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Control of soft joint robots for safe physical HRI

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Summary

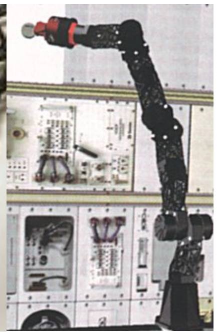
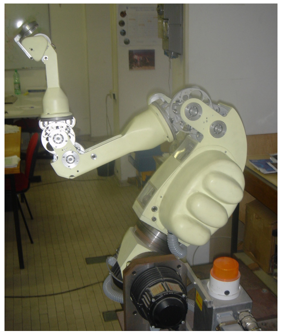
- A world of soft robots
  - manipulators with flexible joints, serial elastic actuation (SEA), variable stiffness actuation (VSA), distributed link flexibility, bio-inspired continuum robots …
  - lightweight robots with flexible joints in safe physical Human-Robot Interaction (pHRI)
- Dynamic modeling of flexible joint manipulators
  - … with few comments on their structural properties
- Classical control tasks and their solution
  - inverse dynamics and feedback linearization design for trajectory tracking
  - regulation with partial/full state feedback and gravity compensation
- Model-based design based on feedback equivalence
  - exact gravity cancellation
  - damping injection on the link side of the flexible transmission
  - environment interaction via generalized impedance control
  - regulation and trajectory tracking in curvature space
- Outlook

Classes of soft robots

Robots with elastic joints

- design of lightweight robots with stiff links for end-effector accuracy
- compliant elements absorb impact energy
  - soft coverage of links (safe bags)
  - elastic transmissions (HD, cable-driven, ...)


- elastic joints decouple instantaneously the *larger* inertia of the driving motors from *smaller* inertia of the links (involved in contacts/collisions!)
- *relatively* soft joints need more sensing (e.g., joint torque) and better control to compensate for static deflections and dynamic vibrations

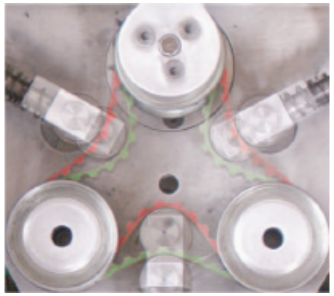


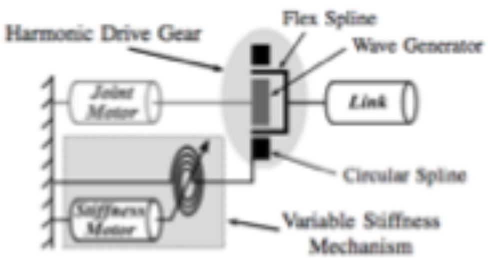
➡ torque-controlled robots (DLR LWR-III, KUKA LWR-IV & iiwa, Franka, ...)

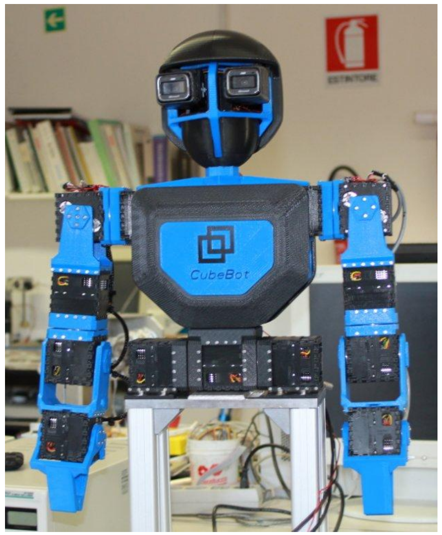
Classes of soft robots

Robots with Variable Stiffness Actuation (VSA)

- uncertain/dynamic interaction with the environment requires to adjust the compliant behavior of the robot and/or to control contact forces
  - passive joint elasticity & active impedance control used in parallel
- nonlinear flexible joints with variable (controlled) stiffness work at best:
  - can be made *stiff when moving slow* (performance), *soft when fast* (safety)
  - enlarge the set of achievable task-oriented compliance matrices
  - feature also: **robustness**, **optimal energy** use, **explosive motion** tasks, ...





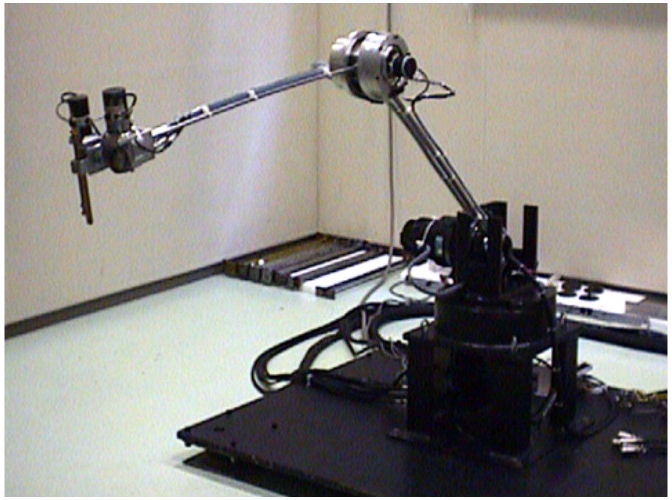
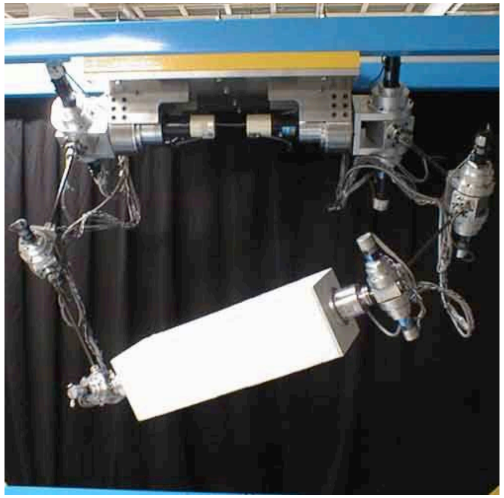


Classes of soft robots

Robots with flexible links

- distributed link deformations
  - design of very long and slender arms (e.g., Euler beam) needed in applications
  - use of lightweight materials to save weight/costs
  - due to large payloads (viz. large contact forces) and/or high motion speed
- as for joint elasticity, neglecting link flexibility will limit static (steady-state error) or dynamic (vibrations, poor tracking) performance
- extra control issue due to non-minimum phase nature of the outputs of interest w.r.t. the command inputs … “move in the opposite direction!”

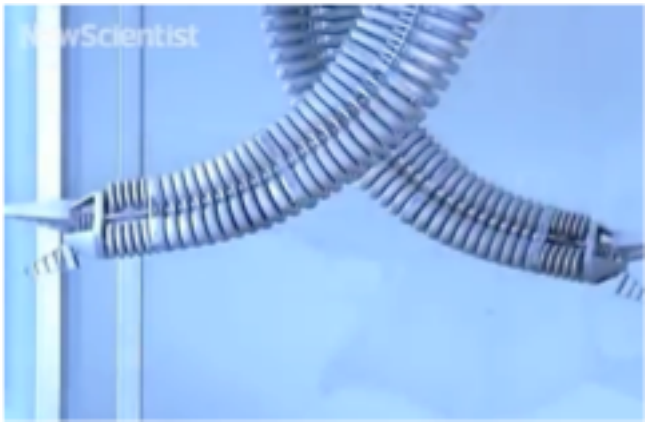


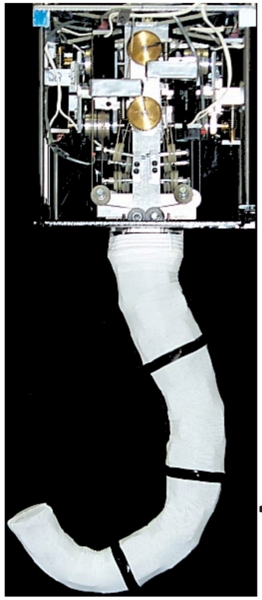



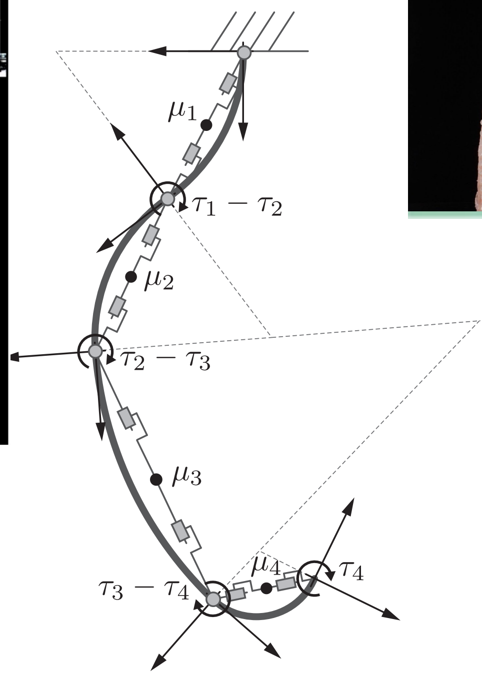
Classes of soft robots

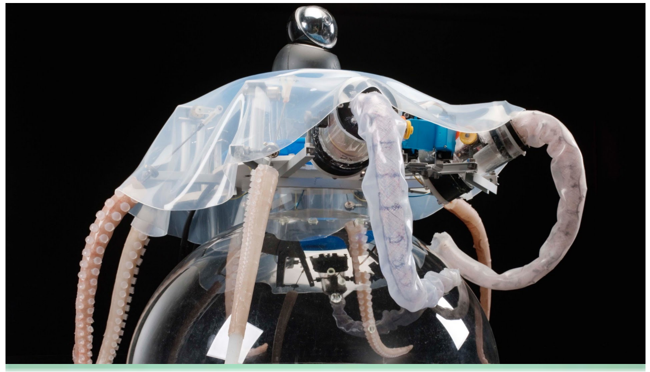
Bio-inspired **continuum** robots

- hyper-redundant degrees of freedom, with distributed deformations
  - approximate finite dimensional models, under some geometric assumptions
  - e.g., a fixed number of segments with (variable) constant curvature
- typically, with multiple distributed/embedded (small) actuation devices









A matter of terminology ...

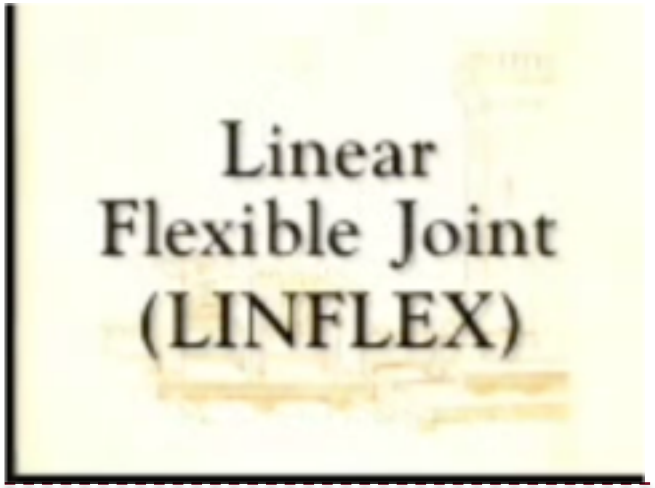
Different sources of softness/flexibility, though similar robotic systems

- elastic joints vs. SEA (Serial Elastic Actuators)
  - based on the same physical phenomenon: compliance in actuation
  - compliance added on purpose in SEA, mostly a disturbance in elastic joints
  - different range of stiffness: 5-10K Nm/rad down to 0.2-1K Nm/rad in SEA
- joint deformation is often considered in the linear domain
  - modeled as a concentrated torsional spring with constant stiffness at the joint
  - nonlinear flexible joints share similar control properties
  - nonlinear stiffness characteristics are needed instead in VSA
  - a (serial or antagonistic) VSA working at constant stiffness is an elastic joint
- flexible joint robots are classified as underactuated mechanical systems
  - have less commands than generalized coordinates
  - non-collocation of command inputs and of dynamic behaviors to be controlled
  - however, they are controllable in the first approximation (the *easier* case!)
  - also continuum soft robots are most of the times underactuated

