*Robotics 2*

# Linear parametrization and identification of robot dynamics

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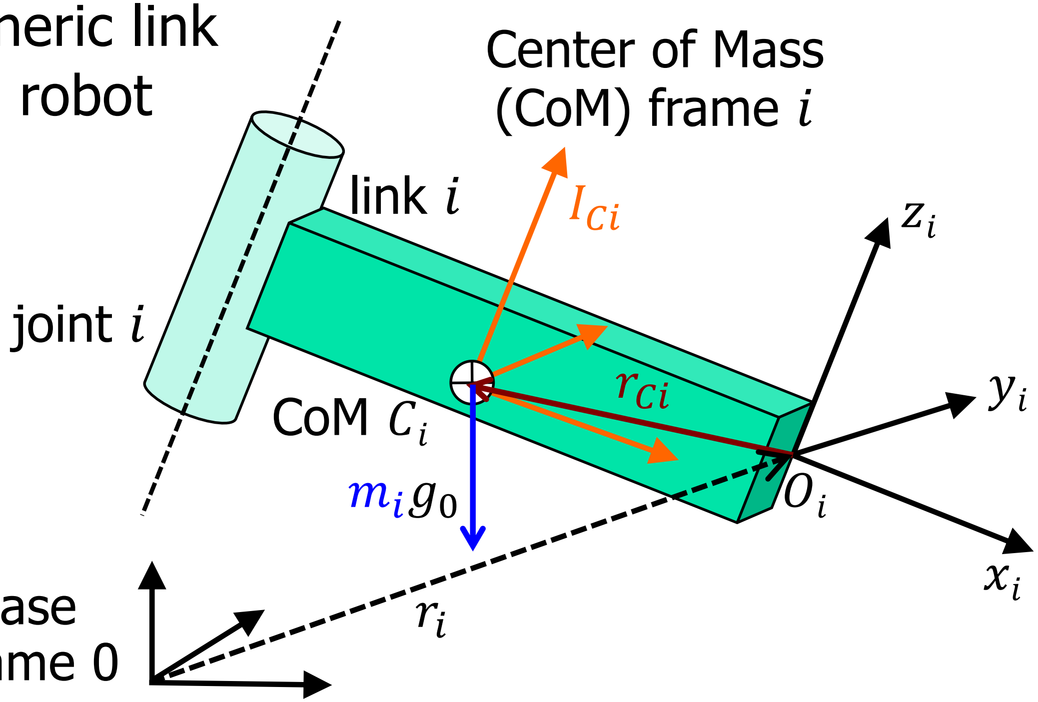
# Dynamic parameters of robot links

- consider a generic link of a fully rigid robot




- each link is characterized by 10 dynamic parameters

$$\left\{ \quad m_i \quad r_{Ci} = \begin{pmatrix} r_{xi} \\ r_{yi} \\ r_{zi} \end{pmatrix} \quad I_{Ci} = \begin{pmatrix} I_{Ci,xx} & I_{Ci,xy} & I_{Ci,xz} \\ & I_{Ci,yy} & I_{Ci,yz} \\ \text{symm} & & I_{Ci,zz} \end{pmatrix} \right.$$

- however, the robot dynamics depends in a nonlinear way on some of these parameters (e.g., through the combination $I_{ci,zz} + m_i r_{xi}^2$)

# Dynamic parameters of robots

- kinetic energy and gravity potential energy can both be rewritten so that a new set of dynamic parameters appears only in a linear way
  - need to re-express link inertia and CoM position in (any) known kinematic frame attached to the link (same orientation as the barycentric frame)
- fundamental kinematic relation

$$v_{ci} = v_i + \omega_i \times r_{Ci} = v_i + S(\omega_i)\, r_{Ci} = v_i - S(r_{Ci})\, \omega_i$$

- kinetic energy of link $i$

$$\begin{aligned} T_i &= \frac{1}{2} m_i v_{Ci}^T v_{Ci} + \frac{1}{2} \omega_i^T I_{Ci} \omega_i \\ &= \frac{1}{2} m_i (v_i - S(r_{Ci})\omega_i)^T (v_i - S(r_{Ci})\omega_i) + \frac{1}{2} \omega_i^T I_{Ci} \omega_i \\ &= \frac{1}{2} m_i v_i^T v_i + \frac{1}{2} \omega_i^T \big(\underbrace{I_{Ci} + m_i S^T(r_{Ci}) S(r_{Ci})}_{\text{Steiner theorem}}\big)\omega_i - v_i^T S(m_i r_{Ci})\omega_i \end{aligned}$$

$$I_i = \begin{pmatrix} I_{i,xx} & I_{i,xy} & I_{i,xz} \\ & I_{i,yy} & I_{i,yz} \\ \text{symm} & & I_{i,zz} \end{pmatrix}$$

# Standard dynamic parameters of robots

- gravitational potential energy of link $i$

$$U_i = -m_i g_0^T r_{0,Ci} = -m_i g_0^T (r_i + r_{Ci}) = -m_i g_0^T r_i - g_0^T (m_i r_{Ci})$$

- by expressing vectors and matrices in frame $i$, both $T_i$ and $U_i$ are linear in the set of 10 (constant) standard parameters $\pi_i \in \mathbb{R}^{10}$

$$T_i = \frac{1}{2} m_i \, {}^i v_i^T \, {}^i v_i + m_i \, {}^i r_{Ci}^T S({}^i v_i) \, {}^i \omega_i + \frac{1}{2} \, {}^i \omega_i^T \, {}^i I_i \, {}^i \omega_i$$

$$U_i = -m_i g_0^T r_i - g_0^T \, {}^0 R_i \, (m_i \, {}^i r_{Ci})$$

$$\boldsymbol{\pi}_i = \begin{pmatrix} m_i \\ m_i \, {}^i r_{Ci} \\ vect\{{}^i I_i\} \end{pmatrix} = (\underbrace{m_i}_{\substack{\text{mass of link } i \\ \text{(0-th order moment )}}} \quad \underbrace{m_i \, {}^i r_{Ci,x} \quad m_i \, {}^i r_{Ci,y} \quad m_i \, {}^i r_{Ci,z}}_{\substack{\text{mass} \times \text{CoM position of link } i \\ \text{(1-st order moment )}}} \quad \underbrace{{}^i I_{i,xx} \quad {}^i I_{i,xy} \quad {}^i I_{i,xz} \quad {}^i I_{i,yy} \quad {}^i I_{i,yz} \quad {}^i I_{i,zz}}_{\substack{\text{inertia of link } i \\ \text{(2-nd order moment )}}})^T$$

