to compensate for static deflections and dynamic vibrations

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**torque-controlled** robots (DLR LWR-III, KUKA LWR 4, KUKA iiwa, ...)
Classes of soft robots

Robots with Variable Stiffness Actuation (VSA)

- uncertain/dynamic interaction with the environment requires to adjust the compliant behavior of the robot and/or to control contact forces
  - passive joint elasticity & active impedance control used in parallel
- nonlinear flexible joints with variable (controlled) stiffness do their best:
  - can be made stiff when moving slow (performance), soft when fast (safety)
  - enlarge the set of achievable task-oriented compliance matrices
  - feature also: robustness, energy optimization, explosive motion tasks, ...

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Classes of soft robots
Robots with flexible links

- **distributed link deformations in robots**
  - need to design *very long* and *slender* arms for the application
  - use of *lightweight* materials to save weight/costs
  - due to large payloads and/or high motion speed (or large contact forces)
- as for joint elasticity, neglecting link flexibility will limit *static* (steady-state error) or *dynamic* (vibrations, poor tracking) performance
- additional control problems due to the *non-collocation* of typical output quantities of interest w.r.t. the input commands
Additional notes

...also terminology varies for the considered robots

- elastic joints vs. SEA (Serial Elastic Actuators)
  - consider/use the same physical phenomenon: compliance in actuation
  - compliance added on purpose in SEA, mostly is a disturbance in elastic joints
  - different range of stiffness: 5-10K Nm/rad down to 0.2-1K Nm/rad in SEA

- joint torque sensors introduce joint elasticity!

- joint deformation is often considered in the linear domain
  - modeled as a concentrated torsional spring with constant stiffness at the joint
  - nonlinear flexible joints are handled too, and share similar control properties
  - viscosity may also be present (visco-elastic joints)
  - nonlinear stiffness characteristics are needed in VSA

- (serial or antagonistic) VSA working at constant stiffness are elastic joints

- often classified as underactuated mechanical systems
  - have less commands than generalized coordinates
  - however, are controllable in the first approximation (the easy case!)
Dynamic modeling

Lagrangian formulation for the complete model

- open chain robot with \( N \) (rotary or prismatic) elastic joints and \( N \) rigid links, driven by electrical actuators
- use \( N \) motor variables \( \theta \) (as reflected through the gear ratios) and \( N \) link variables \( q \)
- standing assumptions
  A1) small displacements at joints
  A2) axis-balanced motors
  A3) each motor is mounted on the robot in a position preceding the driven link

\[
\begin{bmatrix}
M_L(q) & S(q) \\
S^T(q) & B
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
c_1(q, \dot{q}, \dot{\theta}) \\
c_2(q, \dot{q})
\end{bmatrix}
+ \begin{bmatrix}
g(q) \\
0
\end{bmatrix}
+ \begin{bmatrix}
K(q - \theta) \\
K(\theta - q)
\end{bmatrix}
= \begin{bmatrix}
0 \\
\tau_m
\end{bmatrix}
\]

2\(N\times2N\) (full) inertia matrix

A2) \(\Rightarrow\) inertia matrix and gravity vector independent from \(\theta\)

center of mass of rotors on rotation axes
Dynamic modeling

Approximation for the reduced model (Spong 87)

- **simplifying assumption**
  
  A4) the angular kinetic energy of each motor is due only to its own spinning

\[
S(q) = 0
\]

\[
\begin{bmatrix}
    M(q) & 0 \\
    0 & B
\end{bmatrix}
\begin{bmatrix}
    \ddot{q} \\
    \ddot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
    C(q, \dot{q}) \dot{q} \\
    0
\end{bmatrix}
+ \begin{bmatrix}
    g(q) \\
    0
\end{bmatrix}
+ \begin{bmatrix}
    K(q - \theta) \\
    K(\theta - q)
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    \tau_m
\end{bmatrix}
\]

**link equation**

**motor equation**

<table>
<thead>
<tr>
<th>complete model</th>
<th>reduced model</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertial and stiffness couplings</td>
<td>only stiffness couplings</td>
</tr>
<tr>
<td>linearizable by <strong>dynamic</strong> state feedback [De Luca, Lucibello 98]</td>
<td>linearizable by <strong>static</strong> state feedback [Spong 87]</td>
</tr>
<tr>
<td>always valid (under assumptions A1-A3)</td>
<td>A4 valid when gear ratios are very high</td>
</tr>
</tbody>
</table>
Single elastic joint
Transfer functions of interest

$$\tau_i = K_i(\theta_i - q_i)$$

- with **viscous friction** on motor and/or link, complex pole/zero pairs are moved to the Lhs of the $C$-plane

$$P_{motor}(s) = \frac{\theta(s)}{\tau_m(s)} = \frac{M s^2 + K}{M B s^2 + (M + B)K} \frac{1}{s^2}$$

