Robotics 1

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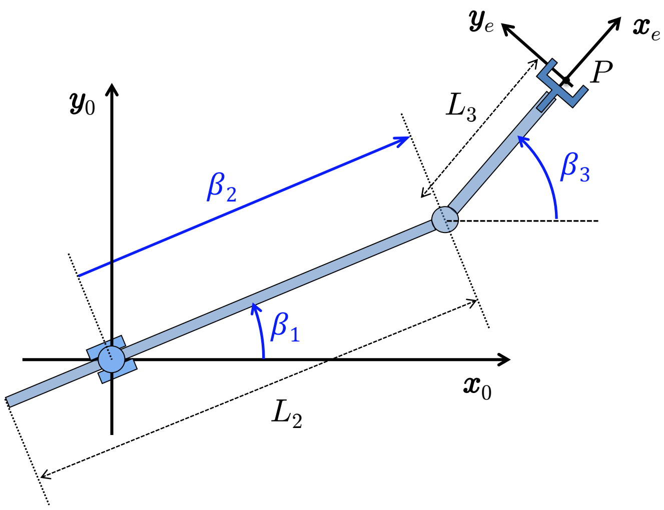

Figure 1: A 3-dof planar robot, with the definition of the joint variables β_i, $i = 1, 2, 3$.

Consider the 3-dof planar robot with one prismatic and two revolute joints shown in Fig. 1. The joint variables $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)$ are defined therein. The prismatic joint has a limited range, with $\beta_2 \in [-L_2, L_2]$, while the revolute joints are unlimited.

1. Sketch the primary workspace of this robot.
2. Compute the direct kinematics $\boldsymbol{r} = (\boldsymbol{p}, \alpha) = \boldsymbol{f}(\boldsymbol{\beta})$ for the position $\boldsymbol{p} \in \mathbb{R}^2$ of point P and the orientation $\alpha \in \mathbb{R}$ of the end-effector frame w.r.t. the $\boldsymbol{x}_0$ axis.
3. Given a value of $\boldsymbol{r} \in \mathbb{R}^3$ solve the inverse kinematics problem in analytic form, taking into account the limited range of the prismatic joint.
4. Assign the frames for this robot according to the standard Denavit-Hartenberg (DH) convention and fill in the corresponding table of parameters. Denote by $\boldsymbol{q} = (q_1, q_2, q_3)$ the DH joint variables.
5. Compute the direct kinematics of $\boldsymbol{r}$ as a function of $\boldsymbol{q}$, i.e., $\boldsymbol{r} = \boldsymbol{k}(\boldsymbol{q})$. Find the transformation between the two sets of joint variables, in its direct form $\boldsymbol{q} = \boldsymbol{t}(\boldsymbol{\beta})$ and inverse form $\boldsymbol{\beta} = \boldsymbol{t}^{-1}(\boldsymbol{q})$, such that $\boldsymbol{r} = \boldsymbol{f}(\boldsymbol{\beta}) = \boldsymbol{k}(\boldsymbol{t}(\boldsymbol{\beta}))$ or, equivalently, $\boldsymbol{r} = \boldsymbol{k}(\boldsymbol{q}) = \boldsymbol{f}\left(\boldsymbol{t}^{-1}(\boldsymbol{q})\right)$.
6. Determine the singularities of the 3×3 Jacobian $\boldsymbol{J}(\boldsymbol{q})$ in $\dot{\boldsymbol{r}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}$.
7. In a singular configuration $\boldsymbol{q}_s$, determine a basis for each of the following four subspaces of $\mathbb{R}^3$: $\mathcal{R}(\boldsymbol{J}(\boldsymbol{q}_s))$, $\mathcal{N}(\boldsymbol{J}(\boldsymbol{q}_s))$, $\mathcal{R}(\boldsymbol{J}^T(\boldsymbol{q}_s))$, and $\mathcal{N}(\boldsymbol{J}^T(\boldsymbol{q}_s))$.
8. Let $L_2 = L_3 = L$. Plan a rest-to-rest trajectory in time T between $\boldsymbol{r}(0) = (L/2, L/2, \pi/2)$ and $\boldsymbol{r}(T) = (-L/2, -L/2, -\pi/2) (= -\boldsymbol{r}(0))$ without violating the joint limits. Is it possible to follow a linear Cartesian path in this case?

[180 minutes, open books]