- since the E-L equations involve only linear operations on $T$ and $U$, also the robot dynamic model is linear in the standard parameters $\boldsymbol{\pi} \in \mathbb{R}^{10N}$

# Linearity in the dynamic parameters

- using a $N \times 10N$ regression matrix $Y_\pi$ that depends only on kinematic quantities, the robot dynamic equations can always be rewritten linearly in the standard dynamic parameters as

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = Y_\pi(q,\dot{q},\ddot{q})\,\pi = u$$

$$\pi^T = \begin{pmatrix} \pi_1^T & \pi_2^T & \cdots & \pi_N^T \end{pmatrix}$$

- the open kinematic chain structure of the manipulator implies that the $i$-th dynamic equation can depend only on the standard dynamic parameters of links $i$ to $N$ $\Rightarrow$ $Y_\pi$ has a block upper triangular structure

$$Y_\pi(q,\dot{q},\ddot{q}) = \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ 0 & Y_{22} & \cdots & Y_{2N} \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & Y_{NN} \end{pmatrix}$$

with row vectors $Y_{i,j}$ of size $1\times 10$

Property: element $m_{ij}$ of $M(q)$ is a function at most of $(q_{k+1}, \cdots, q_N)$, for $k = \min\{i,j\}$ , and of the inertial parameters of at most links $r$ to $N$, with $r = \max\{i,j\}$

# Linearity in the dynamic coefficients

- many standard parameters do not appear ("play no role") in the dynamic model of a given robot ⇒ the associated columns of $Y_\pi$ are 0!
- some standard parameters may appear only in fixed combinations with others ⇒ the associated columns of $Y_\pi$ are linearly dependent!
- one can isolate $p \ll 10N$ independent groups of parameters $\pi$ (associated to $p$ functionally independent columns $Y_{indep}$ of $Y_\pi$) and partition matrix $Y_\pi$ in two blocks, the second containing dependent (or zero) columns as $Y_{dep} = Y_{indep}T$, for a suitable constant $p \times (10N - p)$ matrix $T$

$$Y_\pi(q,\dot{q},\ddot{q})\,\pi = \begin{pmatrix} Y_{indep} & Y_{dep} \end{pmatrix} \begin{pmatrix} \pi_{indep} \\ \pi_{dep} \end{pmatrix} = \begin{pmatrix} Y_{indep} & Y_{indep}T \end{pmatrix} \begin{pmatrix} \pi_{indep} \\ \pi_{dep} \end{pmatrix}$$

$$= Y_{indep}\left(\pi_{indep} + T\,\pi_{dep}\right) = \boxed{Y(q,\dot{q},\ddot{q})\,a}$$

- these grouped parameters are called dynamic coefficients $a \in \mathbb{R}^p$, "the only that matter" in robot dynamics (= base parameters by W. Khalil)
- the minimal number $p$ of dynamic coefficients that is needed can also be checked numerically (see later → Identification)

# Linear parametrization of robot dynamics

it is always possible to rewrite the dynamic model in the form

regression matrix — $a$ = vector of dynamic coefficients

$$\boxed{M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \underbrace{Y(q,\dot{q},\ddot{q})}_{N \times p}\,\underbrace{a}_{p \times 1} = u}$$

e.g., the heuristic grouping (found by inspection) on a 2R planar robot

$$\begin{pmatrix} \ddot{q}_1 & c_2(2\ddot{q}_1+\ddot{q}_2) - s_2(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) & \ddot{q}_2 & c_1 & c_{12} \\ 0 & c_2\ddot{q}_1 + s_2\dot{q}_1^2 & \ddot{q}_1+\ddot{q}_2 & 0 & c_{12} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{aligned} a_1 &= I_{c1,zz} + m_1 d_1^2 + I_{c2,zz} + m_2 d_2^2 + m_2 l_1^2 \\ a_2 &= m_2 l_1 d_2 \\ a_3 &= I_{c2,zz} + m_2 d_2^2 \\ a_4 &= g_0(m_1 d_1 + m_2 l_1) \\ a_5 &= g_0 m_2 d_2 \end{aligned}$$

**NOTE:** 4 more coefficients are added when including the coefficients $F_{V,i}$ and $F_{C,i}$ of viscous and Coulomb friction at the joints ("decoupled" terms appearing only in the relative equations $i = 1,2$)

# Linear parametrization
## of a 2R planar robot ($N = 2$)

- being the kinematics known (i.e., $l_1$ and $g_0$), the number of dynamic coefficients can be reduced since we can merge the two coefficients

$$a_2 = m_2 l_1 d_2 \;\;\&\;\; a_5 = g_0 m_2 d_2 \quad \Rightarrow \quad a_2 = m_2 d_2 \quad \text{(factoring out } l_1 \text{ and } g_0\text{)}$$

- therefore, after regrouping, $\boldsymbol{p = 4}$ dynamic coefficients are sufficient

$$\begin{pmatrix} \ddot{q}_1 & l_1 c_2 (2\ddot{q}_1 + \ddot{q}_2) - l_1 s_2 (\dot{q}_2^2 + 2\dot{q}_1 \dot{q}_2) + g_0 c_{12} & \ddot{q}_2 & g_0 c_1 \\ 0 & l_1 (c_2 \ddot{q}_1 + s_2 \dot{q}_1^2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = Y\, a = u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$a_1 = I_{c1,zz} + m_1 d_1^2 + I_{c2,zz} + m_2 d_2^2 + m_2 l_1^2 \qquad a_3 = I_{c2,zz} + m_2 d_2^2$$

$$a_2 = m_2 d_2 \qquad a_4 = m_1 d_1 + m_2 l_1$$

- this (minimal) linear parametrization of robot dynamics is not unique, both in terms of the chosen set of dynamic coefficients $a$ and for the associated regression matrix $Y$
  - a systematic procedure for its derivation would be preferable

