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ABSTRACTS

of the

CONTRIBUTED LECTURES

EQUILIBRIUM POINTS IN TWO-PERSON GAMES ON POLYHEDRAL SETS OF CONNECTED STRATEGIES

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Two-person games in which the payoff function is a sum of a bilinear and two linear functions, and player strategies are chosen from a polyhedral set described by a compatible system of linear inequalities are considered. Examples of games of such a kind that appear in economics, transportation, and trade are demonstrated, and some problems in elections and finance that are reducible to games under consideration are discussed. The conception of equilibrium in games of the kind is a natural generalization of the Nash equilibrium in games with disjoint strategies. Since all the games under consideration that are of interest in practical applications are those of sizable dimensions, it seems expedient to detect classes of the games in which establishing their solvability and finding equilibriums can be done with the use of standard optimization software possessing substantial computational capacity such as, for instance, that for solving linear and quadratic programming problems. It is shown that this is the case in some games of the considered kind. That is, verifiable necessary and sufficient conditions for the existence of the equilibriums in the games are proposed, and it is shown that their verification in the games on polyhedral sets described by compatible systems of linear equations is reducible to solving two quadratic programming problems, whereas in the games on polyhedral sets described by compatible systems of linear inequalities, the verification is reducible to solving some quadratic optimization problems. Finite methods for finding equilibriums that stem from the above-mentioned verifiable necessary and sufficient equilibrium conditions are also discussed.

AN EXPLICIT ALGORITHM FOR MONOTONE VARIATIONAL INEQUALITIES

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We introduce a fully explicit method for solving monotone variational inequalities in Hilbert spaces, where orthogonal projections onto the feasible set are replaced by projections onto suitable hyperplanes. We prove weak convergence of the whole generated sequence to a solution of the problem, under the only assumptions of continuity and monotonicity of the operator and existence of solutions.

BEYOND CANONICAL DC PROGRAMMING

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Many nonconvex optimization problems can be reduced to the so-called canonical DC problem, in which the objective function is linear while the constraint can be expressed as the set difference of two convex sets. Relying on a polar characterization of the constraint and therefore adding "polar" variables, an alternative equivalent formulation can be given in terms of a program whose nonconvexity is due only to a single bilinear constraint. In this talk we consider generalizations of this latter formulation and we analyse their properties and their connections with important classes of nonconvex problems. We define a unified algorithmic framework for outer approximation (cutting plane) type algorithms and we study different sets of conditions which guarantee convergence to an (approximated) optimal solution.

CERTIFICATES FOR COPOSITIVE PROGRAMMING

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Many global and some combinatorial optimization problems have a copositive reformulation, shifting complexity entirely to the question whether a given matrix is copositive, i.e. generates a quadratic form taking no negative values over the positive orthant. Either an affirmative answer or a negative certificate (a violating vector) is required. The dual involves completely positive matrices, so-called because they are symmetric-factorizable with a nonnegative rectangular matrix. Knowledge of this factor yields the explicit solution of the given problem, which is a positive certificate. This talk presents new methods for obtaining either certificate.

A NEWTON-TYPE FEASIBLE METHOD FOR LARGE SCALE MINIMIZATION PROBLEMS WITH BOUND CONSTRAINTS

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We propose an algorithm for the solution of large bound-constrained optimization problems, which maintains feasibility at each iteration and uses only matrix-vector product, thus being well suited for large scale problems. The algorithm follows a nonmonotone stabilization strategy, combining a Newton direction for the estimated free components and a Barzilai-Borwein Gradient direction for the estimated active components. Numerical results on simple bound-constrained problems from the CUTEr library are presented and a comparison with existing solvers is made.

MACHINE LEARNING FOR GLOBAL OPTIMIZATION

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In this talk we introduce the LeGO (Learning for Global Optimization) approach for global optimization in which machine learning is used to predict the outcome of a computationally expensive global optimization run, based upon a suitable training performed by standard runs of the same global optimization method. We propose to use a Support Vector Machine (although different machine learning tools might be employed) to learn the relationship between the starting point of an algorithm and the final outcome (which is usually related to the function value at the point returned by the procedure). Numerical experiments performed both on classical test functions and on difficult space trajectory planning problems show that the proposed approach can be very effective in identifying good starting points for global optimization. Moreover, we will mention some aspects of the LeGO procedure that can be improved to further increase the algorithm performance.

