Robotics II

February 12, 2020

Exercise 1

Consider the 3-dof planar robot in Fig. 1, with one prismatic and two revolute joints, moving in the vertical plane. The coordinates $\boldsymbol{q}$ to be used are defined in the figure. Each link of the robot has uniformly distributed mass $m_i > 0$, $i = 1, 2, 3$, with center of mass on its physical link axis, and a purely diagonal barycentric link inertia matrix. The prismatic joint range is limited as $q_2 \in [q_{2,min}, q_{2,max}]$, with $0 < q_{2,min} < q_{2,max}$. The robot is commanded by a generalized vector of joint forces/torques $\boldsymbol{\tau} \in \mathbb{R}^3$.

a) Derive the robot inertia matrix $\boldsymbol{M}(\boldsymbol{q}) > 0$.

b) Derive the gravity term $\boldsymbol{g}(\boldsymbol{q})$ and find all free equilibrium configurations of the robot.

c) Provide a linear parametrization of $\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}_g(\boldsymbol{q})\,\boldsymbol{a}_g$, in terms of a vector $\boldsymbol{a}_g \in \mathbb{R}^p$ of unknown dynamic coefficients and a $3 \times p$ regressor matrix $\boldsymbol{Y}_g(\boldsymbol{q})$. Assume that the gravity acceleration is known, $g_0 = 9.81$ [m/s^2]. Discuss the minimality of p.

d) Determine which of the 9 non-zero inertia parameters of the three links are irrelevant for the describing the motion of the robot.

e) Provide an upper bound $\alpha > 0$ for the norm of the gradient of the gravity vector, $\|\partial \boldsymbol{g}(\boldsymbol{q})/\partial \boldsymbol{q}\| < \alpha$ for all *feasible* $\boldsymbol{q}$, expressed in terms of the dynamic parameters of the robot.

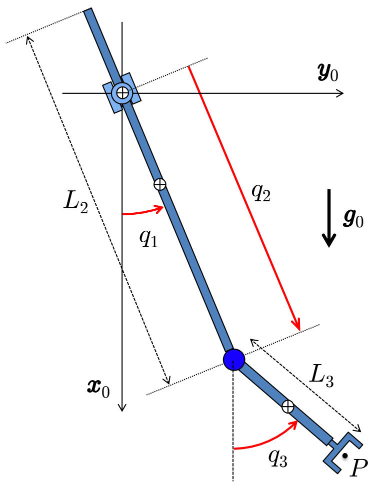

Figure 1: A planar RPR robot, with the definition of the coordinates to be used $\boldsymbol{q} = (q_1\ q_2\ q_3)^T$.

Exercise 2

Consider the same planar RPR robot as in Exercise 1. Assume that the robot can be commanded directly by the generalized vector of joint velocities $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ defined in Fig. 1, thanks to a low-level control action that guarantees their accurate reproduction. For a desired smooth motion of the end-effector position $\boldsymbol{p} = \boldsymbol{p}_d(t)$ of its end-effector point P, provide the explicit expression of the instantaneous joint velocity command that executes the Cartesian motion while minimizing $\frac{1}{2}\|\dot{\boldsymbol{q}}\|^2$. Modify this scheme in order to keep possibly the prismatic joint close to the center of its limited range $[q_{2,min}, q_{2,max}]$, by using two alternative methods: weighted pseudoinversion and projected gradient in the null space.

[turn for the next exercise]

Exercise 3

With reference to Fig. 2, we consider a control problem for a mechanical system made by a first mass $m_r > 0$, representing the robot moved by a force F, and a second mass $m_e > 0$, anchored to a rigid wall by a spring of stiffness $k_e > 0$, representing as a whole a compliant dynamic environment. When in contact, the two masses are connected by another spring of stiffness $k_s > 0$, representing a force sensor. The positional coordinates x_r and x_e of the two masses have their respective zero reference when the system has no stored elastic energy, i.e., when there is no compression (nor extension) of the two springs. As a result, the force measured by the sensor is $F_s = k_s(x_r - x_e) \geq 0$, when $x_r \geq x_e$ (the robot is in contact), or $F_s = 0$, when $x_r < x_e$ (no contact).

We would like to regulate the contact force to a constant value $F_d > 0$ by means of four alternative feedback/feedforward schemes $F = F_i$, $i = 1, \ldots, 4$, defined as follows:

$$F_1 = k_1(F_d - F_s), \qquad P \text{ control}, \tag{1}$$

$$F_2 = F_d + k_2(F_d - F_s), \qquad P\text{+}\mathit{ffw} \text{ control}, \tag{2}$$

$$F_3 = k_3 \int (F_d - F_s)\, dt, \qquad I \text{ control}, \tag{3}$$

$$F_4 = F_d + k_4 \int (F_d - F_s)\, dt, \qquad I\text{+}\mathit{ffw} \text{ control}, \tag{4}$$

for suitable choices of $k_i > 0$, $i = 1, \ldots, 4$.

a) Determine the dynamic model of the open-loop system in the two situations $x_r(t) \geq x_e(t)$ (contact) and $x_r(t) < x_e(t)$ (no contact).

b) For each of the control laws (1) to (4), provide the equilibrium conditions of the closed-loop system in terms of positions and contact forces. Which schemes satisfy zero force error at the equilibrium? Are the equilibria unique in each case?

c) Is there any case in which a steady-state condition is not reached? Using any preferred analysis technique (e.g., Lyapunov-based, or Routh criterion in the Laplace domain, or even qualitatively), study the asymptotic stability of the closed-loop equilibria for at least two of the control laws.

d) Determine the initial motion of mass m_r under the action of the different control laws, when starting with $x_r(0) < x_e(0)$ and with the system at rest. What problem would be encountered during the non-contact phase and how could this be milden/resolved by the addition, when needed, of a damping term $-d_v \dot{x}_r$, $d_v > 0$, in the control law?

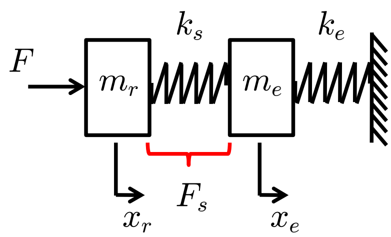

Figure 2: The model of a mechanical system used for the design of force control laws.

[210 minutes, open books
(but no internet, no smartphone, and no communication with others!)]