# Linear parametrization
## of a 2R planar robot ($N = 2$)

- as alternative to the previous heuristic method, apply the general procedure
  - $10N = 20$ standard parameters are defined for the two links
  - from the assumptions made on CoM locations, only 5 such parameters actually appear, namely (with $d_i = r_{ci,x}$)

link 1: $m_1 d_1 \quad I_{1,zz} = I_{c1,zz} + m_1 d_1^2$ link 2: $m_2 \quad m_2 d_2 \quad I_{2,zz} = I_{c2,zz} + m_2 d_2^2$

  - in the $2{\times}5$ matrix $Y_\pi$, the 3rd column (associated to $\boxed{m_2}$) is $Y_{\pi 3} = Y_{\pi 1} l_1 + Y_{\pi 2} l_1^2$
  - after regrouping/reordering, $\boldsymbol{p} = \mathbf{4}$ dynamic coefficients are again sufficient

$$\begin{pmatrix} g_0 c_1 & \ddot{q}_1 & l_1 c_2(2\ddot{q}_1 + \ddot{q}_2) - l_1 s_2(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 \\ 0 & 0 & l_1(c_2\ddot{q}_1 + s_2\dot{q}_1^2) + g_0 c_{12} & \ddot{q}_1 + \ddot{q}_2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = Y\,a = u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$a_1 = m_1 d_1 + \boxed{m_2} l_1 \qquad a_2 = I_{1,zz} + \boxed{m_2} l_1^2 = \left(I_{c1,zz} + m_1 d_1^2\right) + m_1 l_1^2 \qquad a_3 = m_2 d_2 \qquad a_4 = I_{2,zz} = I_{c2,zz} + m_2 d_2^2$$

- determining a minimal parameterization (i.e., minimizing $p$) is important for
  - experimental identification of dynamic coefficients
  - adaptive/robust control design in the presence of uncertain parameters

# Identification of dynamic coefficients

- in order to "use" the model, one needs to know the numeric values of the robot dynamic coefficients
  - robot manufacturers provide at most only a few principal dynamic parameters (e.g., link masses)
- estimates can be found with CAD tools (e.g., assuming uniform mass)
- friction coefficients are (slowly) varying over time
  - lubrication of joints/transmissions
- for an added payload (attached to the E-E)
  - a change in the 10 dynamic parameters of last link
  - this implies a variation of (almost) all robot dynamic coefficients
- preliminary identification experiments are needed
  - robot in motion (dynamic issues, not just static or geometric ones!)
  - only the robot dynamic coefficients can be identified (and not all the link standard parameters!)

# Identification experiments

1. choose a motion trajectory $q_d(t)$ that is sufficiently "exciting", i.e.,
   - explores the robot workspace and involves all components in the robot dynamic model
   - is periodic, with multiple frequency components
2. execute this motion (approximately) by means of a control law
   - taking advantage of any available information on the robot model
   - often $u = K_P(q_d - q) + K_D(\dot{q}_d - \dot{q})$ (PD, no model information used)
3. measure $q$ (encoders) in $n_c$ time instants (and, if available, also $\dot{q}$)
   - joint velocity $\dot{q}$ and acceleration $\ddot{q}$ can be later estimated off line by numerical differentiation (use of non-causal filters is feasible)
4. with such measures/estimates, evaluate the regression matrix $Y$ (on the left) and use the applied commands $u$ (on the right) in the expression

$$Y\big(q(t_k), \dot{q}(t_k), \ddot{q}(t_k)\big)\, a = u(t_k) \qquad k = 1, \cdots, n_c$$

# Least Squares (LS) identification

- set up the system of linear equations

$$n_c \times N \left\{ \begin{pmatrix} Y\big(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)\big) \\ \vdots \\ Y\big(q(t_{n_c}), \dot{q}(t_{n_c}), \ddot{q}(t_{n_c})\big) \end{pmatrix} a = \begin{pmatrix} u(t_1) \\ \vdots \\ u(t_{n_c}) \end{pmatrix} \right. \quad \Leftrightarrow \quad \boxed{\bar{Y} a = \bar{u}}$$

- sufficiently "exciting" trajectories, large enough number of samples ($n_c \times N \gg p$), and their suitable selection/position, guarantee rank($\bar{Y}$) $= p$ (full column rank)
- solution by pseudoinversion of matrix $\bar{Y}$

$$a = \bar{Y}^{\#}\bar{u} = (\bar{Y}^T\bar{Y})^{-1}\bar{Y}^T\bar{u} \quad (\in \mathbb{R}^p)$$

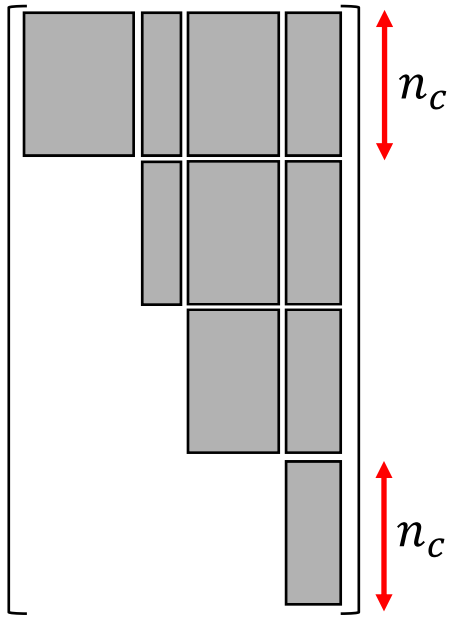
- one can also use a weighted pseudoinverse, to take into account different levels of noise in the collected measures

