Robotics I

February 12, 2020

Exercise 1

Consider the 4-dof manipulator in Fig. 1. The robot has the first joint prismatic and the other three revolute. Determine a frame assignment and the associated table of parameters following the Denavit-Hartenberg (DH) convention. Assign the given geometric data l_A and l_B to the corresponding constant DH parameters. The origin of the first DH frame RF_0 is already specified, while the origin of the last frame RF_4 should be placed in P. Use the provided Extra Sheet #1 to draw the frames, and complete the DH table there. Add your name on the sheet and return it.

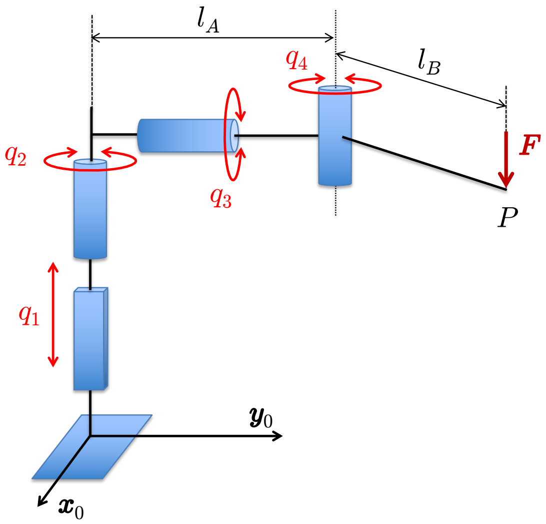

Figure 1: A 4-dof PRRR spatial manipulator.

Exercise 2

For the 4-dof manipulator of Exercise 1:

a) Determine the direct kinematics $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q})$ of point P in symbolic form. Using the numerical values $l_A = 0.5$ [m] and $l_B = 0.75$ [m], evaluate the position $\boldsymbol{p}$ in the two configurations $\boldsymbol{q}^I = \boldsymbol{0}$ and $\boldsymbol{q}^{II} = \begin{pmatrix} 1 & 0 & -\pi/2 & \pi/2 \end{pmatrix}^T$.

b) Find the symbolic expression of the generalized joint force/torque $\boldsymbol{\tau} \in \mathbb{R}^4$ that balances the purely vertical, downward force $\boldsymbol{F} \in \mathbb{R}^3$ applied at the point P as shown in Fig. 1, so that the robot remains in a static equilibrium. Using the same previous numerical values for l_A and l_B, evaluate $\boldsymbol{\tau}$ at the two given configurations $\boldsymbol{q}^I$ and $\boldsymbol{q}^{II}$.

c) Determine the angular part of the geometric Jacobian $\boldsymbol{J}_A(\boldsymbol{q})$ that relates the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^4$ of the robot to the angular velocity $\boldsymbol{\omega} \in \mathbb{R}^3$ of its end-effector frame RF_4. Study the singular configurations of $\boldsymbol{J}_A(\boldsymbol{q})$. Find a basis for all possible $\dot{\boldsymbol{q}} \in \mathbb{R}^4$ that produce $\boldsymbol{\omega} = \boldsymbol{0}$ when the Jacobian $\boldsymbol{J}_A(\boldsymbol{q})$ looses rank.

Exercise 3

Consider the 3-dof, planar PPR robot in Fig. 2, with a third link of length $L > 0$. The robot end-effector should move at a constant speed $v > 0$, tracing counterclockwise a full circle of radius $R > 0$ centered in $\boldsymbol{P}_c$, and keep its end-effector always aligned with the normal to the surface, pointing toward the circle center. Neglect for simplicity any possible collision between the robot body and the circle (e.g., they may live on two parallel, but different horizontal planes). At time $t = 0$, the robot end-effector is correctly at the initial point $\boldsymbol{A}$ with the right orientation. Suppose that the two prismatic joints have a (common) maximum velocity limit $|\dot{q}_i| \leq V$, $i = 1, 2$, while the revolute joint velocity is limited by $|\dot{q}_3| \leq \Omega$, with $V > 0$ and $\Omega > 0$. Similarly, for the joint accelerations, there are the bounds $|\ddot{q}_i| \leq A$, $i = 1, 2$, and $|\ddot{q}_3| \leq \Psi$, with $A > 0$ and $\Psi > 0$.