- controllable system with zeros
- passive (zeros always precede poles on the imaginary axis)
- stabilization can be achieved via output $\theta$ feedback

$$P_{link}(s) = \frac{q(s)}{\tau_m(s)} = \frac{K}{M B s^2 + (M + B)K} \frac{1}{s^2}$$

- NO zeros!!
- maximum relative degree

![Diagram of single elastic joint](Image)
**Single elastic joint**

Transfer functions of interest (with some added damping...)

- typical antiresonance/resonance behavior on **motor velocity** output
- pure resonance on **link velocity** output (weak or no zeros)
Visco-elasticity of the joints

Introduces a structural change ...

\[
\begin{bmatrix}
    M(q) & 0 \\
    0 & B
\end{bmatrix}
\begin{bmatrix}
    \ddot{q} \\
    \ddot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
    C(q, \dot{q}) \dot{q} \\
    0
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0
\end{bmatrix}
+ \begin{bmatrix}
    K(q - \theta) + D(\dot{q} - \dot{\theta}) \\
    K(\theta - q) + D(\dot{\theta} - \dot{q})
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    \tau_m
\end{bmatrix}
\]

<table>
<thead>
<tr>
<th>coupling type</th>
<th>consequence for the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>stiffness</td>
<td>basic static coupling, maximum relative degree (= 4) of output q</td>
</tr>
<tr>
<td>damping</td>
<td>reduced relative degree, static I/O linearization</td>
</tr>
<tr>
<td>inertia</td>
<td>reduced relative degree, only dynamic I/O linearization</td>
</tr>
</tbody>
</table>
Regulation task

Using a minimal PD action on motor side

for a desired constant link position $q_d$

- evaluate the associated desired motor position at steady state
- collocated (partial state) feedback preserves passivity, with stiff $K_\theta$ gain dominating gravity
- focus on the term for gravity compensation (acting on link side) from motor measurements

$$\theta_d = q_d + K^{-1}g(q_d) \quad \tau_m = \tau_g + K_\theta(\theta_d - \theta) - D_\theta \dot{\theta}$$

<table>
<thead>
<tr>
<th>$\tau_g$</th>
<th>gain criteria for stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(q_d)$</td>
<td>$\lambda_{\text{min}} \begin{bmatrix} K &amp; -K \ -K &amp; K + K_\theta \end{bmatrix} &gt; \alpha$ [Tomei 91]</td>
</tr>
<tr>
<td>$g(\theta - K^{-1}g(q_d))$</td>
<td>$\lambda_{\text{min}} \begin{bmatrix} K &amp; -K \ -K &amp; K + K_\theta \end{bmatrix} &gt; \alpha$ [De Luca, Siciliano, Zollo 04]</td>
</tr>
<tr>
<td>$g(\vec{q}(\theta))$, $\vec{q}(\theta)$: $g(\vec{q}) = K(\theta - \vec{q})$</td>
<td>$K_\theta &gt; 0$, $\lambda_{\text{min}}(K) &gt; \alpha$ [Ott, Albu-Schäffer 04]</td>
</tr>
<tr>
<td>$g(q) + BK^{-1}\dddot{q}(q)$</td>
<td>$K_\theta &gt; 0$, $K &gt; 0$ [De Luca 10]</td>
</tr>
</tbody>
</table>

gravity cancellation (with full state feedback): more on this later...

$$\alpha = \max(\left\| \frac{\partial g(q)}{\partial q} \right\|)$$
Inverse dynamics

Feedforward action for following a desired trajectory in nominal conditions

given a desired smooth link trajectory \( q_d(t) \in C^4 \)

- compute symbolically the desired motor acceleration and, therefore, also the desired link jerk (i.e., up to the fourth time derivative of the desired motion)

\[
\begin{bmatrix}
M(q) & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
C(q, \dot{q}) \dot{q} \\
0
\end{bmatrix}
+ \begin{bmatrix}
g(q) \\
0
\end{bmatrix}
+ \begin{bmatrix}
K(q - \theta) \\
K(\theta - q)
\end{bmatrix}
= \begin{bmatrix}
0 \\
\tau_m
\end{bmatrix}
\]