# Additional remarks on LS identification

- it is convenient to preserve the block (upper) triangular structure of the regression matrix, by "stacking" all time evaluations in row by row sequence of the original $Y$ matrix

$$N\times\left\{\begin{matrix} n_c \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \\ \\ \\ n_c \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right. \end{matrix}\right. \begin{pmatrix} Y_1\big(q(t_1),\dot{q}(t_1),\ddot{q}(t_1)\big) \\ \vdots \\ Y_1\big(q(t_{n_c}),\dot{q}(t_{n_c}),\ddot{q}(t_{n_c})\big) \\ Y_2\big(q(t_1),\dot{q}(t_1),\ddot{q}(t_1)\big) \\ \vdots \\ Y_2\big(q(t_{n_c}),\dot{q}(t_{n_c}),\ddot{q}(t_{n_c})\big) \\ \vdots \\ Y_N\big(q(t_1),\dot{q}(t_1),\ddot{q}(t_1)\big) \\ \vdots \\ Y_N\big(q(t_{n_c}),\dot{q}(t_{n_c}),\ddot{q}(t_{n_c})\big) \end{pmatrix} a = \begin{pmatrix} u_1(t_1) \\ \vdots \\ u_1(t_{n_c}) \\ u_2(t_1) \\ \vdots \\ u_2(t_{n_c}) \\ \vdots \\ u_N(t_1) \\ \vdots \\ u_N(t_{n_c}) \end{pmatrix} \Leftrightarrow \boxed{\bar{Y}a = \bar{u}}$$

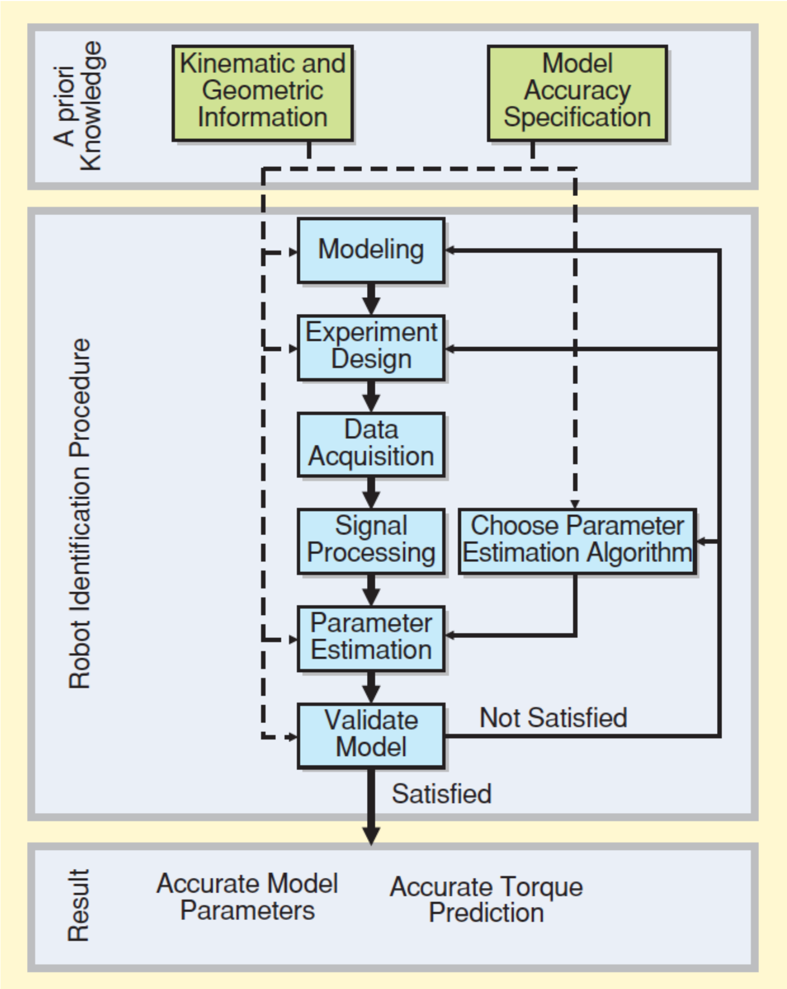
$\bar{Y} =$



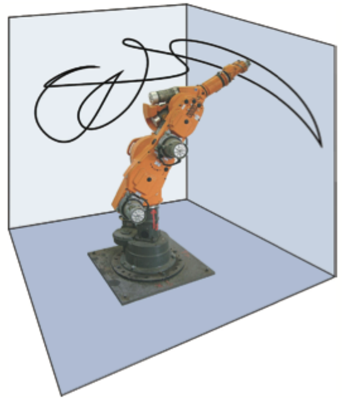

- numerical check of full column rank is more robust $\Leftrightarrow$ rank = $p$ (# of col's)

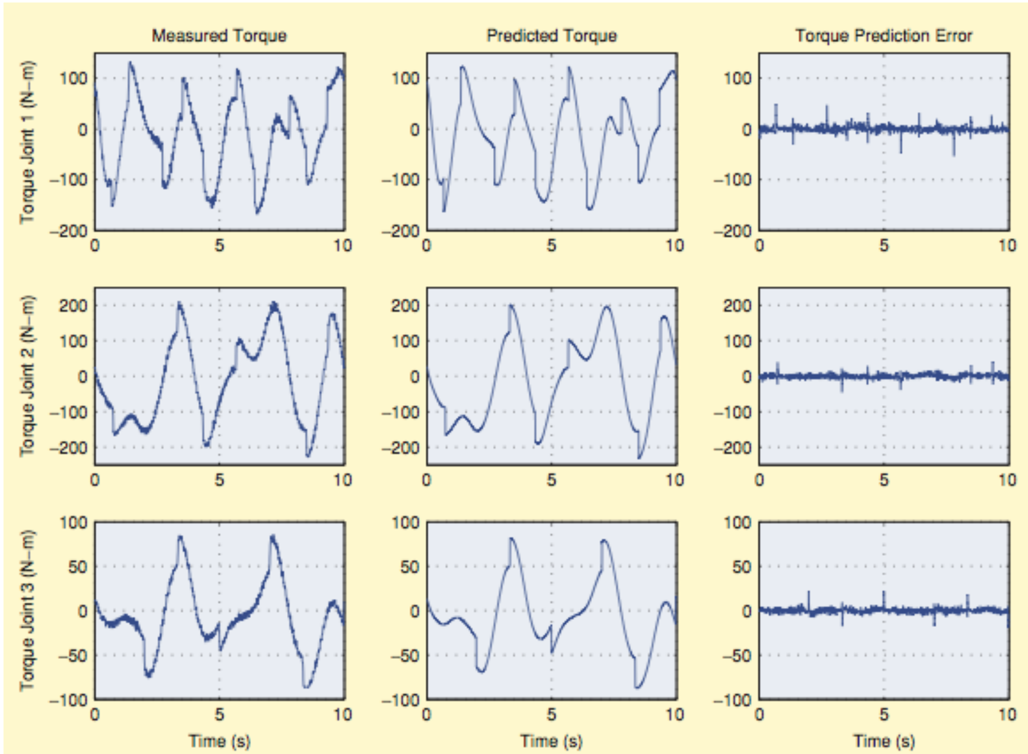
- further practical hints
  - outlier data can be eliminated in advance (when building $Y$)
  - if sufficiently rich friction models are not included in $Ya$, discard the data collected at joint velocities close to zero