a) Provide the expressions of the time evolution of the end-effector position $\boldsymbol{p} \in \mathbb{R}^2$, velocity $\dot{\boldsymbol{p}}$ and acceleration $\ddot{\boldsymbol{p}}$, possibly using separation in space and time, when the given task is perfectly executed. Similarly, provide the expressions of the time evolution of the end-effector absolute orientation angle $\phi \in \mathbb{R}$ and of its derivatives $\dot{\phi}$ and $\ddot{\phi}$. Sketch qualitative plots of all these quantities (keep symbolic values).

b) Provide the associated expressions of the time evolution of the joint position $\boldsymbol{q} \in \mathbb{R}^3$, velocity $\dot{\boldsymbol{q}}$ and acceleration $\ddot{\boldsymbol{q}}$. Sketch qualitative plots also of these quantities.

c) Determine, as a function of the parametric data, the expression of the maximum constant speed v at which the task can be completed without violating any of the physical limits of the robot. Accordingly, give the minimum time T for completing one full round of the circle while remaining feasible.

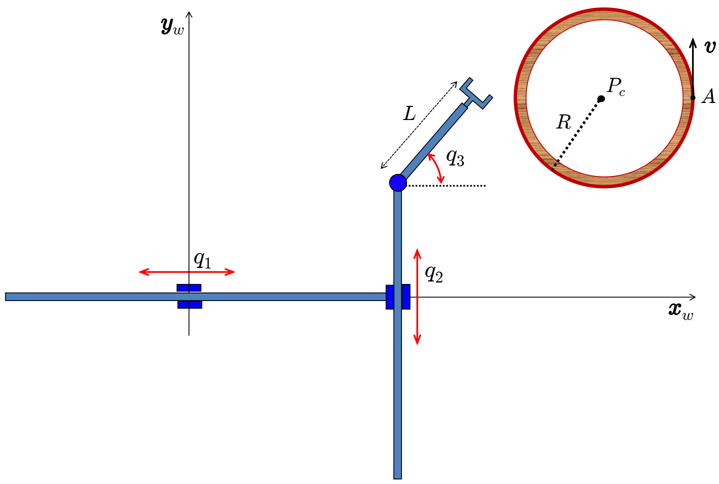

Figure 2: The planar PPR robot and the Cartesian task to be executed.

Exercise 4

A number of questions and statements are reported on the Extra Sheet #2. Fill in your answers on the same sheet, providing also a short motivation/explanation for each item. Add your name on the sheet and return it.

[240 minutes, open books]

Solution

February 12, 2020

Exercise 1

A possible Denavit-Hartenberg frame assignment for the 4-dof PRRR manipulator of Fig. 1 is shown in Fig. 3, with the associated parameters reported in Tab. 1. In the shown picture, the robot is in a configuration with a (generic) $q_1 > 0$, $q_2 = 0$, $q_3 = 0$, and $q_4 > 0$ (about 70°).

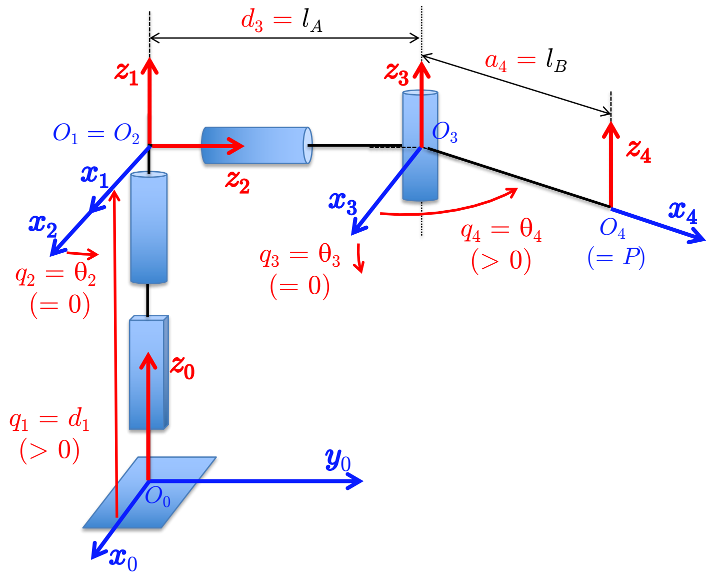

Figure 3: Assignment of DH frames for a PRRR robot.

i	α_i	a_i	d_i	θ_i
1	0	0	q_1	0
2	$-\pi/2$	0	0	q_2
3	$\pi/2$	0	$l_A > 0$	q_3
4	0	$l_B > 0$	0	q_4

Table 1: The DH table of parameters for the frame assignment in Fig. 3.