\( \tau_{m,d} = B\ddot{\theta}_d + K(\theta_d - qd) \)

\[
= BK^{-1} \left[ M(q_d) q_d^{(4)} + 2\dot{M}(q_d) q_d^{(3)} + \ddot{M}(q_d) \ddot{q}_d + \frac{d^2}{dt^2} (C(q_d, \dot{q}_d) \dot{q}_d + g(q_d)) \right]
+ [M(q_d) + B] \ddot{q}_d + C(q_d, \dot{q}_d) \dot{q}_d + g(q_d)
\]

- the inverse dynamics can be efficiently computed using a modified Newton-Euler algorithm (with link recursions up to the fourth order) running in \( O(N) \)

- the feedforward command can be used in combination with a PD feedback control on the motor position/velocity error, so as to obtain a local but simple trajectory tracking controller
Feedback linearization
For accurate trajectory tracking tasks

- the link position $q$ is a linearizing (flat) output

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} C(q, \dot{q}) \dot{q} \\ g(q) \end{bmatrix} + \begin{bmatrix} K(q - \theta) \\ K(\theta - q) \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_m \end{bmatrix} \quad \leftrightarrow \quad q^{(4)} = u$$

- differentiating twice the link equation and using the motor acceleration yields

$$\tau_m = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1} \left( 2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q)) \right)$$

- an exactly linear and I/O decoupled closed-loop dynamics is obtained
  - to be stabilized with standard linear techniques (pole placement, LQ, ...)
  - requires higher derivatives of $q \quad q, \dot{q}, \ddot{q}, q^{(3)}$
  - however, these can be computed from the model using the state measurements
  - requires higher derivatives of the dynamics components \(\dddot{M}, \dddot{C}, \dddot{\theta} \)
  - A $O(N^3)$ Newton-Euler recursive numerical algorithm is available also for this problem
Torque control

A different set of state measurements can be used directly for tracking control

\[
\begin{bmatrix}
M(q) & 0 \\
0 & B
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
C(q, \dot{q})\dot{q} \\
g(q)\dot{q}
\end{bmatrix}
+ \begin{bmatrix}
K(q - \theta) \\
K(\theta - q)
\end{bmatrix}
= \begin{bmatrix}
0 \\
\tau_m
\end{bmatrix}
\]

\[\tau = K(\theta - q) \text{ measurable by a joint torque sensor}\]

\[
BK^{-1}\ddot{\tau} + \tau = \tau_m - B\ddot{q}
\]

rewriting the motor dynamics

\[\tau_m = BK^{-1}\ddot{\tau}_d + \tau_d + KT(\tau_d - \tau) + KS(\dot{\tau}_d - \dot{\tau}) + \alpha B\ddot{q}\]

- \(\alpha < 1\) for avoiding over-compensation
- useful for designing a motor side disturbance observer, e.g., to realize friction compensation
- basis for many cascaded controller designs that start from a rigid body control law \(\tau_d(q, \dot{q})\)
- higher derivatives are still required \((\dddot{\tau}_d, \ddot{q})\)
Torque feedback
An inner loop that largely reduces motor inertia and friction

consider a pure proportional torque feedback (+ a derivative term for the visco-elastic case)

\[
\tau_m = BB_d^{-1}u + (I - BB_d^{-1})\tau + (I - BB_d^{-1})DK^{-1}\dot{\tau}
\]

\[
\frac{\ddot{\theta}}{K_T} + \frac{\tau}{K_S} = \tau_m
\]

physical interpretation:
scaling of the motor inertia and motor friction!
[Ott, Albu-Schäffer 08]

original motor dynamics  
visco-elastic case

\[
B\ddot{\theta} + K(\theta - q) = \tau_m
\]

\[
B\ddot{\theta} + \tau + DK^{-1}\dot{\theta} = \tau_m
\]

After the torque feedback

\[
B_d\ddot{\theta} + K(\theta - q) = u
\]

\[
B_d\ddot{\theta} + \tau + DK^{-1}\dot{\theta} = u
\]
# Full-state feedback

Combining torque feedback with a motor PD regulation law

- Inertia scaling via torque feedback: \[ \tau_m = (I + K_T)u - K_T \tau - K_S \dot{\tau} \]
- Regulation via motor PD, e.g. with: \[ u = g(\bar{q}(\theta)) + K_\theta (\theta_d - \theta) - D_\theta \dot{\theta} \]