# Summary on dynamic identification




KUKA IR 361 robot and optimal excitation trajectory





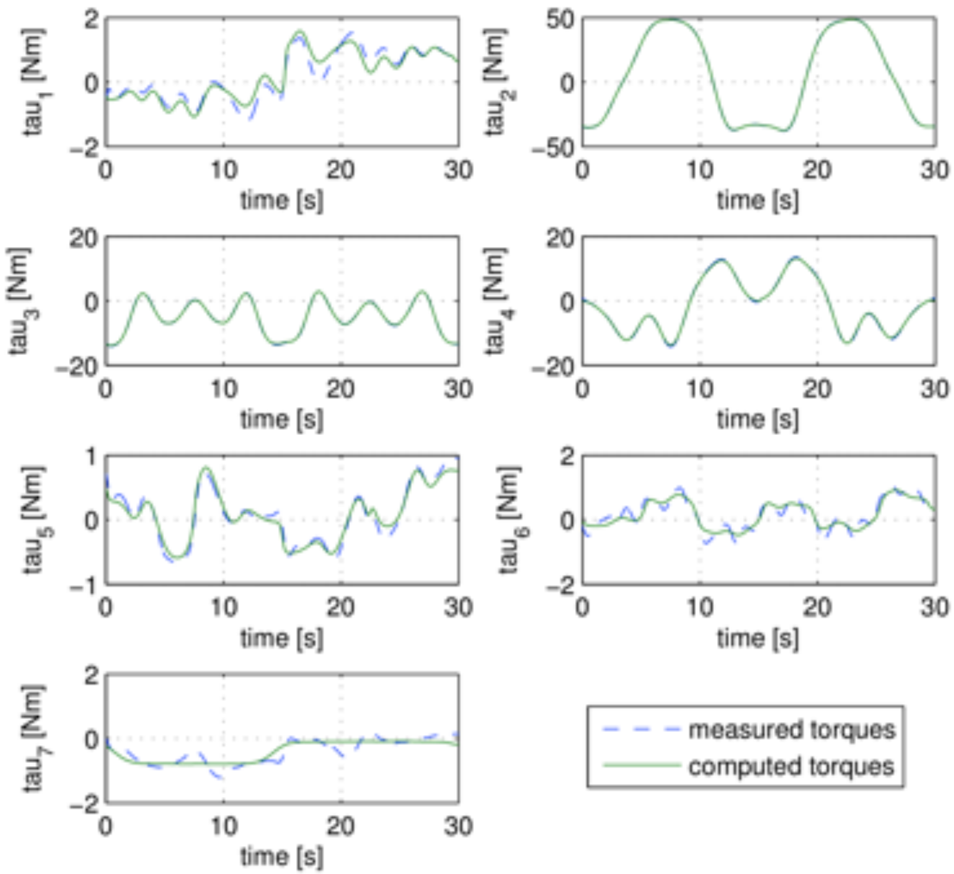

results after identification (first three joints only)

J. Swevers, W. Verdonck, and J. De Schutter: "Dynamic model identification for industrial robots" IEEE Control Systems Mag., Oct 2007

# Dynamic identification of KUKA LWR4

video


data acquisition for identification




dynamic coefficients: 30 inertial, 12 for gravity

C. Gaz, F. Flacco, A. De Luca:
"Identifying the dynamic model used by the KUKA LWR:
A reverse engineering approach"
IEEE ICRA 2014

validation after identification (for all 7 joints):
on new desired trajectories, compare
torques computed with the identified model
and torques measured by joint torque sensors

# Identification of LWR4 gravity terms

using the linear parametrization, gravity terms can also be identified separately

$$\boxed{\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}_g(\boldsymbol{q})\boldsymbol{\pi}_g}$$