Exercise 2

Based on Tab. 1, in order to determine the position $\boldsymbol{p}$ of point P, i.e., the position of the origin O_4, we need the following DH homogenous transformation matrices:

$$
{}^0\boldsymbol{A}_1(q_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad {}^1\boldsymbol{A}_2(q_2) = \begin{pmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} {}^1\boldsymbol{R}_2(q_2) & \mathbf{0} \\ \mathbf{0}^T & 1 \end{pmatrix},
$$

$$
{}^2\boldsymbol{A}_3(q_3) = \begin{pmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & l_A \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} {}^2\boldsymbol{R}_3(q_3) & {}^2\boldsymbol{p}_3 \\ \boldsymbol{0}^T & 1 \end{pmatrix},
$$

$$
{}^3\boldsymbol{A}_4(q_4) = \begin{pmatrix} c_4 & -s_4 & 0 & l_B c_4 \\ s_4 & c_4 & 0 & l_B s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} {}^3\boldsymbol{R}_4(q_4) & {}^3\boldsymbol{p}_4(q_4) \\ \boldsymbol{0}^T & 1 \end{pmatrix},
$$

where the shorthand notations $s_i = \sin q_i$, $c_i = \cos q_i$ have been used.

The position $\boldsymbol{p}$ is computed from

$$
\boldsymbol{p}_H = \begin{pmatrix} \boldsymbol{p} \\ 1 \end{pmatrix} = {}^0\boldsymbol{A}_1(q_1) \left({}^1\boldsymbol{A}_2(q_2) \left({}^2\boldsymbol{A}_3(q_3) \left({}^3\boldsymbol{A}_4(q_4) \begin{pmatrix} \boldsymbol{0} \\ 1 \end{pmatrix} \right) \right) \right),
$$

giving

$$
\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} -l_A s_2 - l_B s_2 s_4 + l_B c_2 c_3 c_4 \\ l_A c_2 + l_B c_2 s_4 + l_B s_2 c_3 c_4 \\ q_1 - l_B s_3 c_4 \end{pmatrix}. \tag{1}
$$

Using the numerical values $l_A = 0.5$ and $l_B = 0.75$, we evaluate (1) in the two configurations $\boldsymbol{q}^I = \boldsymbol{0}$ and $\boldsymbol{q}^{II} = \begin{pmatrix} 1 & 0 & -\pi/2 & \pi/2 \end{pmatrix}^T$ yielding

$$
\boldsymbol{p}^I = \boldsymbol{f}(\boldsymbol{q}^I) = \begin{pmatrix} 0.75 \\ 0.5 \\ 0 \end{pmatrix} \text{ [m]}, \qquad \boldsymbol{p}^{II} = \boldsymbol{f}(\boldsymbol{q}^{II}) = \begin{pmatrix} 0 \\ 1.25 \\ 1 \end{pmatrix} \text{ [m]}.
$$

For the force balancing problem in static conditions, we need the joint torque

$$
\boldsymbol{\tau} = -\boldsymbol{J}_L^T(\boldsymbol{q})\boldsymbol{F}, \quad \text{with } \boldsymbol{F} = \begin{pmatrix} 0 \\ 0 \\ f_z \end{pmatrix}, \quad f_z < 0.
$$

Because of the structure of the Cartesian force $\boldsymbol{F}$, we just have to compute the last row in the 3×4 Jacobian matrix $\boldsymbol{J}_L$,

$$
\boldsymbol{J}_L(\boldsymbol{q}) = \frac{\partial \boldsymbol{p}(\boldsymbol{q})}{\partial \boldsymbol{q}} = \begin{pmatrix} \frac{\partial p_1(\boldsymbol{q})}{\partial \boldsymbol{q}} \\ \frac{\partial p_2(\boldsymbol{q})}{\partial \boldsymbol{q}} \\ \frac{\partial p_3(\boldsymbol{q})}{\partial \boldsymbol{q}} \end{pmatrix} = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ 1 & 0 & -l_B c_3 c_4 & l_B s_3 s_4 \end{pmatrix},
$$

and thus

$$
\boldsymbol{\tau} = -\boldsymbol{J}_L^T(\boldsymbol{q})\boldsymbol{F} = \begin{pmatrix} 1 \\ 0 \\ -l_B c_3 c_4 \\ l_B s_3 s_4 \end{pmatrix} |f_z|. \tag{2}
$$