Control drawbacks due to joint elasticity

Neglecting softness may generate vibrations and trajectory oscillations

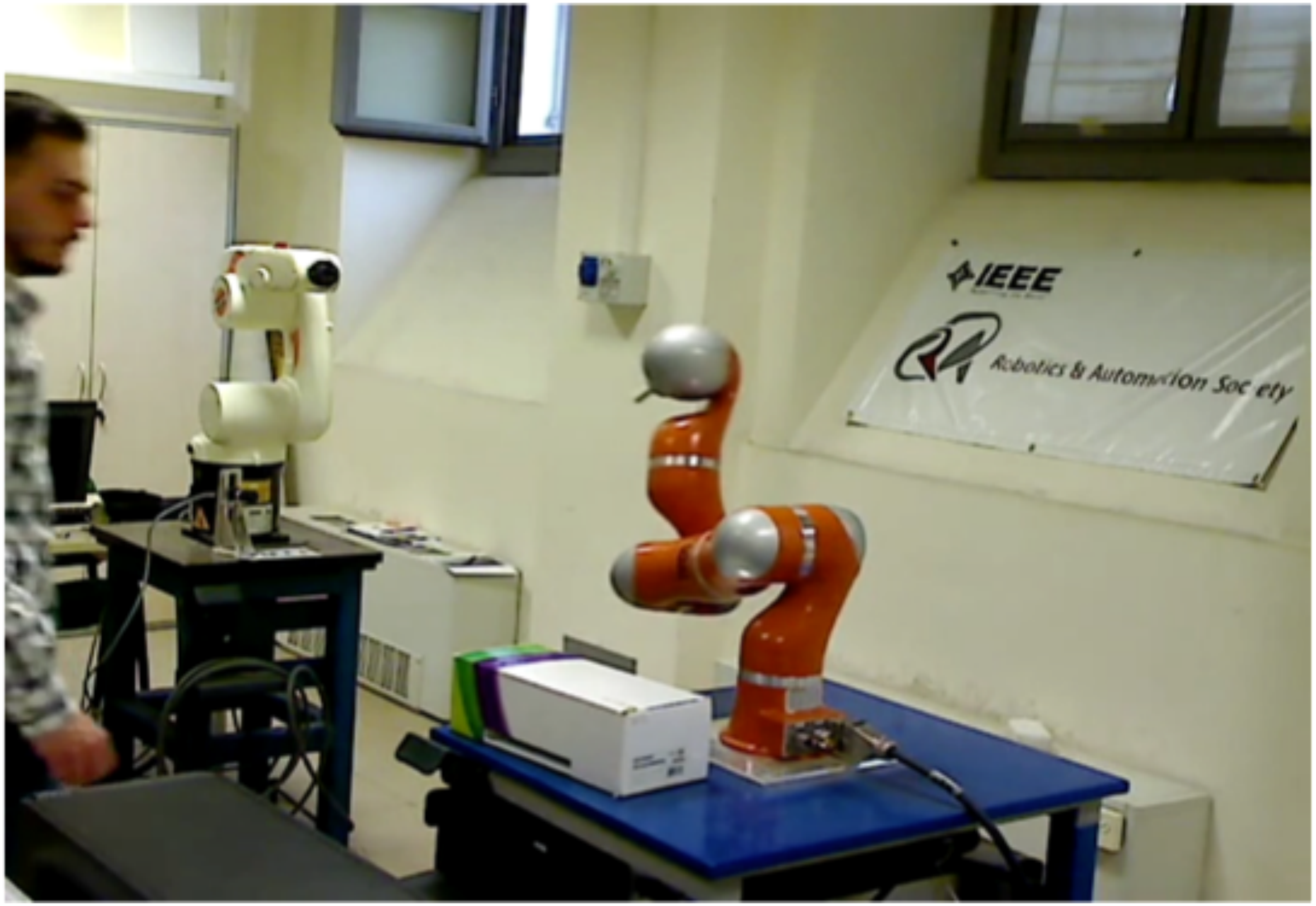
- anywhere: conventional/massive industrial manipulators, lightweight (loaded) research-oriented robots, educational devices, …



Exploiting joint elasticity in pHRI

Detection and selective reaction in torque control mode, based on residuals

- collision detection & reaction for safety (model-based + joint torque sensing)

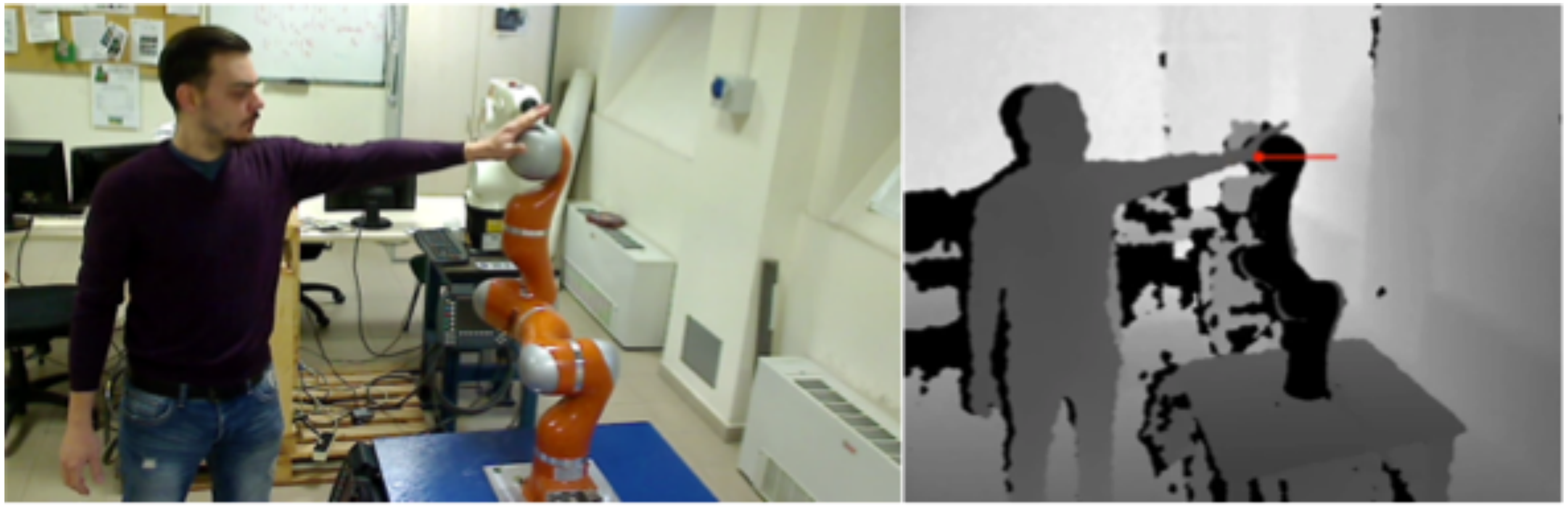


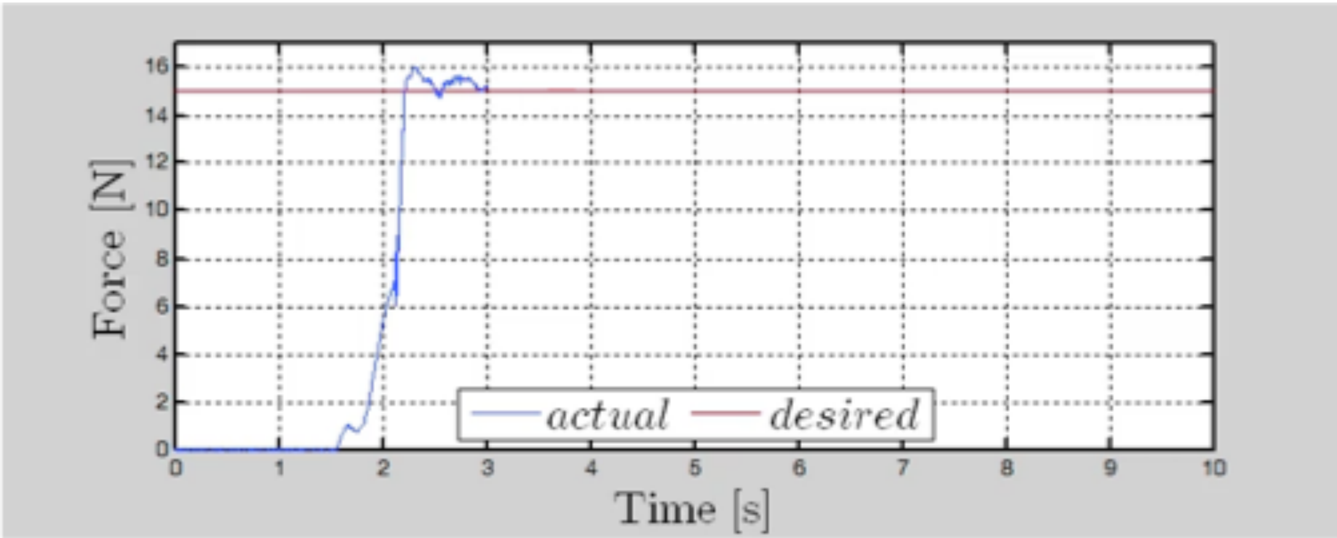
[De Luca *et al,* IROS 2006; Haddadin *et al,* T-RO 2017]

Exploiting joint elasticity in pHRI

Human-robot collaboration in torque control mode

- contact force estimation & control (virtual force sensor, anywhere/anytime)





[Magrini *et al*, ICRA 2015]

Dynamic modeling

Lagrangian formulation (so-called reduced model of Spong)

- open chain robot with N elastic joints and N rigid links, driven by electrical actuators
- use N motor variables θ (as reflected through the gear ratios) and N link variables q
- assumptions

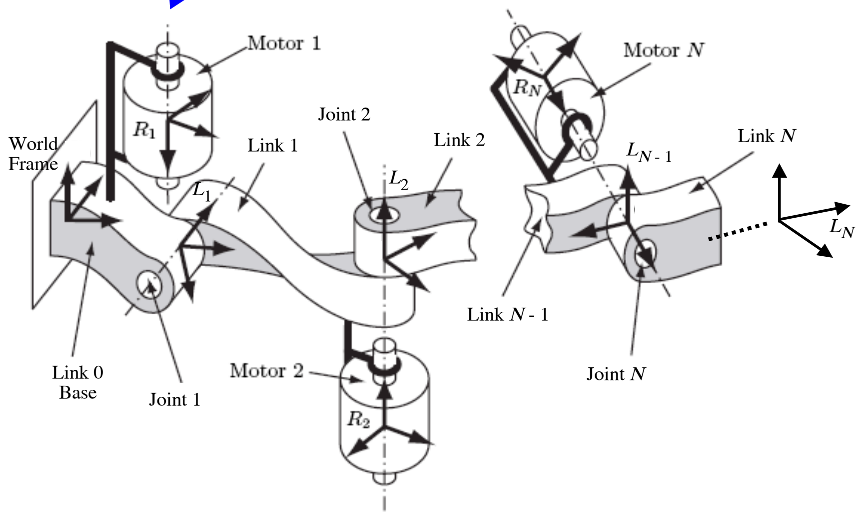
  A1) small displacements at joints

  A2) axis-balanced motors

  A3) each motor is mounted on the robot in a position preceding the driven link

  A4) no inertial couplings between motors and links

center of mass of rotors on rotation axes



A4) ⇒ 2N × 2N inertia matrix Is block diagonal

A2) ⇒ inertia matrix and gravity vector are independent from θ

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

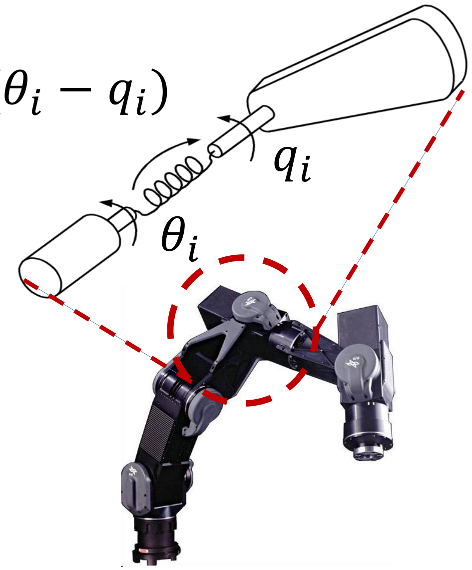
link equation

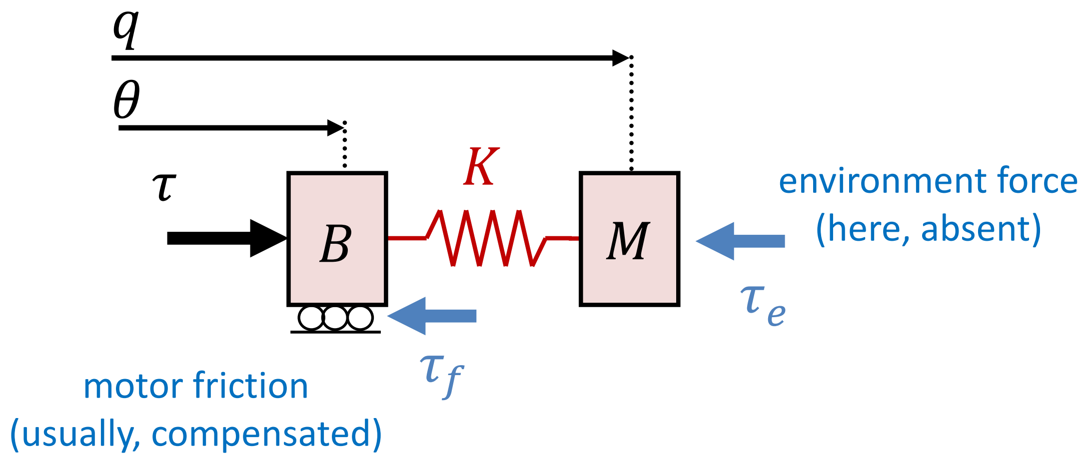
motor equation

Single elastic joint

Transfer functions of interest

$$\tau_{ei} = K_i(\theta_i - q_i)$$





we often look rather at the torque-to-velocity mappings …
(eliminating one integrator)

$$P_{\text{motor}}(s) = \frac{\theta(s)}{\tau(s)} = \frac{Ms^2 + K}{MBs^2 + (M+B)K}\,\frac{1}{s^2}$$