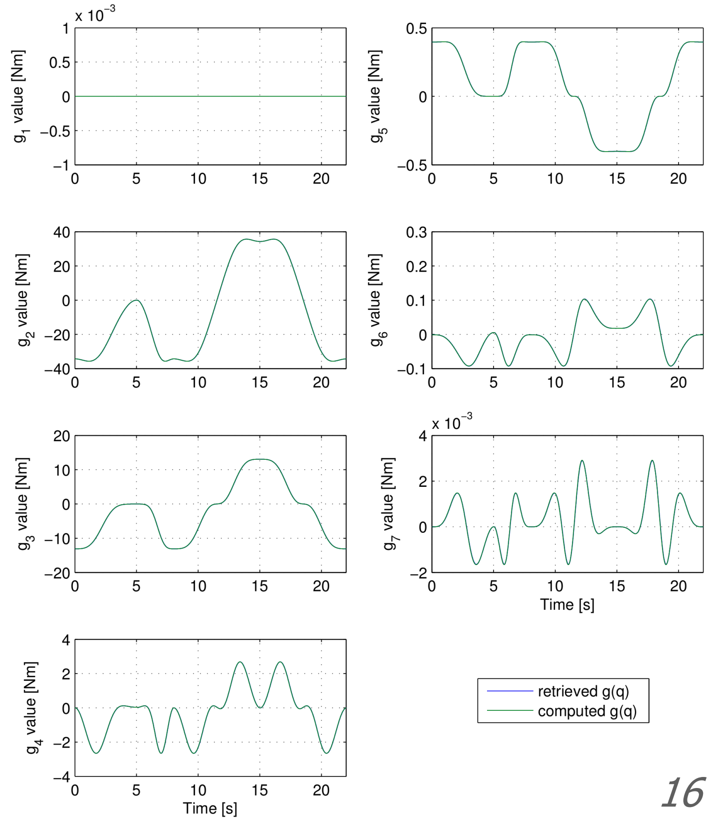
$$\boldsymbol{\pi}_g = \begin{pmatrix} c_{7y}m_7 \\ c_{7x}m_7 \\ c_{6x}m_6 \\ c_{6z}m_6 + c_{7z}m_7 \\ c_{5z}m_5 - c_{6y}m_6 \\ c_{5x}m_5 \\ c_{5y}m_5 + c_{4z}m_4 + d_2(m_5 + m_6 + m_7) \\ c_{4x}m_4 \\ c_{4y}m_4 + c_{3z}m_3 \\ c_{2x}m_2 \\ c_{3x}m_3 \\ c_{2z}m_2 - c_{3y}m_3 + d_1(m_3 + m_4 + m_5 + m_6 + m_7) \end{pmatrix}$$

symbolic expressions of gravity-related dynamic coefficients

$$\hat{\boldsymbol{\pi}}_g = \begin{pmatrix} 9.5457 \times 10^{-4} \\ -2.9826 \times 10^{-4} \\ 8.3524 \times 10^{-4} \\ 0.0286 \\ -0.0407 \\ -6.5637 \times 10^{-4} \\ 1.334 \\ -0.0035 \\ -4.7258 \times 10^{-4} \\ 0.0014 \\ 9.4532 \times 10^{-4} \\ 3.4568 \end{pmatrix}$$

numerical values identified through experiments

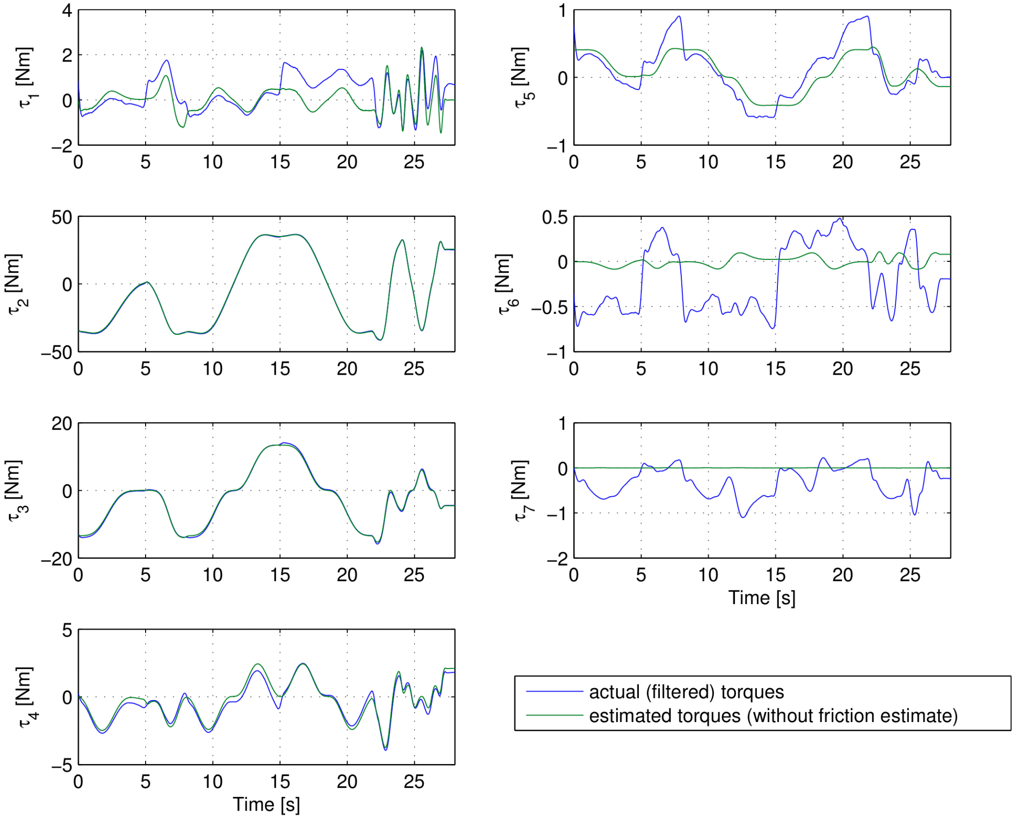
gravity joint torques prediction/evaluation on new validation trajectory