⇒ **Joint level control structure** of the DLR (and KUKA) lightweight robots

- Dynamics feedforward and desired torque command
- Setpoint control
- Motor inertia scaling
- Vibration damping
- Friction compensation and/or disturbance observer

### Torque control

- \( K_P = 0 \)
- \( K_D = 0 \)
- \( K_T > 0 \)
- \( K_S > 0 \)
- \( \tau_d \)

### Position control

- \( K_P > 0 \)
- \( K_D > 0 \)
- \( K_T > 0 \)
- \( K_S > 0 \)
- \( \tau_d = g(q) \)

### Impedance control

- \( K_P = K_T K_\theta \)
- \( K_D = K_T D_\theta \)
- \( K_T = (BB_d^{-1} - I) \)
- \( K_S = (BB_d^{-1} - I)DK^{-1} \)
- \( \tau_d = g(\bar{q}(\theta)) \)
Exact gravity cancellation
A slightly different view

- for rigid robots this is trivial, due to collocation

\[ M(q)\ddot{q} + c(q, \dot{q}) + g(q) = \tau \]

\[ \tau = \tau_g + \tau_0 \]

\[ \tau_g = g(q) \]

\[ q \equiv q_0 \]

\[ M(q)\ddot{q} + c(q, \dot{q}) = \tau_0 \]
Exact gravity cancellation
... based on the concept of **feedback equivalence** between nonlinear systems

- for elastic joint robots, **non-collocation** of input torque and gravity term

\[ M(q)\ddot{q} + c(q, \dot{q}) + g(q) + K(q - \theta) = 0 \]
\[ B\ddot{\theta} + K(\theta - q) = \tau \]
Exact gravity cancellation
... generalized also to VSA robots

- same problem formulation holds also for VSA robots (here, in antagonistic configuration), with the additional consideration of the internal stiffness state.

\[ \phi_i = q - \theta_i \quad i = 1, 2 \]

\[ M(q)\ddot{q} + c(q, \dot{q}) + g(q) + \tau_{e1}(\phi_1) + \tau_{e2}(\phi_2) = 0 \]

\[ B_1\dddot{\theta}_1 - \tau_{e1}(\phi_1) = \tau_1 \]

\[ B_2\dddot{\theta}_2 - \tau_{e2}(\phi_2) = \tau_2 \]
Feedback equivalence

Exploit the system property of being feedback linearizable (without forcing it!)

\[ u = \tilde{\alpha}(x) + \tilde{\beta}(x)u_0 \]
\[ x_0 = T_0^{-1}(T(x)) = \tilde{T}(x) \]

\[ \dot{x} = f(x) + G(x)u \]

gravity-loaded system

feedback transformations
static state feedback
+ change of coordinates
both invertible

linear, controllable system

\[ \dot{x}_0 = f_0(x_0) + G_0(x_0)u_0 \]

gravity-free system

\[ u_0 = \alpha_0(x_0) + \beta_0(x_0)v \]
\[ z = T_0(x_0) \]

\[ z = Az + Bv \]

\[ z \approx \text{linearizing outputs} \]
Flexible joint robots are feedback linearizable...

... with linearizing outputs of suitable relative degrees

- robots with elastic joints
  - also with joints having nonlinear flexibility
- robots with VSA-based actuation
  - antagonistic VSA-II
  - serial DLR-VS joint
  - ...

linearizing output = \text{link position (4)}

linearizing output = \text{link position (4)} + \text{joint stiffness (2)}
Exact gravity cancellation

Elastic joint robots (including link/motor damping)

\[ M(q)\ddot{q} + c(q, \dot{q}) + g(q) + D_q \dot{q} + K(q - \theta) = 0 \]
\[ B\ddot{\theta} + D_\theta \dot{\theta} + K(\theta - q) = \tau \]

\[ q(t) \equiv q_0(t) \quad \forall t \geq 0 \quad \tau = \tau_g + \tau_0 \]

\[ \tau_g = g(q) + D_\theta K^{-1} \dot{g}(q) + B K^{-1} \ddot{g}(q) \]

\[ \dot{g}(q) = \frac{\partial g(q)}{\partial q} \dot{q} \]
\[ \ddot{g}(q) = \frac{\partial g(q)}{\partial q} M^{-1}(q)(K(\theta - q) - c(q, \dot{q}) - g(q) - D_q \dot{q}) + \sum_{i=1}^{n} \frac{\partial^2 g(q)}{\partial q \partial q_i} \dot{q} \dot{q}_i \]

