Robotics 2

July 10, 2023

Exercise 1

A robot with n degrees of freedom and dynamics (with no gravity)

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau}$$

is redundant with respect to a m-dimensional task ($m < n$) described at the second-order differential level by

$$\ddot{\boldsymbol{y}} = \boldsymbol{J}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}},$$

where the $m \times n$ task Jacobian $\boldsymbol{J}$ is assumed to be full row rank. In the redundant case, the joint torque $\boldsymbol{\tau} \in \mathbb{R}^n$ can always be decomposed as

$$\boldsymbol{\tau} = \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{F} + \left(\boldsymbol{I} - \boldsymbol{J}^T(\boldsymbol{q})\boldsymbol{H}(\boldsymbol{q})\right)\boldsymbol{\tau}_0,$$

where $\boldsymbol{F} \in \mathbb{R}^m$ is the task-space generalized force performing work on $\dot{\boldsymbol{y}}$, matrix $\boldsymbol{H}$ is any generalized inverse of $\boldsymbol{J}^T$ (i.e., such that $\boldsymbol{J}^T\boldsymbol{H}\boldsymbol{J}^T = \boldsymbol{J}^T$), and $\boldsymbol{\tau}_0 \in \mathbb{R}^n$.

With the robot in the state $(\boldsymbol{q},\dot{\boldsymbol{q}})$, prove the following two statements.

a) In order for an arbitrary $\boldsymbol{\tau}_0 \neq \boldsymbol{0}$ not to produce any task acceleration ($\ddot{\boldsymbol{y}} = \boldsymbol{0}$), the only choice for $\boldsymbol{H}$ is

$$\boldsymbol{H}(\boldsymbol{q}) = \left(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})\right)^{-1}\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q}), \tag{1}$$

namely the weighted pseudoinverse of $\boldsymbol{J}^T$, with the inverse of the robot inertia as weight.

b) Based on (1), the m-dimensional dynamic model of the robot in the task space is given by

$$\boldsymbol{M}_{\boldsymbol{y}}(\boldsymbol{q})\ddot{\boldsymbol{y}} + \boldsymbol{c}_{\boldsymbol{y}}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{F} \tag{2}$$

with the $m \times m$ task-space inertia matrix $\boldsymbol{M}_{\boldsymbol{y}}$ and the task-space Coriolis and centrifugal terms $\boldsymbol{c}_{\boldsymbol{y}}$ given respectively by

$$\boldsymbol{M}_{\boldsymbol{y}}(\boldsymbol{q}) = \left(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q})\right)^{-1}, \quad \boldsymbol{c}_{\boldsymbol{y}}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{M}_{\boldsymbol{y}}(\boldsymbol{q})\left(\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \dot{\boldsymbol{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}\right).$$

Exercise 2

Consider the 3-dof planar robot in Fig. 1, with one prismatic and two revolute joints, moving in a vertical plane. The coordinates $\boldsymbol{q}$ to be used are defined in the figure. Each link of the robot has uniformly distributed mass $m_i > 0$, $i = 1, 2, 3$, with center of mass on its geometric axis, and a diagonal barycentric inertia matrix. The prismatic joint has a limited range $q_2 \in [-L_2, L_2]$, while the revolute joints are unlimited. The robot is commanded by a joint force/torque $\boldsymbol{\tau} \in \mathbb{R}^3$.

a) Derive the robot inertia matrix $\boldsymbol{M}(\boldsymbol{q})$.

b) Derive the gravity term $\boldsymbol{g}(\boldsymbol{q})$ and find all unforced equilibrium configurations (i.e., with $\boldsymbol{\tau} = \boldsymbol{0}$).

c) Assume that the gravity acceleration g_0 and the kinematic quantities L_2 and L_3 are known, while all other dynamic parameters are unknown. Provide a linear parametrization of the gravity vector $\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}_{\boldsymbol{g}}(\boldsymbol{q})\,\boldsymbol{a}_g$, in terms of a vector $\boldsymbol{a}_g \in \mathbb{R}^p$ of unknown dynamic coefficients and a $3 \times p$ regressor matrix $\boldsymbol{Y}_{\boldsymbol{g}}(\boldsymbol{q})$. Discuss the minimality of p.

d) Provide a symbolic expression (in terms of the robot dynamic parameters and joint limits) of a constant upper bound $\alpha > 0$ for the norm of the gradient of the gravity vector, i.e., such that $\|\partial \boldsymbol{g}(\boldsymbol{q})/\partial \boldsymbol{q}\| \leq \alpha$ for all feasible $\boldsymbol{q}$.

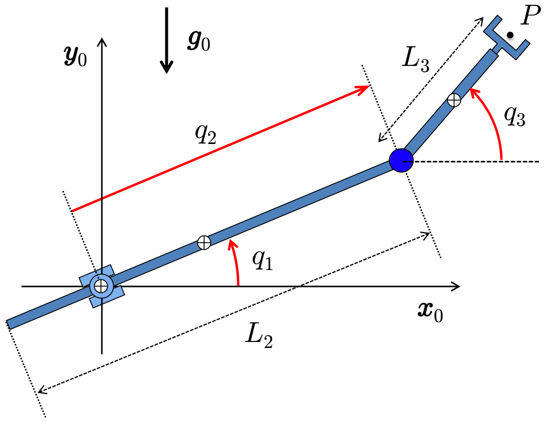

Figure 1: A planar RPR robot, with the definition of the coordinates to be used $\boldsymbol{q} = (q_1\ q_2\ q_3)^T$.

Exercise 3

Consider the robotic task of inserting a sphere in a cylindrical hole having the same size (zero clearance), as shown in Fig. 2. Assuming rigid and frictionless contacts, define a task frame, the natural constraints imposed by the geometry on the generalized velocity/force quantities expressed in this task frame, and the artificial constraints that can be taken as reference values by a hybrid force-velocity control law for the execution of this sphere-in-hole task with minimum effort.

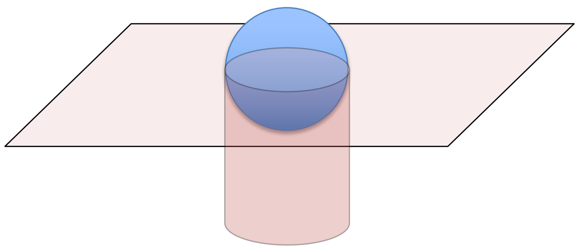

Figure 2: Sphere-in-hole task.

Provide a basis for the space of admissible twists $\boldsymbol{V} = (\boldsymbol{v}^T \boldsymbol{\omega}^T)^T \in \mathbb{R}^6$ and a complementary basis for the space of reaction wrenches $\boldsymbol{F} = (\boldsymbol{f}^T \boldsymbol{m}^T)^T \in \mathbb{R}^6$. Discuss how measurements that are inconsistent with the geometric model are being handled by an hybrid force-velocity control law, and give two examples of such inconsistent measurements, one related to motion and one related to interaction.

[180 minutes; open books]