Evaluating numerically (2) as before yields

$$
\boldsymbol{\tau}^I = \boldsymbol{\tau}|_{\boldsymbol{q}=\boldsymbol{q}^I} = \begin{pmatrix} 1 \\ 0 \\ -0.75 \\ 0 \end{pmatrix} |f_z|, \qquad \boldsymbol{\tau}^{II} = \boldsymbol{\tau}|_{\boldsymbol{q}=\boldsymbol{q}^{II}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -0.75 \end{pmatrix} |f_z|.
$$

The angular part of the geometric Jacobian is computed as

$$\begin{aligned}\boldsymbol{J}_A(\boldsymbol{q}) &= \begin{pmatrix} \boldsymbol{0} & \boldsymbol{z}_1 & \boldsymbol{z}_2 & \boldsymbol{z}_3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{0} & {}^0\boldsymbol{R}_1\,{}^1\boldsymbol{z}_1 & {}^0\boldsymbol{R}_1\,{}^1\boldsymbol{R}_2(q_2)\,{}^2\boldsymbol{z}_2 & {}^0\boldsymbol{R}_1\,{}^1\boldsymbol{R}_2(q_2)\,{}^2\boldsymbol{R}_3(q_1)\,{}^3\boldsymbol{z}_3 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & -s_2 & c_2 s_3 \\ 0 & 0 & c_2 & s_2 s_3 \\ 0 & 1 & 0 & c_3 \end{pmatrix}. \end{aligned} \tag{3}$$

being ${}^i\boldsymbol{z}_i = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$, for all i, and ${}^0\boldsymbol{R}_1 = \boldsymbol{I}_{3\times 3}$. It follows from (3) that the rank of matrix $\boldsymbol{J}_A(\boldsymbol{q})$ is equal to 3, except for $\sin q_3 = 0$, or $q_3 = \{0, \pi\}$ [rad]. When the matrix is singular, it becomes then

$$\boldsymbol{J}_A(q_2)|_{q_3=\{0,\pi\}} = \begin{pmatrix} 0 & 0 & -s_2 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 1 & 0 & \pm 1 \end{pmatrix}, \tag{4}$$

which has always rank equal to 2, for all q_2. Therefore, its null space (i.e., all vectors $\dot{\boldsymbol{q}} \in \mathbb{R}^4$ such that $\boldsymbol{\omega} = \boldsymbol{J}_A(q_2)|_{q_3=\{0,\pi\}}\dot{\boldsymbol{q}} = \boldsymbol{0}$) is spanned by a two-dimensional basis, e.g., by

$$\mathcal{N}\big(\boldsymbol{J}_A(q_2)|_{q_3=\{0,\pi\}}\big) = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ \pm 1 \end{pmatrix} \right\}.$$

Exercise 3

We first describe the Cartesian task in terms of absolute position $\boldsymbol{p} \in \mathbb{R}^2$, relative to the world frame RF_w, and orientation angle $\phi \in \mathbb{R}$, defined w.r.t. the $\boldsymbol{x}_w$ axis. For this, we use separation in space[1] and time:

$$\boldsymbol{p}(s) = P_c + R\begin{pmatrix} \cos s \\ \sin s \end{pmatrix}, \quad \phi(s) = \pi + s, \quad s = [0, 2\pi]; \qquad s = s(t), \quad t = [0, T].$$