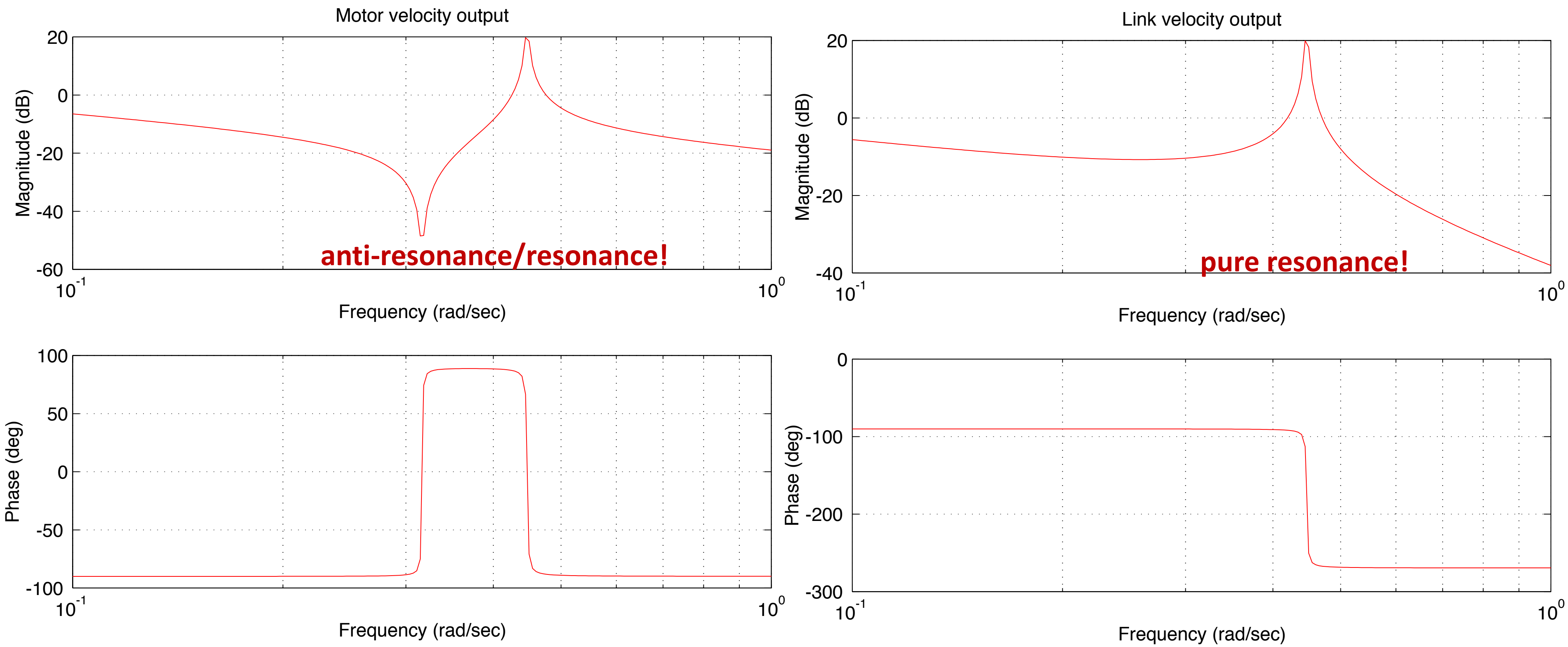
- system with zeros and relative degree = 2
- passive (zeros always *precede* poles on the imaginary axis)
- stabilization can be achieved via output θ feedback

$$P_{\text{link}}(s) = \frac{q(s)}{\tau(s)} = \frac{K}{MBs^2 + (M+B)K}\,\frac{1}{s^2}$$

- NO zeros!!
- maximum relative degree = 4

Single elastic joint

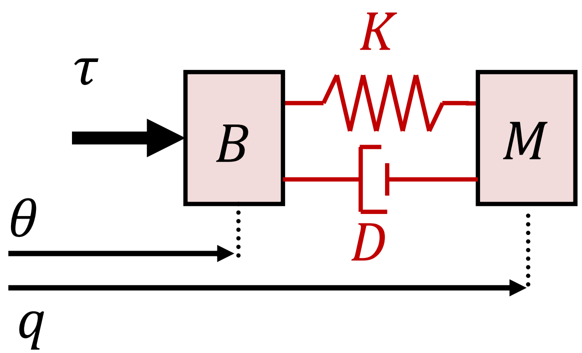
Transfer functions of interest (with added motor and/or link side damping…)



- typical anti-resonance/resonance behavior on motor velocity output
- pure resonance on link velocity output (weak or no zeros)

Visco-elasticity at the joints

Introduces a structural change …



on Spong model

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) + D(\dot{q}-\dot{\theta}) \\ K(\theta-q) + D(\dot{\theta}-\dot{q}) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

coupling type	consequence for the model
stiffness	basic static coupling, maximum relative degree (= 4) of output q
damping	reduced relative degree (= 3), only I/O linearization by static feedback
inertia *	reduced relative degree, I/O linearization needs dynamic feedback

* so-called complete dynamic model

Inverse dynamics

Feedforward action for following a desired trajectory in nominal conditions

given a desired smooth link trajectory $q_d(t) \in C^4$

- compute symbolically the desired motor acceleration and, therefore, also the desired link jerk (i.e., up to the fourth time derivative of the desired motion)

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau_m \end{pmatrix}$$

$$\tau_{m,d} = B\ddot{\theta}_d + K(\theta_d - qd)$$

$$= BK^{-1}\left[M(q_d)\, {q_d}^{(4)} + 2\dot{M}(q_d)\, {q_d}^{(3)} + \ddot{M}(q_d)\ddot{q}_d + \frac{d^2}{dt^2}\left(C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)\right)\right]$$

$$+[M(q_d) + B]\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d)$$

- the inverse dynamics can be efficiently computed using a modified Newton-Euler algorithm (with link recursions up to the fourth order) running in $O(N)$
- the feedforward command can be used in combination with a PD feedback control on the motor position/velocity error, so as to obtain a local but simple trajectory tracking controller

Feedback linearization

For accurate trajectory tracking tasks

- the link position q is a linearizing (a.k.a. **flat**) output

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} \Longleftrightarrow \boxed{q^{(4)} = u}$$

- differentiating twice the link equation and using the motor acceleration yields

$$\tau = BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}\left(C\dot{q} + g(q)\right)\right)$$

- an exactly linear and I/O decoupled closed-loop system is obtained
  - to be stabilized with standard techniques for linear dynamics (pole placement, LQ, …)
- requires higher derivatives of q —— $q, \dot{q}, \ddot{q}, q^{(3)}$
- however, these can be computed from the model using the state measurements
- requires higher derivatives of the dynamics components —— $\ddot{M}, \ddot{C}, \ddot{g}$
- A $O(N^3)$ Newton-Euler recursive numerical algorithm is available for this problem

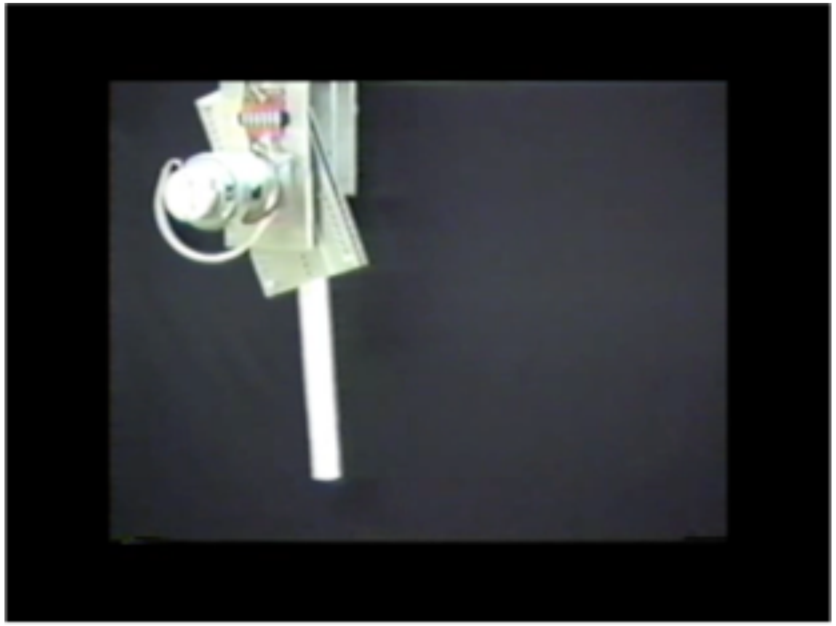
Feedback linearization

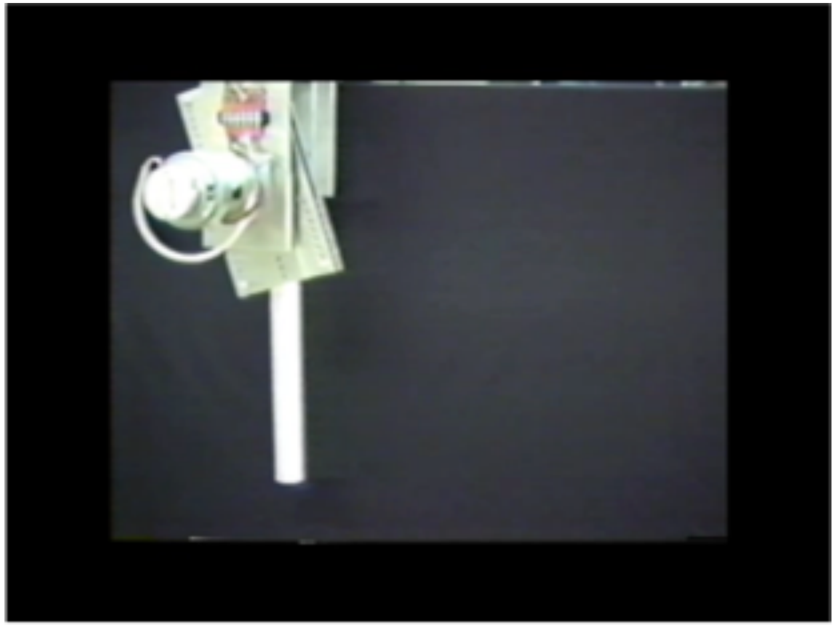
Based on the rigid model only vs. when modeling also joint elasticity

$$\tau = M(q)(\ddot{q}_d + K_D(\dot{q}_d - \dot{q}) + K_P\,(q_d - q)) + C(q,\dot{q})\dot{q} + g(q)$$

$$\tau \;= BK^{-1}M(q)u + K(\theta - q) + B\ddot{q} + BK^{-1}\left(2\dot{M}q^{(3)} + \ddot{M}\ddot{q} + \frac{d^2}{dt^2}(C\dot{q} + g(q))\right)$$

$$u = \left(q_d^{[4]} + K_J(\dddot{q}_d - \dddot{q}) + K_A(\ddot{q}_d - \ddot{q}) + K_D(\dot{q}_d - \dot{q}) + K_P\,(q_d - q)\right)$$





rigid computed torque

[Spong, ASME JDSMC 1986]