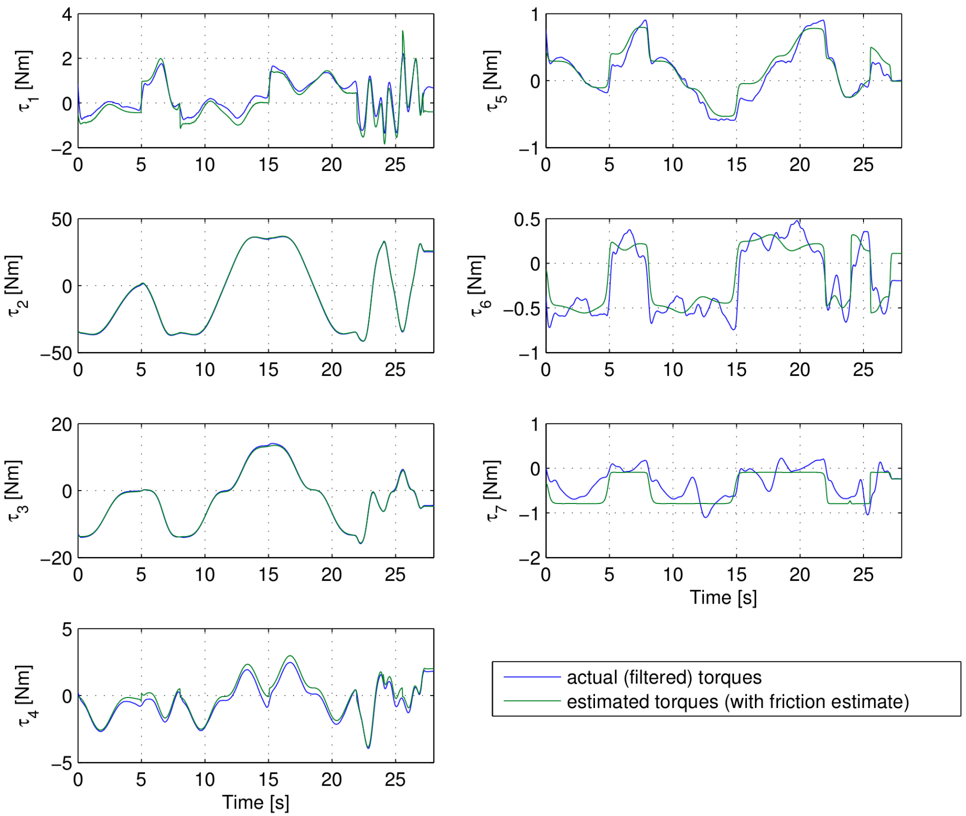


# Role of friction in identification

KUKA LWR4 dynamic model estimation vs. joint torque sensor measurement

$$\boxed{M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \tau} \boxed{- \tau_{friction}} \qquad \boxed{\tau_{meas}}$$




without the use of a joint friction model

including an identified joint friction model

$$\tau_{f,j}(\dot{q}_j) = \frac{\varphi_{1,j}}{1 + e^{-\varphi_{2,j}(\dot{q}_j + \varphi_{3,j})}} - \frac{\varphi_{1,j}}{1 + e^{-\varphi_{2,j}\varphi_{3,j}}}$$

# Dynamic identification of KUKA LWR4


video

using more dynamic robot motions for model identification

J. Hollerbach, W. Khalil, M. Gautier: "Ch. 6: Model Identification", Springer Handbook of Robotics (2nd Ed), 2016
free access to multimedia extension: http://handbookofrobotics.org

# Adding a payload to the robot

- in several industrial applications, changes in the robot payload are often needed
  - using different tools for various technological operations such as polishing, welding, grinding, ...
  - pick-and-place tasks of objects having unknown mass
- what is the rule of change for dynamic parameters when there is an additional payload?
  - do we obtain again a linearly parameterized problem?
  - does this property rely on some specific choice of reference frames (e.g., conventional or modified D-H)?

# Rule of change in dynamic parameters

- only the dynamic parameters of the link where a load is added will change (typically, added to the last one –link $n$– as payload)
  - last link dynamic parameters: $m_n$ (mass), $\boldsymbol{c}_n = (c_{nx} c_{ny} c_{nz})^T$ (center of mass), $\boldsymbol{I}_n$ (inertia tensor expressed w.r.t. frame $n$)
  - payload dynamic parameters: $m_L$ (mass), $\boldsymbol{c}_L = (c_{Lx} c_{Ly} c_{Lz})^T$ (center of mass), $\boldsymbol{I}_L$ (inertia tensor expressed w.r.t. frame $n$)

- mass $\quad m_n \to m_n + m_L$

- center of mass $\quad c_{ni} m_n \to \dfrac{c_{ni} m_n + c_{Li} m_L}{m_n + m_L} (m_n + m_L) = c_{ni} m_n + c_{Li} m_L$

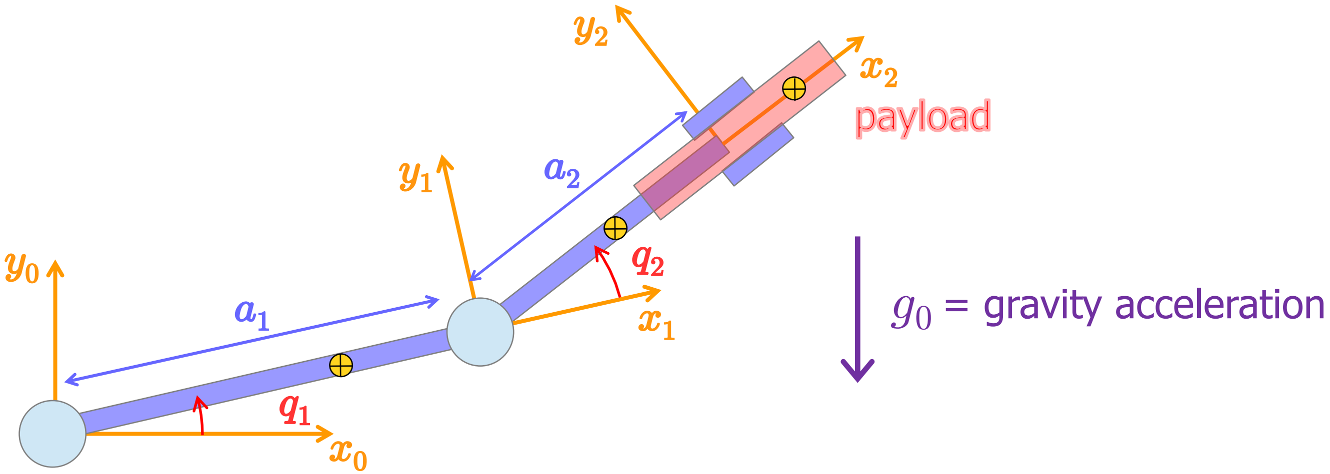
  (weighted average) where $i = x, y, z$

- inertia tensor $\quad \boldsymbol{I}_n \to \boldsymbol{I}_n + \boldsymbol{I}_L$

  valid only if tensors are expressed w.r.t. the same reference frame (i.e., frame $n$)!