Note that the angle $\phi(s)$ can also be evaluated modulo 2π (if we don't care, as here, about the number of rounds traced on the circle). Taking into account that motion along the circle should be performed at a constant speed v, the time derivative of the above quantities is given by

$$\dot{\boldsymbol{p}}(t) = \frac{d\boldsymbol{p}}{ds}\cdot\frac{ds}{dt} = \boldsymbol{p}'(s)\,\dot{s}(t) = R\begin{pmatrix} -\sin s \\ \cos s \end{pmatrix}\dot{s} \quad \Rightarrow \quad \dot{s} = \frac{v}{R} \text{ (constant, and so } \ddot{s} = 0\text{)},$$

and

$$\dot{\phi}(t) = \dot{s} = \frac{v}{R}.$$

As a consequence, $s(t) = (v/R)\,t$, and the time for completing a full circle will be $T = 2\pi R/v$. So, the choice of a maximum speed v that satisfies the robot motion constraints will provide also the minimum feasible time T. Differentiating further w.r.t. time gives

$$\ddot{\boldsymbol{p}}(t) = \boldsymbol{p}'(s)\,\ddot{s}(t) + \boldsymbol{p}''(s)\,\dot{s}^2(t) = -\begin{pmatrix} \cos s \\ \sin s \end{pmatrix}\frac{v^2}{R}, \qquad \ddot{\phi}(t) = 0.$$

[1] The used parametrization specifies s as an angle (in [rad]). One could have used equally well a parametrization in terms of the arc length $\sigma = s/R \in [0, 2\pi R]$, in [m], as the argument of the trigonometric functions. The modifications that would follow are trivial, e.g., $\dot{s} = v$ (in [m/s]).

Note that $\|\ddot{\boldsymbol{p}}(t)\| = v^2/R$, $\forall t \in [0, T]$.

With reference to Fig. 2, we have for the direct kinematics of the PPR robot

$$\begin{pmatrix} \boldsymbol{p} \\ \phi \end{pmatrix} = \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} q_1 + L\cos q_3 \\ q_2 + L\sin q_3 \\ q_3 \end{pmatrix}. \tag{5}$$

Accordingly, the first- and second-order differential maps are

$$\begin{pmatrix} \dot{\boldsymbol{p}} \\ \dot{\phi} \end{pmatrix} = \frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial \boldsymbol{q}}\,\dot{\boldsymbol{q}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}} = \begin{pmatrix} 1 & 0 & -L\sin q_3 \\ 0 & 1 & L\cos q_3 \\ 0 & 0 & 1 \end{pmatrix} \dot{\boldsymbol{q}}, \tag{6}$$

and

$$\begin{pmatrix} \ddot{\boldsymbol{p}} \\ \ddot{\phi} \end{pmatrix} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}} = \begin{pmatrix} 1 & 0 & -L\sin q_3 \\ 0 & 1 & L\cos q_3 \\ 0 & 0 & 1 \end{pmatrix} \ddot{\boldsymbol{q}} - \begin{pmatrix} L\cos q_3 \\ L\sin q_3 \\ 0 \end{pmatrix} \dot{q}_3^2 = \ddot{\boldsymbol{q}} - \begin{pmatrix} L \cdot \boldsymbol{R}(q_3) \begin{pmatrix} \dot{q}_3^2 \\ \ddot{q}_3 \end{pmatrix} \\ 0 \end{pmatrix}, \tag{7}$$

where $\boldsymbol{R}(q_3)$ is the 2×2 planar rotation matrix by an angle q_3.

Equation (5) can be easily inverted. Taking into account the parametrization of the task, we have:

$$\begin{aligned} \boldsymbol{q}(t) = \begin{pmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{pmatrix} = \begin{pmatrix} p_x(t) - L\cos\phi(t) \\ p_y(t) - L\sin\phi(t) \\ \phi(t) \end{pmatrix} &= \begin{pmatrix} P_{c,x} + R\cos s(t) - L\cos(\pi + s(t)) \\ P_{c,y} + R\sin s(t) - L\sin(\pi + s(t)) \\ \pi + s(t) \end{pmatrix} \\ &= \begin{pmatrix} P_{c,x} + (R+L)\cos s(t) \\ P_{c,y} + (R+L)\sin s(t) \\ \pi + s(t) \end{pmatrix}. \end{aligned} \tag{8}$$

Equation (8) shows that the tip of the second link (viz., the base of the third link) of the PPR robot should trace a circle of large radius $R + L$.