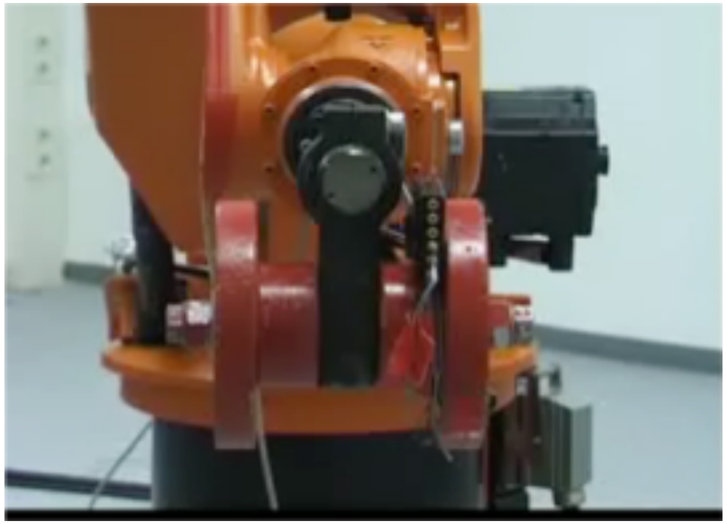
elastic joint feedback linearization

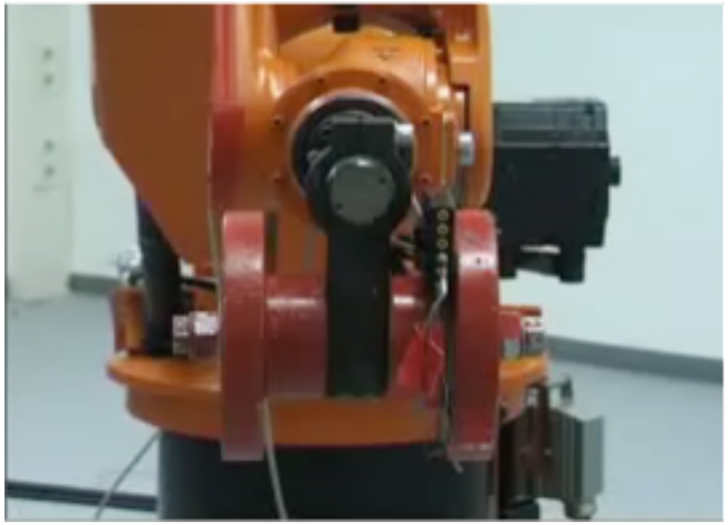
Feedback linearization

Benefits on an industrial KUKA KR-15/2 robot (235 kg) with joint elasticity

conventional industrial robot control

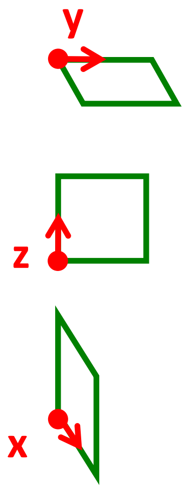
feedback linearization + high-damping





trajectory tracking with model-based control

three squares in:



horizontal plane

vertical front plane

vertical sagittal plane

[Thümmel, PhD@TUM 2007]

Torque control

A different set of state measurements can be used directly for tracking control

$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$\tau_J = K(\theta - q)$ measurable by a joint torque sensor

$BK^{-1}\ddot{\tau}_J + \tau_J = \tau - B\ddot{q}$ rewriting the motor dynamics

$$\tau = BK^{-1}\ddot{\tau}_{J,d} + \tau_{J,d} + K_T(\tau_{J,d} - \tau_J) + K_S(\dot{\tau}_{J,d} - \dot{\tau}_J) + \alpha B\ddot{q}$$

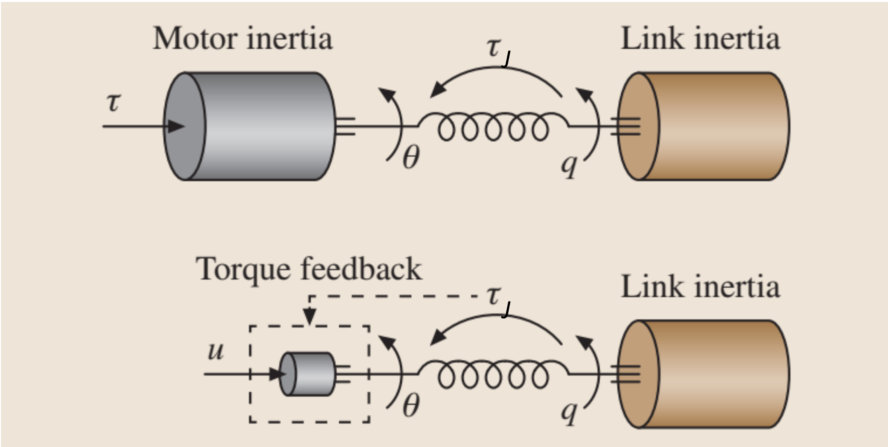
- $\alpha < 1$ for avoiding over-compensation
- useful for designing a motor side disturbance observer, e.g., to realize friction compensation
- basis for many cascaded controller designs, starting from a given rigid body control law $\tau = \tau(q, \dot{q}, \mathrm{t})$ taken as $\tau_{J,d}(\mathrm{t})$ in the above formulas
- higher derivatives are still required (either $\ddot{q}$ or $\ddot{\tau}_J$)

Torque feedback

An inner loop that largely reduces motor inertia (and friction)

Consider a pure proportional torque feedback (+ a derivative term for the visco-elastic case)

$$\tau = BB_d^{-1}u + \underbrace{\left(I - BB_d^{-1}\right)}_{-K_T}\tau_J \; + \underbrace{\left(I - BB_d^{-1}\right)DK^{-1}}_{-K_S}\dot{\tau}_J$$



physical interpretation:

scaling of the motor inertia and motor friction!

[Ott, Albu-Schäffer, 2008]

original motor dynamics	visco-elastic case
$B\ddot{\theta} + K(\theta - q) = \tau$	$B\ddot{\theta} + \tau_J + DK^{-1}\dot{\tau}_J = \tau$

after the torque feedback

$$B_d\ddot{\theta} + K(\theta - q) = u \qquad B_d\ddot{\theta} + \tau_J + DK^{-1}\dot{\tau}_J = u$$

Full-state feedback

Combining torque feedback with a motor PD regulation law

[Albu-Schäffer *et al*, IJRR 2007]

inertia scaling via torque feedback $\tau = (I + K_T)u - K_T\,\tau_J - K_S\dot{\tau}_J$

regulation via motor PD, e.g., with $u = g(\bar{q}(\theta)) + K_\theta(\theta_d - \theta) - D_\theta\dot{\theta}$

⇒ **joint level control structure** of the DLR (and KUKA) lightweight robots

dynamics feedforward and desired torque command ($\tau_{J,d}$)

setpoint control ($K_P(\theta_d - \theta)$)

$$\tau = \tau_{J,d} - K_T\left(\tau_J - \tau_{J,d}\right) - K_S\dot{\tau}_J - K_P(\theta_d - \theta) - K_D\dot{\theta} + \tau_f + \tau_{dob}$$ (+ integral actions)

motor inertia scaling (K_T)

vibration damping ($K_S\dot{\tau}_J$, $K_D\dot{\theta}$)

friction compensation and/or disturbance observer (τ_f, τ_{dob})

torque control	**position control**	**impedance control**
$K_P = 0$	$K_P > 0$	$K_P = K_T K_\theta$
$K_D = 0$	$K_D > 0$	$K_D = K_T D_\theta$
$K_T > 0$	$K_T > 0$	$K_T = (BB_d^{-1} - I)$
$K_S > 0$	$K_S > 0$	$K_S = \left(BB_d^{-1} - I\right)DK^{-1}$
$\tau_{J,d} = \tau_d$	$\tau_{J,d} = g(q)$	$\tau_{j,d} = g(\bar{q}(\theta))$

Regulation tasks

Using a minimal PD+ action on the motor side

for a desired constant link position q_d

- evaluate the associated desired motor position θ_d at steady state
- collocated (partial state) feedback preserves passivity, with stiff K_P gain dominating gravity
- focus on the term for **gravity compensation** (acting on link side) from motor measurements

$$\theta_d = q_d + K^{-1} g(q_d) \qquad \tau = \tau_g + K_P(\theta_d - \theta) - K_D \dot{\theta} \qquad K_D > 0$$

τ_g	gain criteria for stability	
$g(q_d)$	$\lambda_{min} \begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} > \alpha$	[Tomei, 1991]
$g(\theta - K^{-1} g(q_d))$	$\lambda_{min} \begin{bmatrix} K & -K \\ -K & K + K_P \end{bmatrix} > \alpha$	[De Luca, Siciliano, Zollo, 2004]
$g(\bar{q}(\theta))$, $\bar{q}(\theta)$: $g(\bar{q}) = K(\theta - \bar{q})$	$K_P > 0, \quad \lambda_{min}(K) > \alpha$	[Ott, Albu-Schäffer, 2004]
$g(q) + BK^{-1} \ddot{g}(q)$	$K_P > 0, \quad K > 0$	[De Luca, Flacco, 2010]

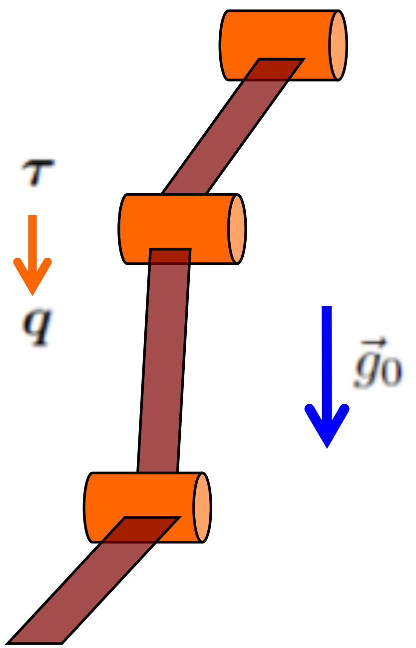
exact gravity cancellation
(with **full state** feedback)
more on this next…

$$\alpha = \max\left(\left\|\frac{\partial g(q)}{\partial q}\right\|\right)$$

Exact gravity cancellation

A slightly different view

- for rigid robots this is trivial, due to full actuation and collocation



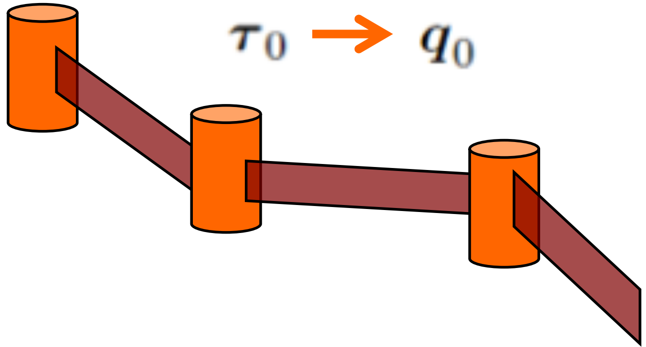
$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \tau$$

$$\tau = \tau_g + \tau_0$$

⇒

$$\tau_g = g(q)$$

$$q \equiv q_0$$

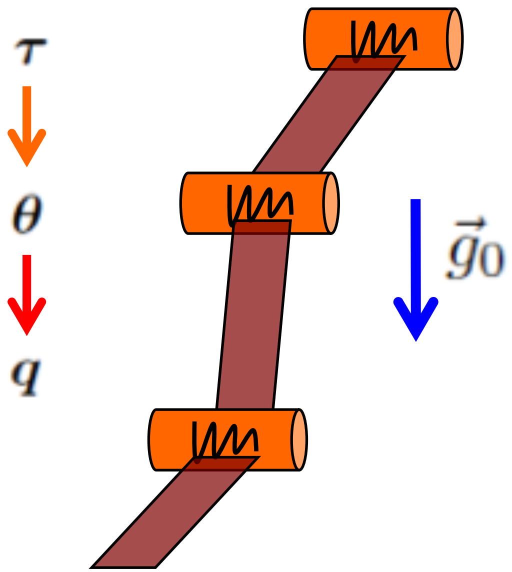


$$M(q)\ddot{q} + c(q,\dot{q}) = \tau_0$$

Exact gravity cancellation

… exploiting the concept of feedback equivalence between nonlinear systems

- for elastic joint robots, non-collocation of input torque and gravity term



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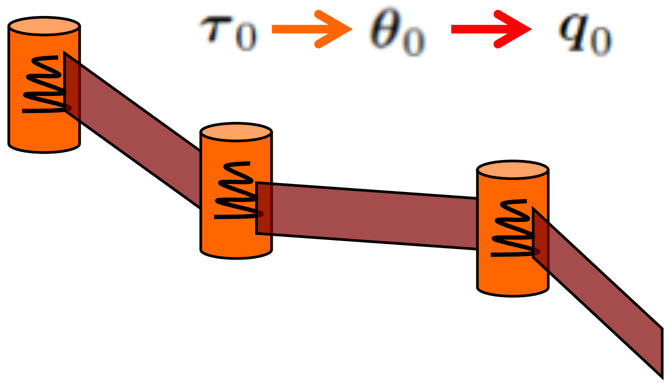
$$\tau = \tau_g + \tau_0$$

$$\Rightarrow$$

$$\boxed{q \equiv q_0}$$

??