- linear parametrization is preserved with any kinematic convention (the parameters of the last link will always appear in the form shown above)

# Example: 2R planar robot with payload




unloaded robot dynamics $\boldsymbol{Y\pi} = \boldsymbol{\tau}$

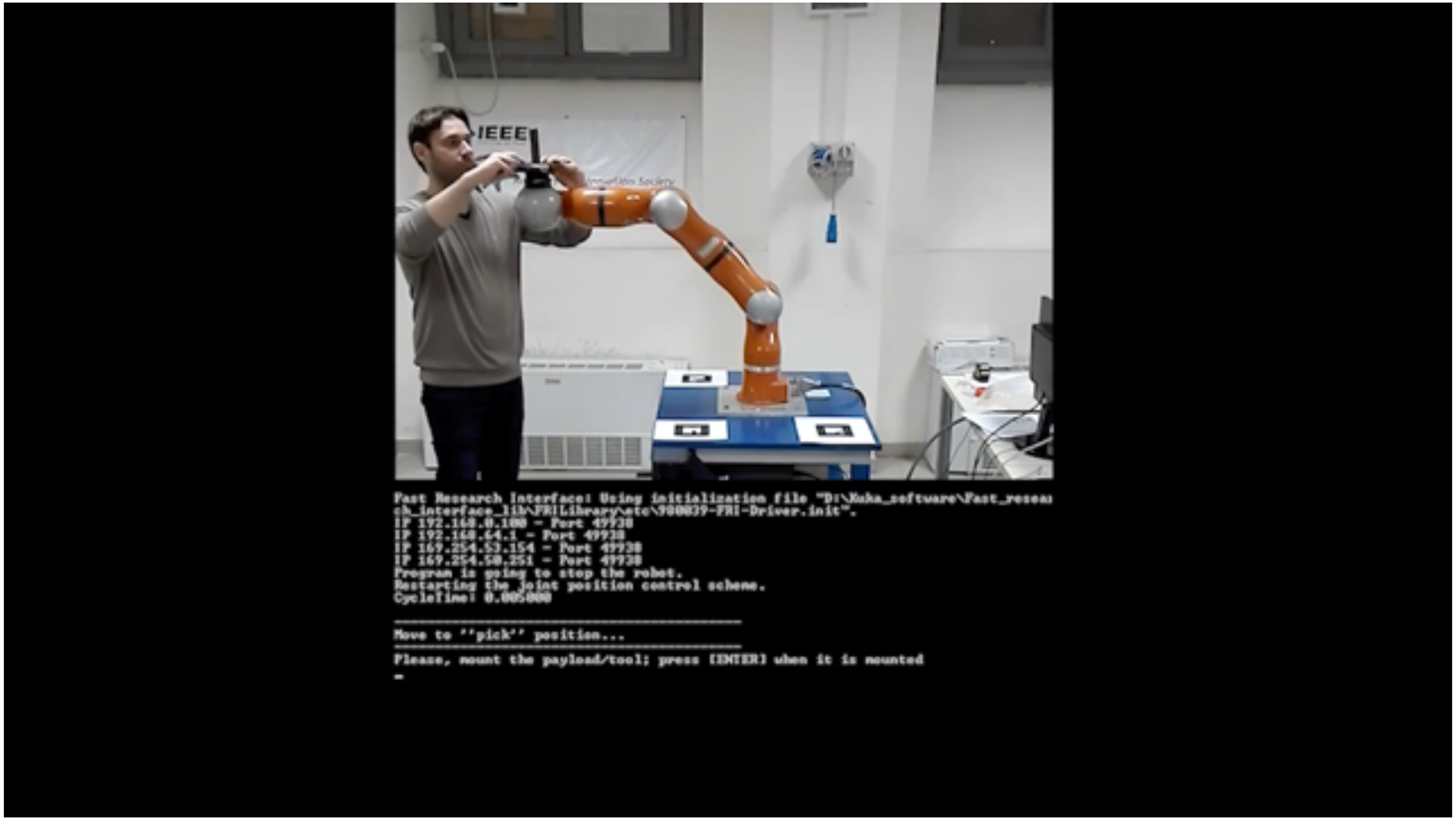
loaded robot dynamics $\boldsymbol{Y\pi}^L = \boldsymbol{\tau}^L$

$$\boldsymbol{\pi} = \begin{pmatrix} \frac{1}{2}\left(m_2 a_2^2 + I_{2zz}\right) + a_2 c_{2x} m_2 \\ c_{2x} m_2 + a_2 m_2 \\ c_{2y} m_2 \\ \frac{1}{2}\left(I_{1zz} + a_1^2 m_1 + a_1^2 m_2\right) + a_1 c_{1x} m_1 \\ c_{1x} m_1 + a_1 m_1 + a_1 m_2 \\ c_{1y} m_1 \end{pmatrix} \Rightarrow \boldsymbol{\pi}^L = \begin{pmatrix} \frac{1}{2}\left(a_2^2\left(m_2 + m_L\right) + I_{2zz} + I_{Lzz}\right) + a_2\left(c_{2x} m_2 + c_{Lx} m_L\right) \\ c_{2x} m_2 + c_{Lx} m_L + a_2\left(m_2 + m_L\right) \\ c_{2y} m_2 + c_{Ly} m_L \\ \frac{1}{2}\left(I_{1zz} + a_1^2 m_1 + a_1^2\left(m_2 + m_L\right)\right) + a_1 c_{1x} m_1 \\ c_{1x} m_1 + a_1 m_1 + a_1\left(m_2 + m_L\right) \\ c_{1y} m_1 \end{pmatrix}$$

Note 1: position of the center of mass of the two links and of the payload may also be asymmetric

Note 2: link inertia & center of mass are expressed in the DH kinematic frame attached to the link (e.g., $I_{2zz}$ is the inertia of the second link around the axis $z_2$)

# Validation on the KUKA LWR4 robot




video

C. Gaz, A. De Luca: "Payload estimation based on identified coefficients of robot dynamics – with an application to **collision detection**" IEEE IROS 2017, Vancouver, September 2017

see the block of slides!

# Bibliography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KUKA
LWR4 (7R)

Universal Robot
UR10 (6R)

Franka Emika
Panda (7R)