requires full state feedback
Numerical results

Exact gravity cancellation for a 1-DOF elastic joint

\[ \tau_g = m d g_0 \left\{ (1 - \frac{B}{K} \dot{q}^2) \sin q - \frac{B}{M} \frac{m d g_0}{K} \sin q \cos q + \frac{M D_\theta - B D_q}{K M} \dot{q} \cos q + \frac{B}{M} (\theta - q) \cos q \right\} \]

\[ \tau_0 = \sin \frac{0.1 \pi}{t} \]

\[ g(q) = m d g_0 \sin q. \]

exact reproduction of same link behavior with and without gravity

different motor behavior with and without gravity

\[ \theta = \theta_0 + K^{-1} g(q) \]
A global PD-type regulator

Exact gravity cancellation combined with PD law on modified motor variables

\[
\tau = \tau_g + \tau_0 \\
\tau_g = g(q) + D_\theta K^{-1} \dot{g}(q) + B K^{-1} \ddot{g}(q) \\
\tau_0 = K_P (\theta_d - \theta_0) - K_D \dot{\theta}_0 \\
= K_P (q_d - \theta + K^{-1} g(q)) - K_D (\dot{\theta} - K^{-1} \dot{g}(q))
\]

Global asymptotic stability can be shown using a Lyapunov analysis under “minimal” sufficient conditions (also without viscous friction)

\[
K_P > 0 \quad K > 0 \quad \text{i.e., no strict positive lower bounds} \quad \text{and} \quad K_D > 0
\]
Numerical results

Regulation of a *1-DOF* arm with elastic joint under gravity

- **Identical link behavior**
  - Without Gravity vs Dynamic Gravity Cancellation (with PD)
  - Link position (rad)
  - Link velocity (rad/s)

- **Different motor behavior**
  - Motor position (deg)
  - Motor velocity (rad/s)

- **Total control torque**
  - Gravity-loaded system
  - Gravity-free system

Gravity-loaded system under PD + gravity cancellation
vs.
Gravity-free system under PD (with same gains)
Numerical results

Exact gravity cancellation for the VSA-II of UniPisa

via feedback

applied torques for gravity cancellation

exact reproduction of link behavior

exact reproduction of stiffness behavior
Link vibration damping

DLR method for VSA-driven bimanual humanoid torso David [Keppler et al. 16]

\[
\begin{bmatrix}
M(q) & 0 \\
0 & B
\end{bmatrix}\begin{bmatrix}
\ddot{q} \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
C(q, \dot{q})\dot{q} \\
0
\end{bmatrix} + \begin{bmatrix}
g(q) \\
0
\end{bmatrix} + \begin{bmatrix}
K(q - \theta) \\
K(\theta - q)
\end{bmatrix} = \begin{bmatrix}
0 \\
\tau_m
\end{bmatrix}
\]

\[
\frac{d^2}{dt^2}\tau_m = u - D\dot{q} - BK^{-1}\frac{d^2}{dt^2}D\dot{q}
\]

\[
\begin{bmatrix}
M(q) & 0 \\
0 & B
\end{bmatrix}\begin{bmatrix}
\ddot{q} \\
\dot{\eta}
\end{bmatrix} + \begin{bmatrix}
C(q, \dot{q})\dot{q} \\
0
\end{bmatrix} + \begin{bmatrix}
g(q) \\
0
\end{bmatrix} + \begin{bmatrix}
K(q - \eta) \\
K(\eta - q)
\end{bmatrix} = \begin{bmatrix}
-D\dot{q} \\
u
\end{bmatrix}
\]

- same principle of feedback equivalence (including state transformation)
- ESP = Elastic Structure Preserving control
- generalizations to trajectory tracking, to nonlinear joint flexibility, and to viscoelastic joints
Short outlook

- Mature control field recently revamped by the new “explosion” of interest for compliant and soft robots
  - simpler control laws are always welcome
  - sensing requirements could be a bottleneck
  - iterative learning on repetitive tasks already in place for flexible manipulators

- Control ideas assessed for concentrated elasticity at the joints can migrate to other classes of soft-bodied manipulators
  - but intrinsic constraints and control limitations should be kept in mind (e.g., instabilities in the system inversion of tip trajectories for flexible link robots)

- Emerging notion: not fighting against the natural dynamics!
  - and trying also not to give up too much of the desirable performance …