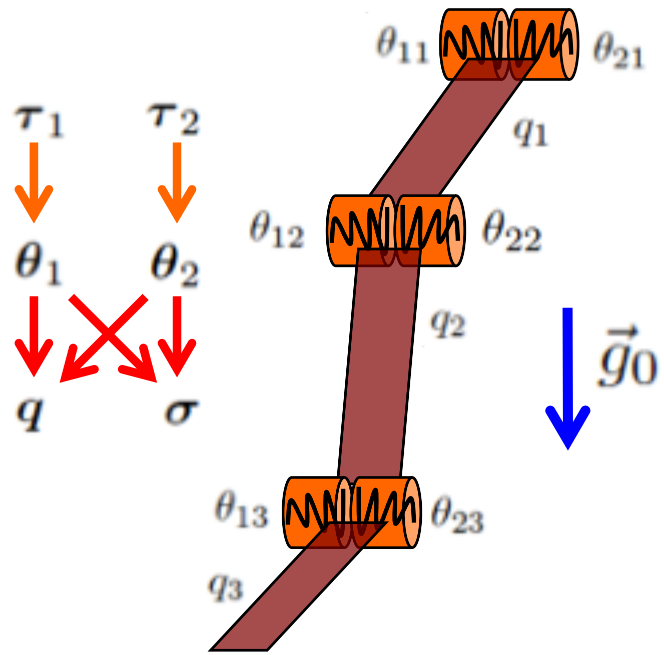


$$\begin{aligned} M(q)\ddot{q} + c(q,\dot{q}) + g(q) + K(q-\theta) &= 0 \\ B\ddot{\theta} + K(\theta - q) &= \tau \end{aligned}$$

Exact gravity cancellation

… generalized also to VSA robots

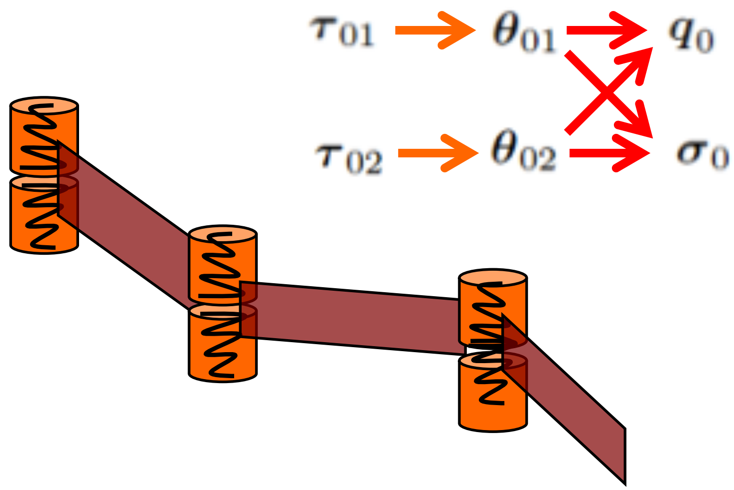
- same problem formulation holds also for VSA robots (here, in antagonistic configuration), with the additional consideration of the internal stiffness state



$\tau_1 \quad \tau_2$??

$q \equiv q_0$??

$\sigma \equiv \sigma_0$??



$\phi_i = q - \theta_i$

$i = 1, 2$

$$
\begin{aligned}
M(q)\ddot{q} + c(q,\dot{q}) + g(q) + \tau_{e1}(\phi_1) + \tau_{e2}(\phi_2) &= 0 \\
B_1\ddot{\theta}_1 - \tau_{e1}(\phi_1) &= \tau_1 \\
B_2\ddot{\theta}_2 - \tau_{e2}(\phi_2) &= \tau_2
\end{aligned}
$$

Feedback equivalence

Use the system property of being feedback linearizable (without forcing it!)

$$u = \tilde{\alpha}(x) + \tilde{\beta}(x) u_0$$
$$x_0 = T_0^{-1}(T(x)) = \tilde{T}(x)$$

$$\dot{x} = f(x) + G(x)u$$

gravity-loaded system

$$\dot{x}_0 = f_0(x_0) + G_0(x_0)u_0$$

gravity-free system

feedback transformations
static **state feedback**
+ **change of coordinates**
both invertible

$$u = \alpha(x) + \beta(x)v$$
$$z = T(x)$$

$$u_0 = \alpha_0(x_0) + \beta_0(x_0)v$$
$$z = T_0(x_0)$$

$$\dot{z} = Az + Bv$$

linear, controllable system

z ≈ linearizing outputs

Exact gravity cancellation

Elastic joint robots (including link/motor damping) [De Luca, Flacco, CDC 2010]

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + D_q\dot{q} + K(q-\theta) = 0$$

$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

$$q(t) \equiv q_0(t) \quad \forall t \geq 0 \qquad \tau = \tau_g + \tau_0$$

$$\tau_g = g(q) + D_\theta K^{-1}\dot{g}(q) + BK^{-1}\ddot{g}(q)$$

$$\dot{g}(q) = \frac{\partial g(q)}{\partial q}\,\dot{q}$$

$$\ddot{g}(q) = \frac{\partial g(q)}{\partial q} M^{-1}(q)\big(K(\theta-q) - c(q,\dot{q}) - g(q) - D_q\dot{q}\big) + \sum_{i=1}^{n} \frac{\partial^2 g(q)}{\partial q\,\partial q_i}\,\dot{q}\,\dot{q}_i$$

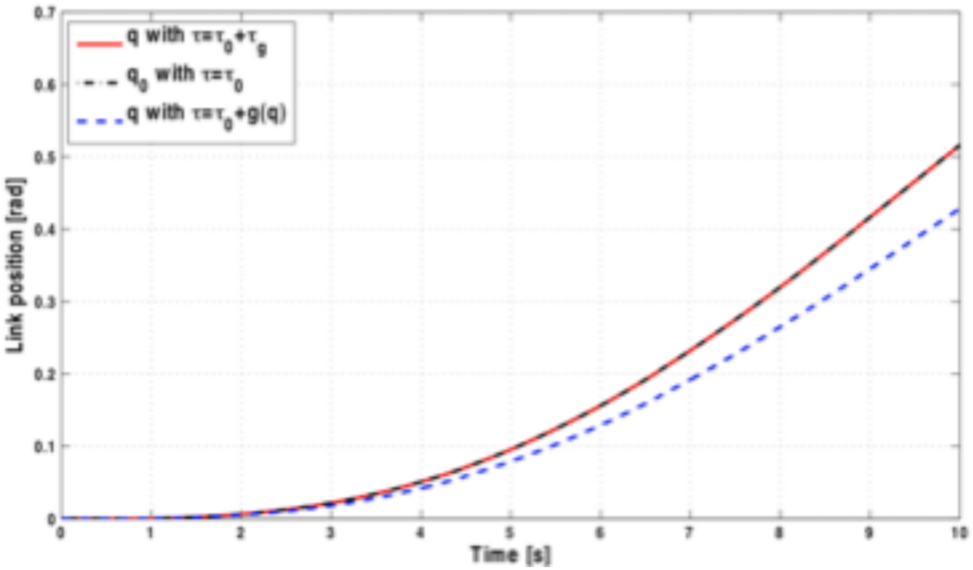
requires (in principle) full state feedback

Numerical results

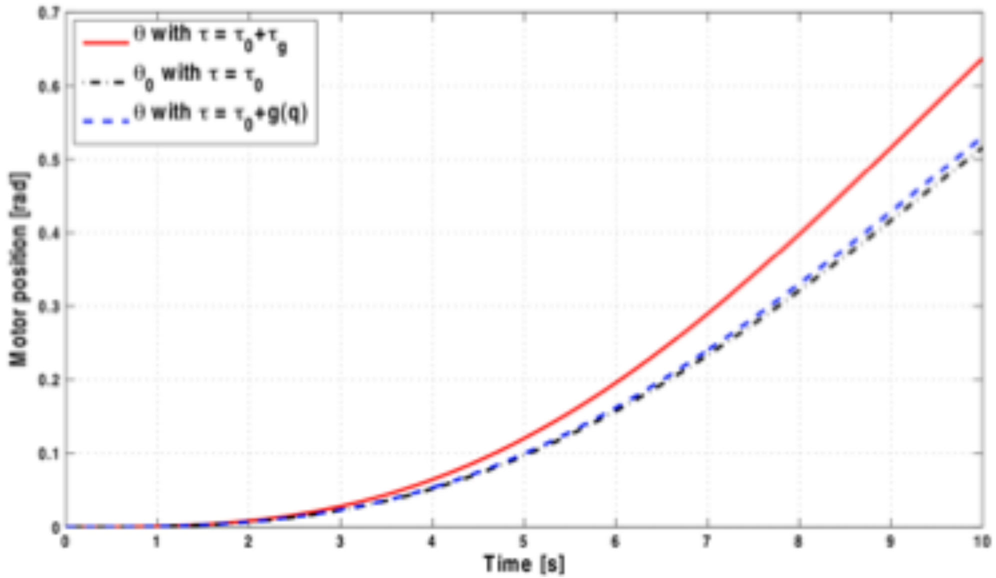
Exact gravity cancellation for a 1-DOF elastic joint

$$\tau_g = m d g_0 \left\{ \left(1 - \frac{B}{K}\dot{q}^2\right)\sin q - \frac{B}{M}\frac{m d g_0}{K}\sin q \cos q + \frac{M D_\theta - B D_q}{K M}\dot{q}\cos q + \frac{B}{M}(\theta - q)\cos q \right\}$$

$$\boxed{\tau_0 = \sin 0.1\pi t} \qquad g(q) = m d g_0 \sin q$$



exact reproduction of same link behavior with and without gravity



different motor behavior with and without gravity

$$\theta = \theta_0 + K^{-1} g(q)$$

A global PD-type regulator

Exact gravity cancellation combined with PD law on modified motor variables

$$\boldsymbol{\tau} = \boldsymbol{\tau}_g + \boldsymbol{\tau}_0$$

$$\boldsymbol{\tau}_g = \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}_\theta \boldsymbol{K}^{-1}\dot{\boldsymbol{g}}(\boldsymbol{q}) + \boldsymbol{B}\boldsymbol{K}^{-1}\ddot{\boldsymbol{g}}(\boldsymbol{q})$$

$$\boldsymbol{\tau}_0 = \boldsymbol{K}_P(\boldsymbol{\theta}_{d0} - \boldsymbol{\theta}_0) - \boldsymbol{K}_D\dot{\boldsymbol{\theta}}_0$$

$$= \boldsymbol{K}_P(\boldsymbol{q}_d - \boldsymbol{\theta} + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q})) - \boldsymbol{K}_D(\dot{\boldsymbol{\theta}} - \boldsymbol{K}^{-1}\dot{\boldsymbol{g}}(\boldsymbol{q}))$$

Global asymptotic stability can be shown using a Lyapunov analysis under "minimal" sufficient conditions (also without viscous friction)

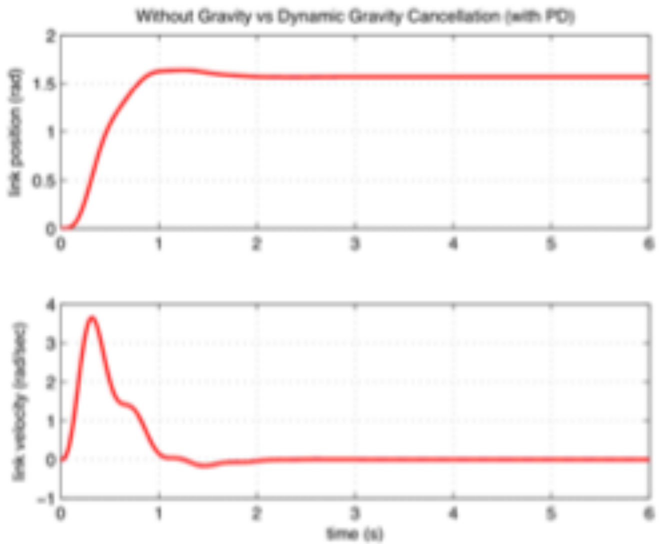
$\boldsymbol{K}_P > 0$ $\quad$ $\boldsymbol{K} > 0$ $\quad$ i.e., **no** strictly positive lower bounds are needed any longer $\quad$ and $\boldsymbol{K}_D > 0$

[De Luca, Flacco, ICRA 2011]