Taking into account the timing law, we can invert also eq. (6) as

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{J}^{-1}(\boldsymbol{q}(t)) \begin{pmatrix} \dot{\boldsymbol{p}}(t) \\ \dot{\phi}(t) \end{pmatrix} = \begin{pmatrix} -(R+L)\sin s(t) \\ (R+L)\cos s(t) \\ 1 \end{pmatrix} \frac{v}{R}.$$

Therefore, a first upper bound on v follows from the joint velocity limits:

$$|\dot{q}_1| \le V, \quad |\dot{q}_2| \le V, \quad |\dot{q}_3| \le \Omega \qquad \Rightarrow \qquad v \le v_v = \min\left\{ \frac{VR}{R+L}, \Omega R \right\}.$$

Finally, inversion of eq. (7) yields for the acceleration of the third joint

$$\ddot{q}_3(t) = \ddot{\phi}(t) = 0,$$

and thus, as a whole

$$\ddot{\boldsymbol{q}}(t) = -\begin{pmatrix} (R+L)\cos s(t) \\ (R+L)\sin s(t) \\ 0 \end{pmatrix} \left(\frac{v}{R}\right)^2.$$

A second upper bound on v follows then from the joint acceleration limits:

$$|\ddot{q}_1| \leq A, \quad |\ddot{q}_2| \leq A, \quad |\ddot{q}_3| \leq \Psi \qquad \Rightarrow \qquad v \leq v_a = \sqrt{\frac{AR^2}{R+L}}.$$

Accordingly, the maximum (constant) speed of feasible execution of the task and the associated minimum time will be

$$v_{max} = \min\{v_v, v_a\} = \min\left\{\frac{VR}{R+L}, \Omega R, \sqrt{\frac{AR^2}{R+L}}\right\}, \qquad T_{min} = \frac{2\pi R}{v_{max}}.$$

Note that the limit Ψ on the acceleration of joint 3 plays no role for the considered task.

In the following figures, we plot the time evolution of all requested quantities in the Cartesian and in the joint space of the PPR robot. In Matlab, the following data were used:

$$L = 1.5 \text{ [m]}, \quad P_c = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ [m]}, \quad R = 0.5 \text{ [m]}, \quad V = 2 \text{ [m/s]}, \quad \Omega = 2 \text{ [rad/s]}, \quad A = 6 \text{ [m/s}^2\text{]}.$$

These numerical values imply

$$v_{max} = 0.5 \text{ [m/s]}, \qquad T_{min} = \frac{\pi}{v_{max}} \simeq 6.28 \text{ [s]}.$$

Indeed, any other set of values would provide qualitatively similar results, i.e., linear quantities would be have like sine and cosine, while angular quantities would be linear, constant, or zero over time (depending on their differential order). The only difference may be on which of the given bounds would be attained in the minimum time solution.

Figures 4–6 show the time evolutions of the coordinates of the Cartesian position $\boldsymbol{p}$, linear velocity $\dot{\boldsymbol{p}}$, and linear acceleration $\ddot{\boldsymbol{p}}$, together with the evolution of the angle ϕ of the normal to the surface, pointing inward to the center of the circle, its speed $\dot{\phi}$ (here constant and $= 1$ [rad/s]) and acceleration $\ddot{\phi}$ (here $= 0$).

Figures 7–9 show the time evolutions of the joint position $\boldsymbol{q}$, joint velocity $\dot{\boldsymbol{q}}$, and joint acceleration $\ddot{\boldsymbol{q}}$, respectively. Note that the bounds that limit the task speed v to v_{max} are those on the velocities of the two prismatic joints ($\dot{q}_1$ and $\dot{q}_2$).

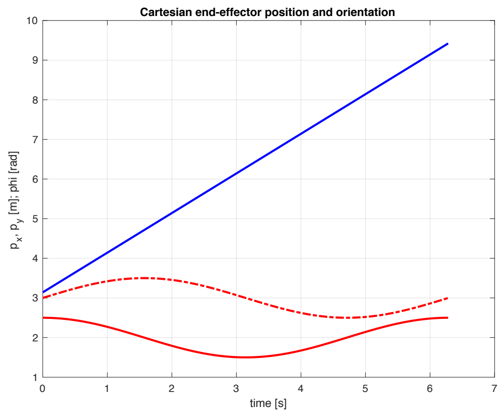

Figure 4: Cartesian position $\boldsymbol{p}(t)$ ($p_x(t)$ [red], $p_y(t)$ [red, dashed]) and angle $\phi(t)$ [blue].

