Robotics 2

October 19, 2021

Exercise #1

The DLR Justin robot in Fig. 1 has a torso with three revolute joints, with joint variables $\boldsymbol{q}_t \in \mathbb{R}^3$, on which two 7R identical left and right arms are mounted, with joint variables $\boldsymbol{q}_l \in \mathbb{R}^7$ and $\boldsymbol{q}_r \in \mathbb{R}^7$, respectively. The robot has thus a total of 17 dofs, neglecting those of the end effectors (two anthropomorphic hands), with a perfectly symmetric body structure. Assume that the robot is commanded by the joint velocity $\dot{\boldsymbol{q}} = (\dot{\boldsymbol{q}}_l, \dot{\boldsymbol{q}}_r, \dot{\boldsymbol{q}}_t) \in \mathbb{R}^{17}$. Consider the following kinematic control problems and introduce in a symbolic way all kinematic terms (matrices, vectors, etc.) that you need for the definition of the control laws.

Figure 1: The DLR Justin robot, a torso with bimanual arms.

a) Let the Cartesian motion tasks for the left and right end effectors be specified in an independent way in terms of a desired position and orientation trajectory $\boldsymbol{r}_{l,d}(t) \in \mathbb{R}^6$ and $\boldsymbol{r}_{r,d}(t) \in \mathbb{R}^6$, respectively. A minimal representation is used for the orientations. Both trajectories are expressed in the base frame of the torso. Define a control strategy $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_a(t)$ for all the robot joints that is able to execute simultaneously the two tasks, whenever possible, recovering also from any (initial or later) Cartesian error and keeping the joints close to their midrange. Specify qualitatively the conditions for the feasibility of these tasks.

b) Is it possible to execute the complete bimanual task as in a), but shaping the solution so that the motion of the torso joints is largely reduced? And blocking completely the motion of the torso joints? Provide the associated control laws $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_b(t)$ and $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_{bb}(t)$, and discuss which may be the difficulties possibly encountered.

c) Consider the same task $\boldsymbol{r}_{l,d}(t) \in \mathbb{R}^6$ as in a) for the left end effector. The right end effector should move now in coordination with the left one, as specified by a desired relative position $\boldsymbol{p}_{lr,d}(t)$ and orientation ${}^{l}\boldsymbol{R}_{r,d}(t)$, both expressed with respect to the frame of the left end effector. Define a control strategy $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_c(t)$ for all the robot joints that addresses this new composite task for Justin, recovering also from any error during task execution.

d) What if the command is moved to the acceleration level, with $\ddot{\boldsymbol{q}} = (\ddot{\boldsymbol{q}}_l, \ddot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_t) \in \mathbb{R}^{17}$? Provide an extension $\ddot{\boldsymbol{q}} = \ddot{\boldsymbol{q}}_d(t)$ for the case in a) that prevents also the internal build up of joint velocities.

Exercise #2

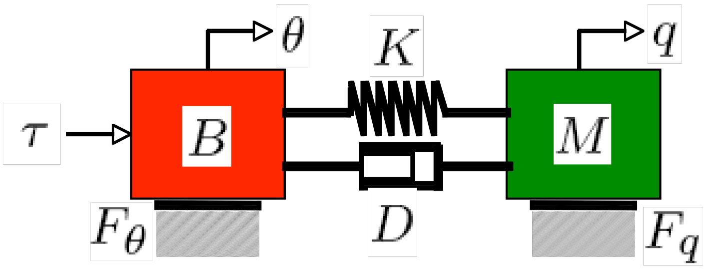

Figure 2: Two masses connected by a visco-elastic spring.

Figure 2 shows a mechanical system made of two masses B and M with a visco-elastic coupling, viscous friction on the motion of the individual masses, and an input force τ acting on the first mass. The zero of the two position variables θ and q is associated to an undeformed spring. The spring has stiffness $K > 0$ and its elastic potential energy is quadratic in the deformation $q - \theta$. This model represents also a visco-elastic joint of a robot, where B and M are, respectively, the motor and the link inertia, while θ and q are their respective (angular) positions.

a) Derive the dynamic model of this system, including all non-conservative terms due to the viscous friction affecting the motion of the two masses (with coefficients $F_\theta > 0$ and $F_q > 0$) and to the viscous damping on the deformation velocity of the spring (with coefficient $D \geq 0$).

b) Provide the simplest feedback law that is able to asymptotically stabilize the position of the second mass to a constant desired value q_d. Prove the result using a Lyapunov/LaSalle technique (or any other preferred method).

c) Set now $D = 0$. Solve the inverse dynamics problem for a desired, sufficiently smooth trajectory $q_d(t)$. Provide the explicit expression of $\tau_d(t)$ as a function of $q_d(t)$ and its (higher order) time derivatives only.

[150 minutes (2.5 hours); open books]