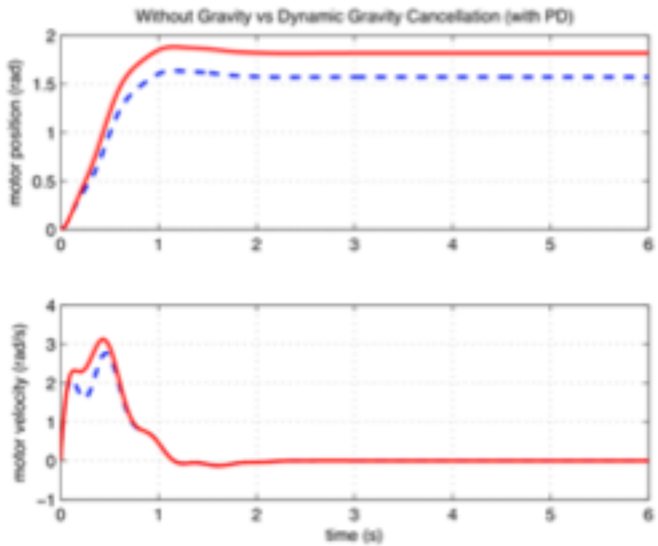
Numerical results

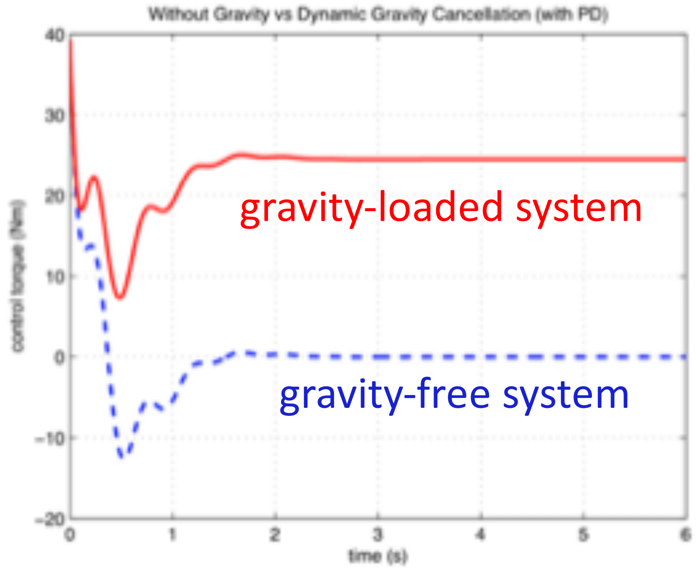
Regulation of a 1-DOF arm with elastic joint under gravity

identical link behavior



different motor behavior



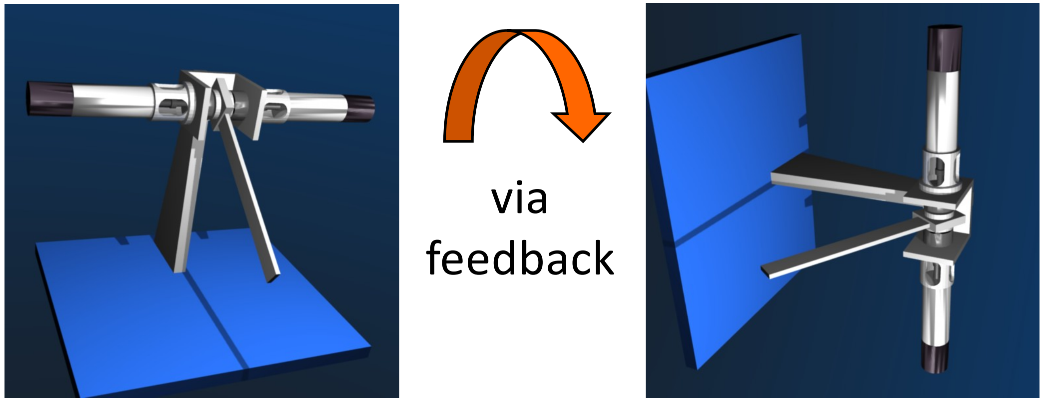


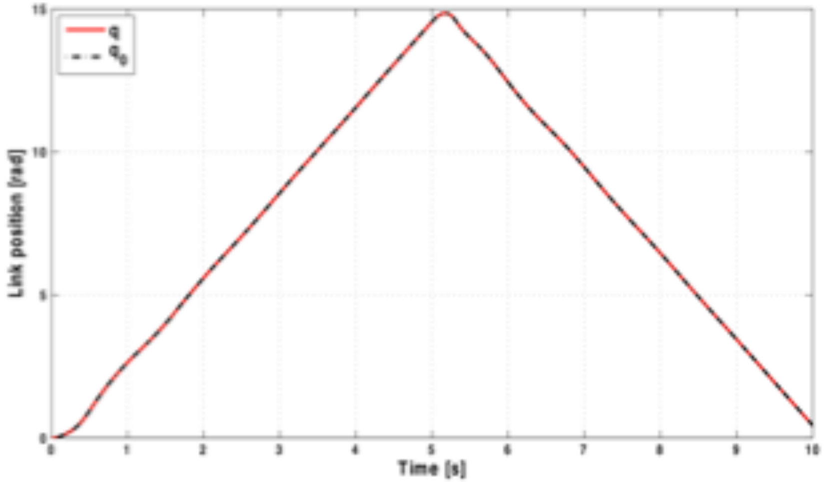
total **control torque**

gravity-loaded system under PD
+ gravity cancellation
vs.
gravity-free system under PD
(with same gains)

Numerical results

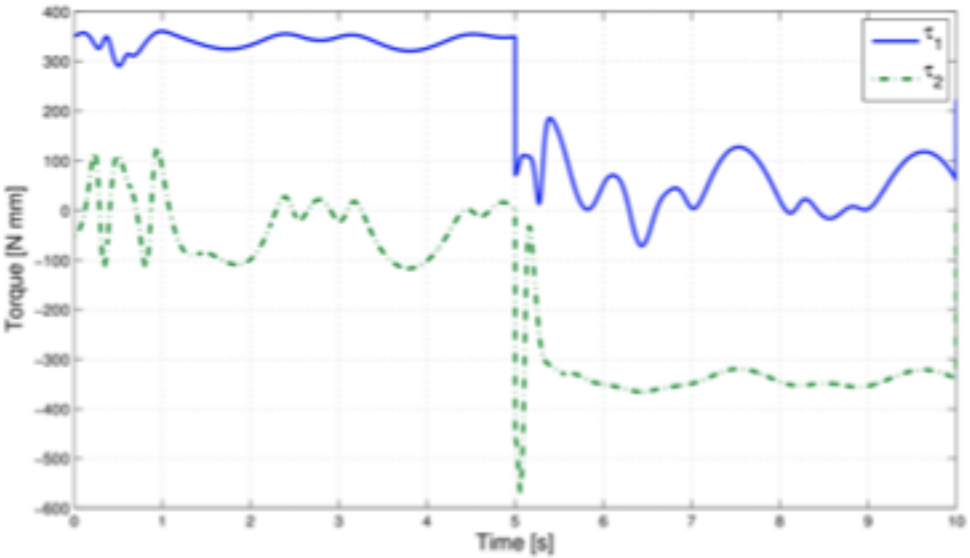
Exact gravity cancellation for the VSA-II of UniPisa



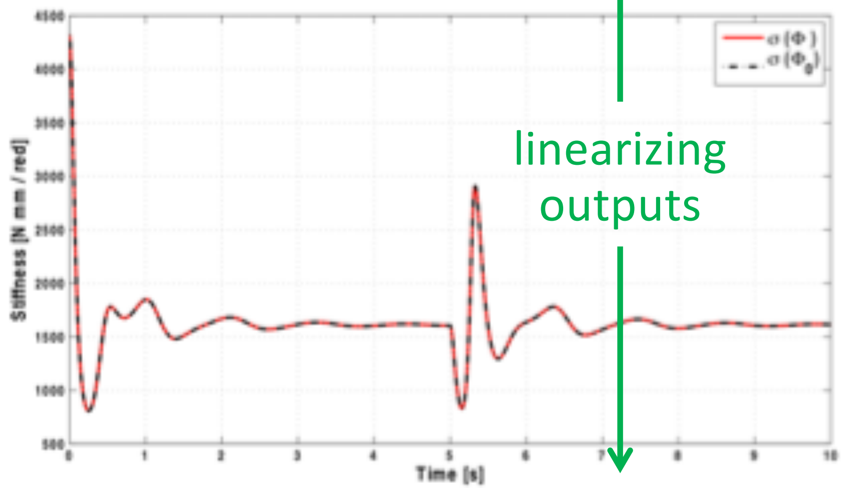


exact reproduction of link behavior

linearizing
outputs



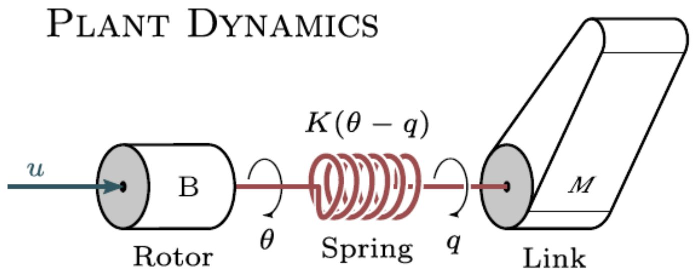
applied torques for gravity cancellation



exact reproduction of stiffness behavior

Damping injection on the link side

Method for the VSA-driven bimanual humanoid torso David



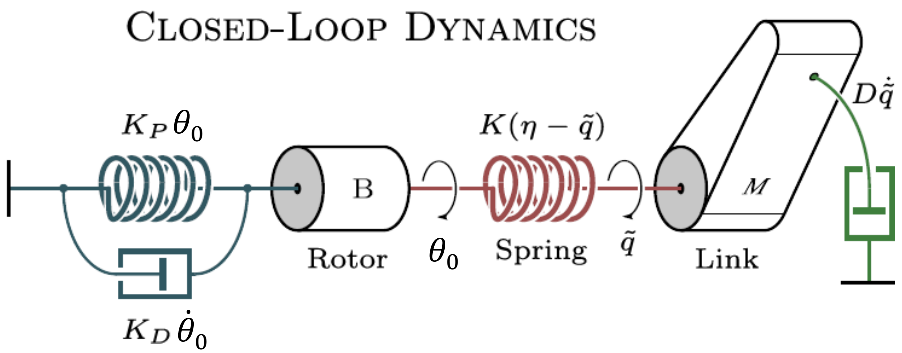
$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\theta) \\ K(\theta-q) \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix}$$

$$\boldsymbol{K(q-\theta) = K(q-\theta_0) + D\dot{q}}$$

state transformation

$$\boldsymbol{\tau = \tau_0 - D\dot{q} - BK^{-1}D\ddot{q}}$$

feedback control

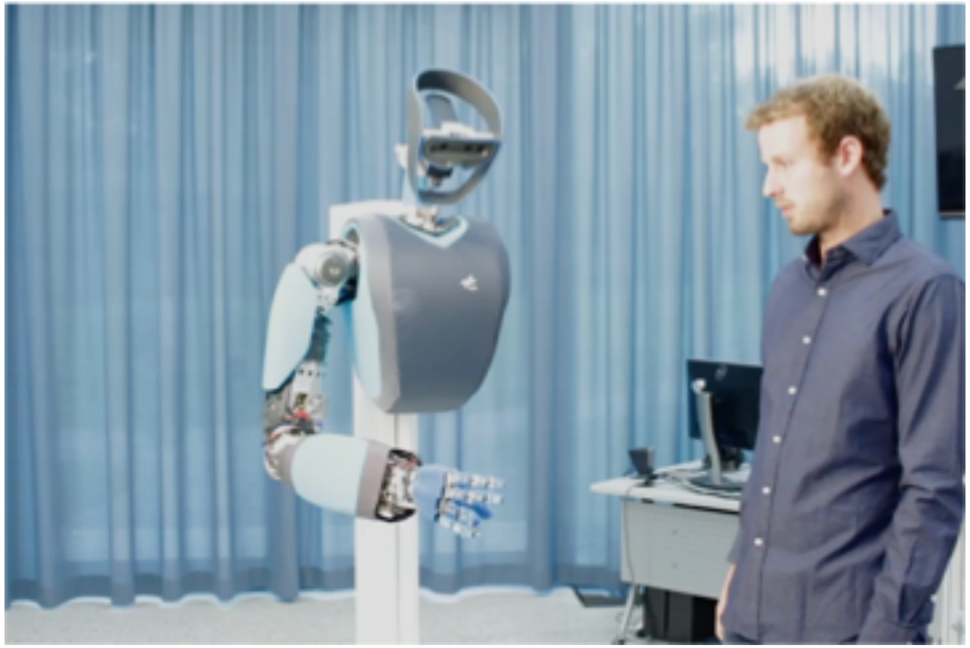


$$\begin{bmatrix} M(q) & 0 \\ 0 & B \end{bmatrix} \begin{pmatrix} \ddot{q} \\ \ddot{\boldsymbol{\theta}}_{\boldsymbol{0}} \end{pmatrix} + \begin{pmatrix} C(q,\dot{q})\dot{q} \\ 0 \end{pmatrix} + \begin{pmatrix} g(q) \\ 0 \end{pmatrix} + \begin{pmatrix} K(q-\boldsymbol{\theta_0}) \\ K(\boldsymbol{\theta_0}-q) \end{pmatrix} = \begin{pmatrix} -\boldsymbol{D\dot{q}} \\ \tau_0 \end{pmatrix}$$

- same principle of **feedback equivalence** (including state transformation)
- **ESP** = Elastic Structure Preserving control by DLR **[Keppler *et* al, T-RO 2018]**
- generalizations to **trajectory tracking**, to nonlinear joint flexibility, and to visco-elastic joints

Damping injection on the link side

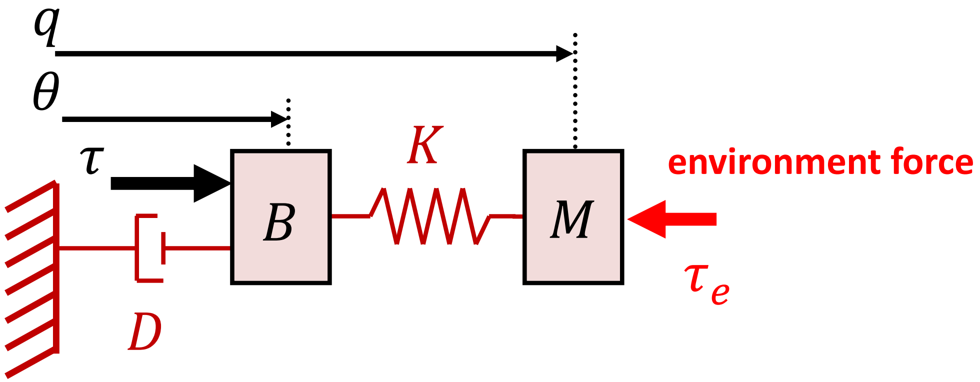
Method for **VSA-driven** bimanual humanoid torso **David** at DLR