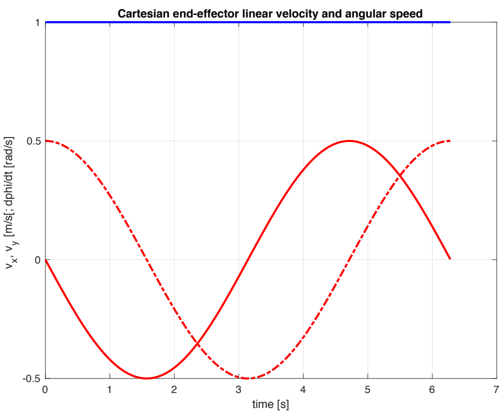

Figure 5: Cartesian linear velocity $\dot{\boldsymbol{p}}(t)$ ($v_x(t)$ [red], $v_y(t)$ [red, dashed]) and angular speed $\dot{\phi}(t)$ [blue].

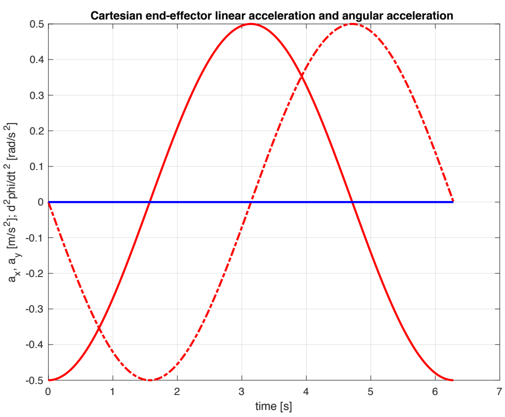

Figure 6: Cartesian linear acceleration $\ddot{\boldsymbol{p}}(t)$ ($a_x(t)$ [red], $a_y(t)$ [red, dashed]) and angular acceleration $\ddot{\phi}(t)$ [blue].

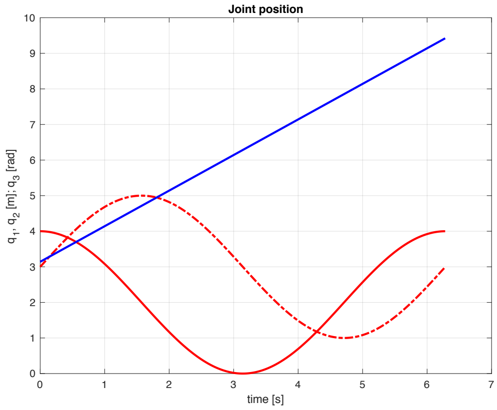

Figure 7: Joint position $\boldsymbol{q}(t)$: $q_1(t)$ [red], $q_2(t)$ [red, dashed], $q_3(t)$ [blue].

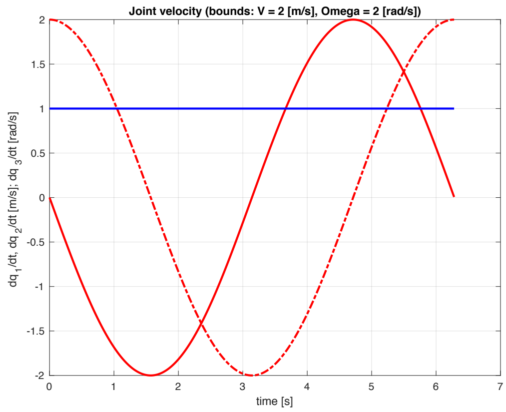

Figure 8: Joint velocity $\dot{\boldsymbol{q}}(t)$: $\dot{q}_1(t)$ [red], $\dot{q}_2(t)$ [red, dashed], $\dot{q}_3(t)$ [blue].

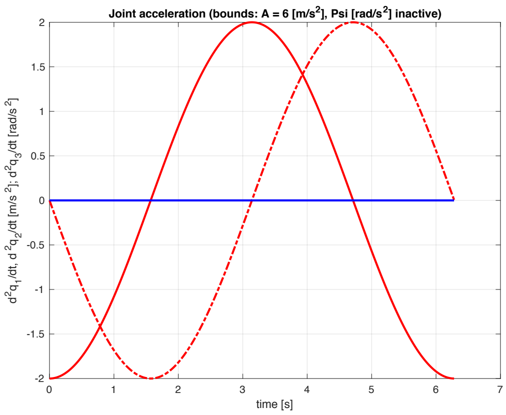

Figure 9: Joint acceleration $\ddot{\boldsymbol{q}}(t)$: $\ddot{q}_1(t)$ [red], $\ddot{q}_2(t)$ [red, dashed], $\ddot{q}_3(t)$ [blue].