[Keppler *et al*, T-RO 2018]

Environment interaction via impedance control

Matching a generalized (fourth order) impedance model: A simple 1-DOF case



$$M\ddot{q} + K(q-\theta) = \tau_e$$

$$B\ddot{\theta} + D\dot{\theta} + K(\theta - q) = \tau$$

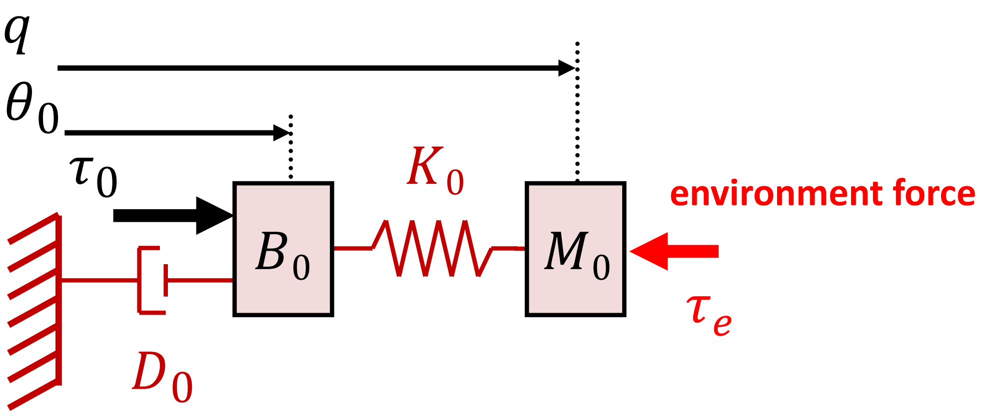
feedback control

assume that $M_0 = M$ in order to avoid **derivatives** of the measured force $\boldsymbol{\tau_e}$

$$\tau = K(\theta - q) + D\dot{\theta} - BK^{-1}\left\{\begin{array}{l}(K-K_0)M^{-1}\left(\boldsymbol{\tau_e} + K(\theta - q)\right) \\ \quad + K_0 B_0^{-1}\left(\tau_0 - D_0\dot{\theta}_0 - K(\theta - q)\right)\end{array}\right\}$$

$$\dot{\theta}_0 = \dot{q} + KK_0^{-1}(\dot{\theta} - \dot{q})$$

state transformation



$$M_0\ddot{q} + K_0(q - \theta_0) = \tau_e$$

$$B_0\ddot{\theta}_0 + D_0\dot{\theta}_0 + K_0(\theta_0 - q) = \tau_0$$

- again, by the principle of **feedback equivalence** (including the state transformation)

Control of a soft robot

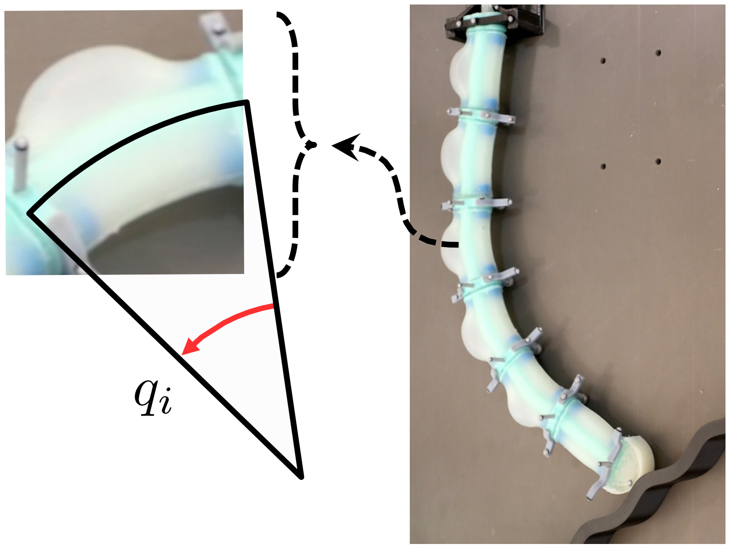
Matching the natural dynamics of the system: **Continuum** robot case

- dynamic modeling assumptions

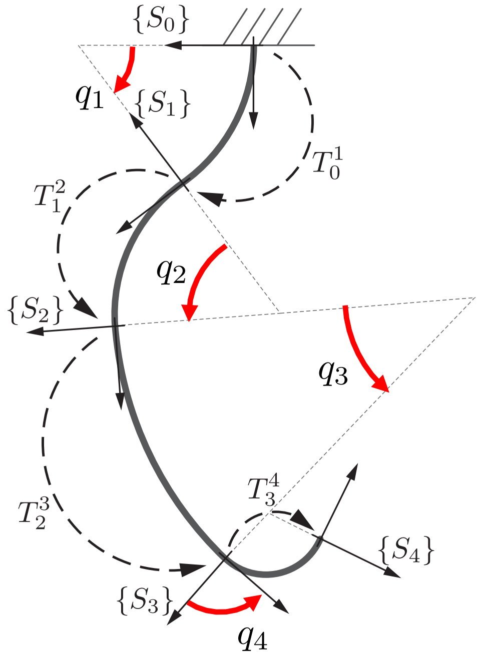
  A1) [kinematics] approximated as a series of n segments with constant curvature

  A2) [inertia] each segment can be described by an equivalent point mass

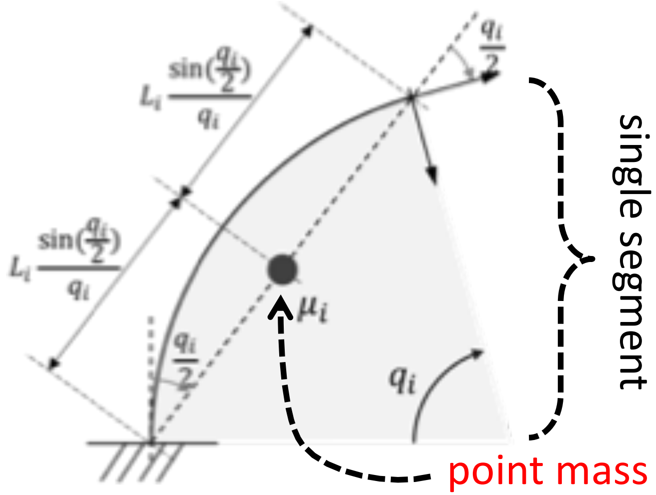
  A3) [impedance] continuous distribution of infinitesimal springs and dampers



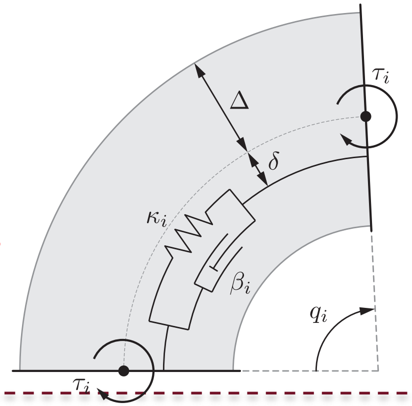
$s \in C^4$



$q \in \mathbb{R}^n$



linear stiffness!

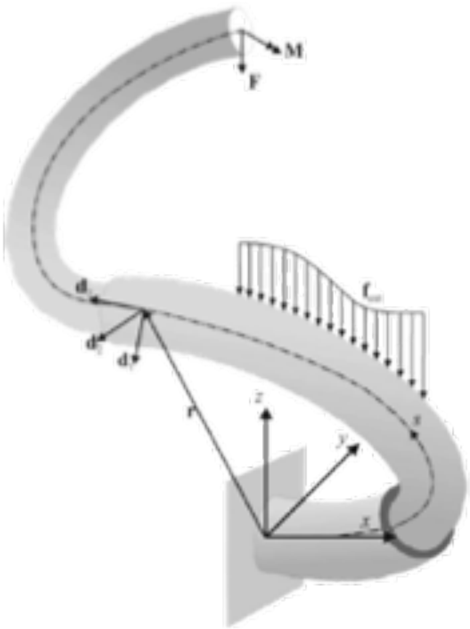


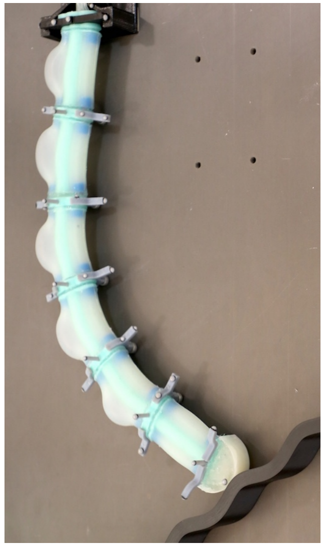
[Della Santina *et al*, IJRR 2018]

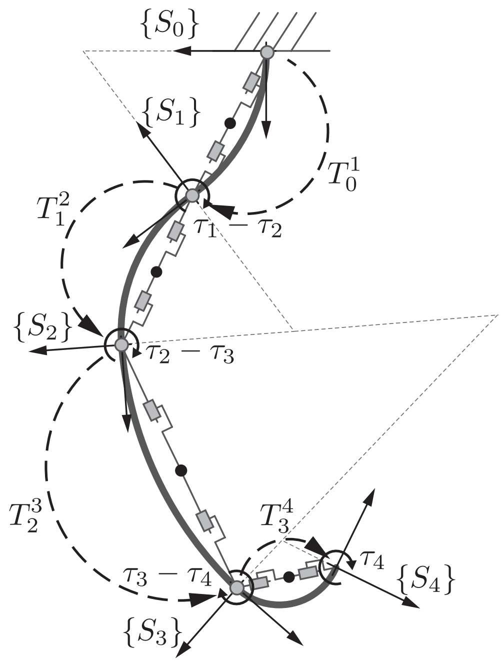
Dynamic modeling of a continuum soft robot

Finite dimensional (arbitrary) approximation

- continuum soft robot
- articulated soft robot (**fully actuated!**)







$$(\mathbf{n} - F_{\text{ex}}\mathbf{d}_3)_{,s} + \mathbf{f} = \rho A_t \mathbf{r}_{,tt}$$

$$\mathbf{m}_{,s} + \mathbf{r}_{,s} \times \mathbf{n} = \mathbf{J}\omega_{,t}$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + Kq + D\dot{q} = \tau$$

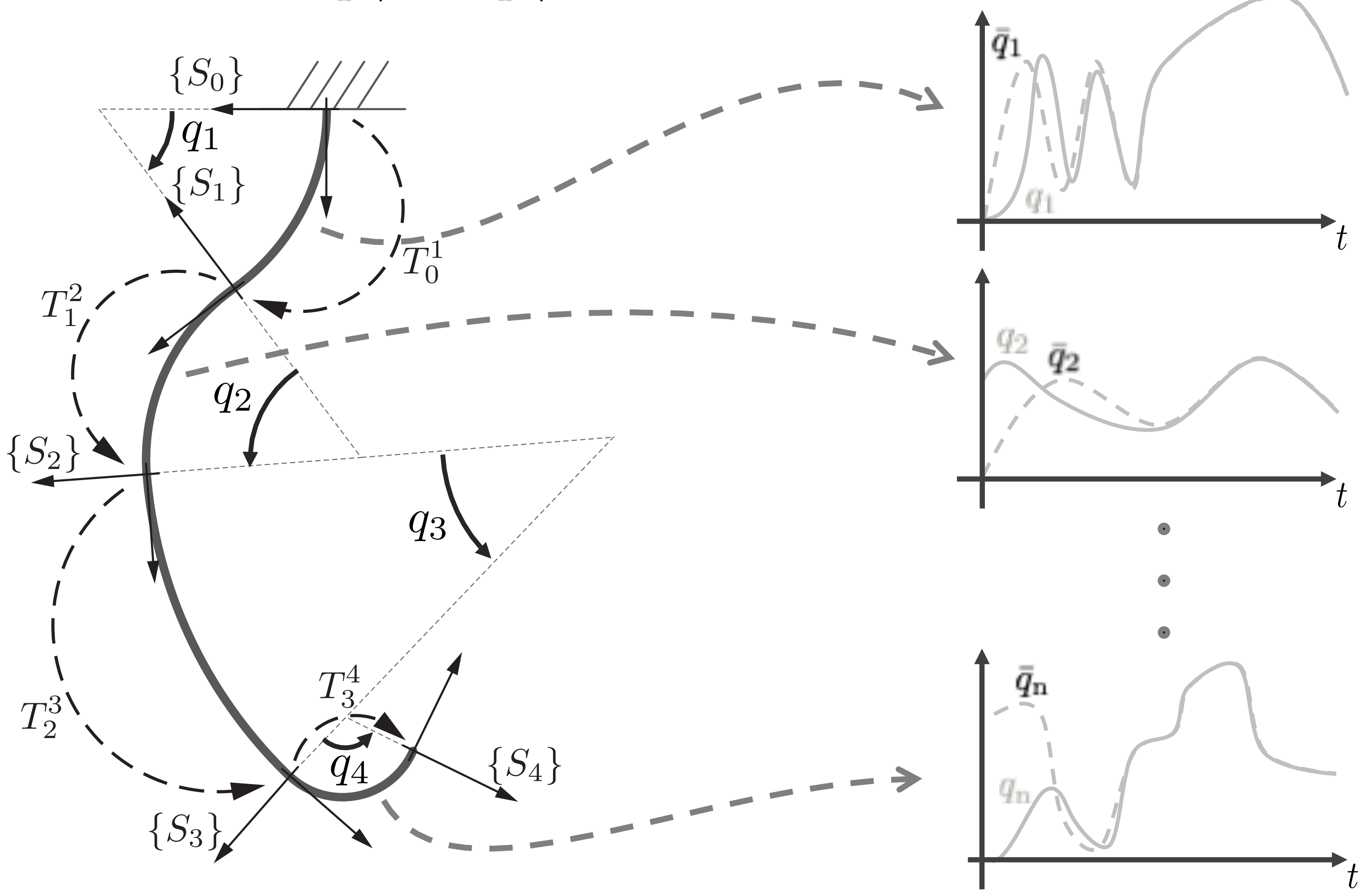
Trivedi et al.
"Geometrically Exact Models for Soft Robotic Manipulators"

Albu-Schaeffer and Bicchi "Actuators for Soft Robotics"
Ch. 21 in *Springer Handbook of Robotics* (Siciliano and Khatib eds.)

Regulation and trajectory tracking in curvature space

Moving from joint configuration space to local curvature space

- tracking case when $\dot{\bar{q}} \neq 0\,,\, \ddot{\bar{q}} \neq 0$



Regulation and trajectory tracking in curvature space

Static, quasi-static, dynamic reference (without and with gravity)

pure feedforward

$$\tau = K\bar{q} + D\dot{\bar{q}}$$

desired curvature

soft robot stiffness

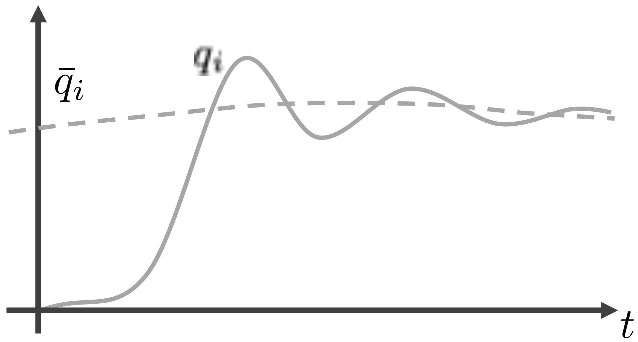
soft robot damping

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + Kq + D\dot{q} = \tau$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} = \underbrace{K(\bar{q} - q) + D(\dot{\bar{q}} - \dot{q})}$$

feedforward + physical impedance

physical PD control!



quasi-static reference $\begin{cases} \dot{\bar{q}} \simeq 0 \\ \ddot{\bar{q}} \simeq 0 \end{cases}$

no gravity $g = 0$

$\Leftrightarrow$ rigid robot controlled through a PD: global asymptotic stability

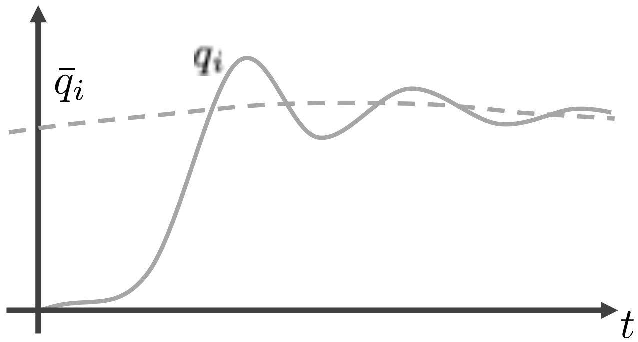
Regulation and trajectory tracking in curvature space

Static, quasi-static, dynamic reference … (without and with gravity)

pure feedforward | feedback

$$\tau = K\bar{q} + D\dot{\bar{q}} + G(q)$$

- K: soft robot stiffness
- $\bar{q}$, $\dot{\bar{q}}$: desired curvature
- D: soft robot damping
- G: gravity
- q: curvature



quasi-static reference

$$\begin{cases} \dot{\bar{q}} \simeq 0 \\ \ddot{\bar{q}} \simeq 0 \end{cases}$$

with gravity $\quad g \neq 0$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + Kq + D\dot{q} = \tau$$

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} = \underbrace{K(\bar{q} - q) + D(\dot{\bar{q}} - \dot{q})} \quad \Leftrightarrow$$

feedforward + physical impedance

physical PD control!

rigid robot controlled through a PD: global asymptotic stability

Regulation and trajectory tracking in curvature space

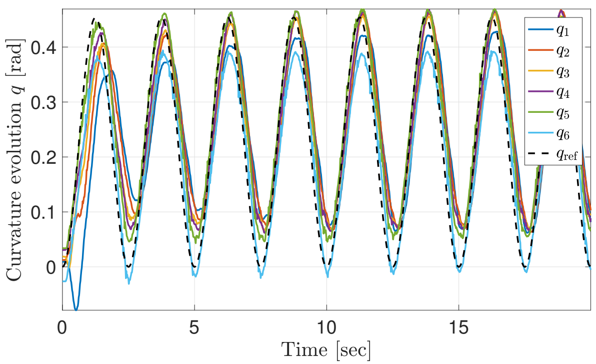
Static, quasi-static, dynamic reference … (without and with gravity)

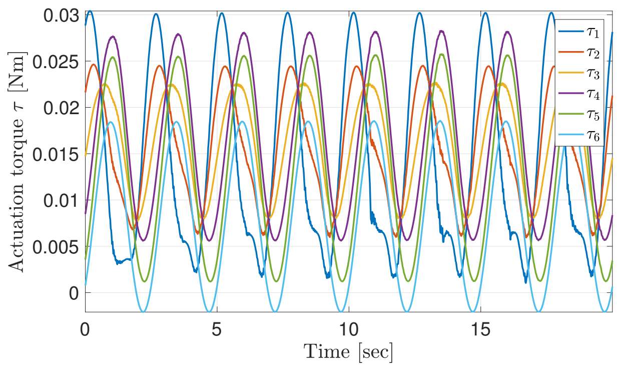
passivity-based nonlinear control, with **physical PD**: global asymptotic stability

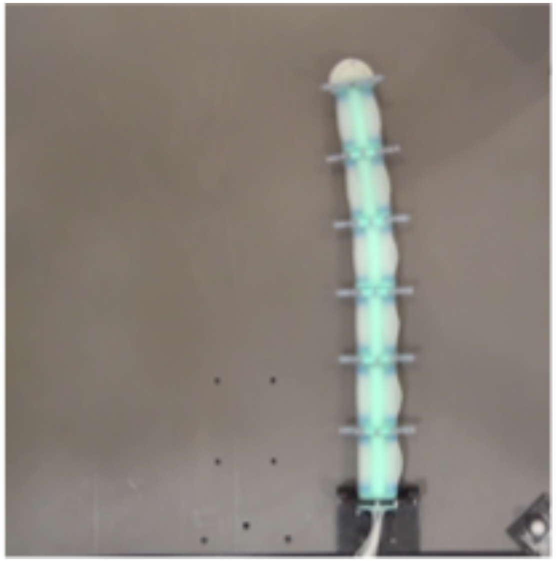
$$\tau = \underbrace{K\bar{q} + D\dot{\bar{q}}}_{\text{pure feedforward}} + \underbrace{G(q) + C(q,\dot{q})\dot{\bar{q}} + B(q)\ddot{\bar{q}}}_{\text{feedback}}$$

- K: soft robot stiffness
- $\bar{q}$: desired curvature
- D: soft robot damping
- G: gravity
- q: curvature
- C: Coriolis and centrifugal
- B: inertia

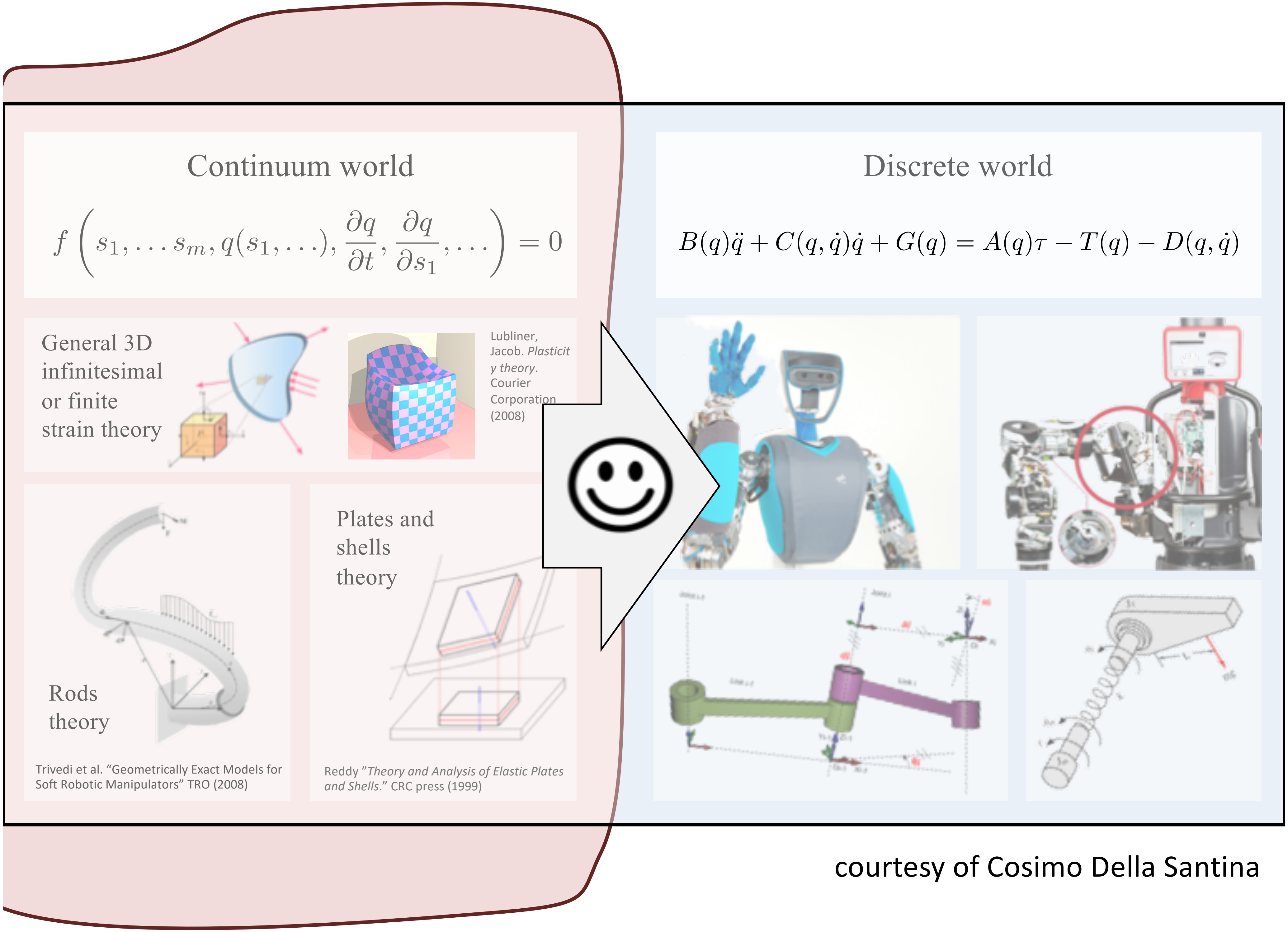
dynamic reference $\begin{cases} \dot{\bar{q}} \neq 0 \\ \ddot{\bar{q}} \neq 0 \end{cases}$







[Della Santina *et al*, IJRR 2018]



courtesy of Cosimo Della Santina

Outlook

Control of soft & flexible robots in 2020+

- Mature field revamped by a new “explosion” of interest
  - simpler control laws for compliant and soft robots are very welcome
  - sensing requirements could be a bottleneck
  - combine (learned) feedforward and feedback to achieve robustness
    - learning on repetitive tasks (ILC) already available for flexible manipulators
  - optimal control (min time, min energy, max force, …) still “open for fun”
- Revisiting model-based control design
  - do not fight against the natural dynamics of the system
    - unwise to stiffen what was designed/intended to be soft on purpose!
  - don’t give up too much of desirable performance (use feedback equivalence)
  - keep in mind under-actuation and control limitations (e.g., instabilities in the system inversion of tip trajectories for flexible link robots, I/O synergies, …)
- Ideas assessed for joint elasticity may migrate to many application domains and other classes of soft-bodied robots
  - locomotion, shared manipulation, physical interaction in complex tasks …