Exercise 4

Answer to the questions or reply/comment on/complete the statements, providing a *short* motivation/explanation for each of the following 8 items.

1. Order the three classes of infrared, laser, and ultrasound proximity sensors in terms of their typical range of measurement.

A: In terms of increasing distance covered: infrared < ultrasound < laser.

2. Order infrared, laser, and ultrasound sensors in terms of their typical angular resolution.

A: In terms of angular resolution at the same distance: laser is better than infrared, which is in general better than ultrasound.

3. Compare the motor-side position resolution of an incremental encoder with 512 pulses per revolution (PPR) and quadrature electronics mounted on the motor with that of an absolute encoder with 16 bits mounted on the link, when the transmission has reduction ratio $n_r = 20$, Which one is better?

A: The incremental encoder has resolution $r_{IE} = 360°/(512 \cdot 4) = 0.18°$ *(as evaluated on the motor side). The resolution of the absolute encoder mounted on the link side, once reflected back to the motor side, is* $r_{AE} = (360°/2^{n_{bit}}) \cdot n_r = (360°/2^{16}) \cdot 20 = 0.11°$. *Thus, the absolute encoder has a (almost twice) better resolution.*

4. Given a desired end-effector position for a planar PPR robot, the gradient method will always provide a solution to the inverse kinematics problem without need of restarting procedures. True or false? Why?

A: True. The 2×3 *Jacobian of this robot has the structure* $\boldsymbol{J}(\boldsymbol{q}) = \begin{pmatrix} \boldsymbol{I}_{2\times 2} & \boldsymbol{j}_3(q_3) \end{pmatrix}$, *and is in fact always full rank. So, being* $\mathcal{N}\left\{\boldsymbol{J}^T(\boldsymbol{q})\right\} = \boldsymbol{0}$, *the iterations of the gradient method will always and only end with a zero end-effector position error, in one of the* ∞^1 *possible configurations that are solutions to the given inverse kinematic problem.*

5. What is the so-called overfly in trajectory planning and which are its pros and cons? Can this concept be applied equally well at the joint level and at the Cartesian level or not? Why?

A: When a sequence of position/orientation knots is assigned (equally well in the Cartesian or in the joint space), performing overfly of a knot allows the robot to getting close to it, yet without passing through it. Continuity of motion is typically gained at the expense of accurate interpolation. If the interpolating path between knots were made of linear segments, passing through the knots would typically require to stop to avoid velocity discontinuity, whereas overfly allows to keep the same motion speed.

6. We have four positional knots to be interpolated in the 3D Cartesian space, plus a number of boundary conditions and continuity requirements. Should we use 4-3-4 polynomials or cubic splines? If both can be used, which choice is better and why?

A: Both classes of functions have sufficient parameters to satisfy the boundary conditions and continuity requirements. The 4-3-4 polynomial asks for an initial and final specification of the second derivative (in space or in time, i.e., acceleration). The cubic splines can be suitably modified to handle also this case. However, cubic splines are the functions providing the minimum total curvature among all possible interpolating functions in space.

7. For a single robot joint, we have computed a spline trajectory interpolating $n = 10$ given knots at some assigned instants of time $t_1 < t_2 < \cdots < t_{10}$. If we modify only one of such time instants, but still satisfying the sequential order —e.g., the kth instant t_k becomes a new $t'_k \in (t_{k-1}, t_{k+1})$, and then redo the computations, will the trajectory change or not? Why?

A: Yes, it will change. The modification of any single or multiple data (time intervals, knot positions) affects the entire trajectory.

8. A robot commanded at the joint velocity level has initially zero position and orientation errors with respect to a desired end-effector trajectory, except along the $\boldsymbol{z}$-component in position. If we apply a Cartesian kinematic control law, the robot will move so that ...

A: . . . the initial error on the $\boldsymbol{z}$*-component of the Cartesian position is recovered at an exponential rate, while the errors on all other components remain always zero (at least in nominal conditions). This is thanks to the decoupling and exact linearization properties of the Cartesian error components achieved when using a Cartesian kinematic control law. The same is* not *true if the kinematic control law is defined on the trajectory errors in the joint space.*

* * * * *