EFFICIENT PRECONDITIONING OF SEQUENCES OF LINEAR SYSTEMS IN OPTIMIZATION

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The solution of sequences of linear systems

$$(A + D_k)x_k = b_k,$$

where A is symmetric and D_k is diagonal, often arises in optimization, e.g. in trust-region and regularization subproblems, where $D_k = \alpha_k I$ with $\alpha_k \geq 0$, or in interior point methods for linear and quadratic programming, where D_k is positive semidefinite.

In large-scale problems, these systems are generally solved by using Krylov methods with appropriate preconditioners. The spectral properties of the matrices in the sequence may considerably change, thus making ineffective reusing the same preconditioner for different matrices; on the other hand, recomputing the preconditioner from scratch for each matrix may result too expensive.

We propose a technique for building preconditioners at a low cost, through a suitable update of a seed preconditioner, i.e. of a preconditioner built for one of the previous matrices in the sequence. We assume that the seed preconditioner is available in the form of an LDL^T factorization. The updated preconditioners are easy to compute and preserve the sparsity pattern of the L factor. Theoretical results as well as numerical experiments show the effectiveness of this approach.

METRIC REGULARITY

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Metric regularity has its roots in the Banach open mapping principle and in the subsequent works of Lyusternik, Graves, Robinson, Ioffe, Borwein and Milyutin. It has been recognized as a basic property in the general area of optimization, which serves as a major constraint qualification condition in deriving optimality conditions, and even more importantly, is very instrumental in obtaining error bounds for perturbed minima and proving convergence of algorithms for solving optimization problems and beyond. In this talk we move into a much wider territory in establishing a general results that links general versions of the Lyusternik-Graves theorem with fixed point theorems for set-valued mappings. As applications we discuss what metric regularity means for Newton's iteration and for discrete approximations of optimal control problems.

APPLYING OPTIMIZATION TECHNIQUES IN COMPUTATIONAL CHEMISTRY

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One of the most basis problems in computational chemistry is to determine the ground state energy of a system of electrons of an atom or a molecule. The mainstream approach for these calculations is to use truncated approximations and heuristic methods. Our group has obtained successful computational results since 2001 [3] which reinvigorated an old dream of physicists/chemists. That is, to compute the ground state energy by a pure optimization method. Surprisingly, this classical approximation problem is what is known as Semidefinite Programming (SDP) problem in optimization [1]. In this talk, we describe how the SDP problem is formulated. Also it is known that this problem

has a close connection to the max-cut problem (in combinatorial optimization). Numerical results by the parallel SDP solver SDPARA and the arbitrary-precision arithmetic SDP solver SDPA-GMP will be presented [2, 4]. Finally, if time permits, we will discuss interesting problems in computational chemistry and quantum information which require solving optimization problems.

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STRAIGHT AND REVERSE REDUCTION OF VARIATIONAL INEQUALITY PROBLEM AND THE PROBLEM ON STRICT SEPARATION OF A ZERO POINT OF EUCLIDIAN SPACE FROM CONVEX POLYHEDRON

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Using the projection-type methods for variational inequalities, especially for complementary problems and vice versa the application of variational inequalities for solving of projecting problems are widely known (e.g. see the papers [1], [2], [3]). Straight and reverse reduction of the variational inequality problem and the problem on strict separation of the zero point of the space from convex polyhedron are considered in this abstract. The investigation are based on proved in [4] strict separability theorem and gives the answers to the following questions:
1) what variational inequality problem can be solved by reduction to the problem on strict separation of the zero point of the Euclidean space from convex polyhedron?
2) to what of variational inequality problem can be reduced the problem on strict separation of zero point of the space from the convex polyhedron?

We begin with some notations. Let be given a set

$$L = \Big\{ z \in R^n : z = \sum_{i \in I} \alpha_i z_i,\ \sum_{i \in I} \alpha_i = 1,\ \alpha_i \geq 0,\ i \in I = \{1, 2, \dots m\} \Big\}$$

By definition, put $\Omega^L = \Big\{ y \in R^n : \langle z_i, y \rangle \geq \|y\|^2 \ \ \forall i \in I \Big\}$. It is evident that Ω^L is convex compact set. Denote by $\mathbf{0}$ the zero point of Euclidean space R^n.

The problem on strict separation of zero point of the space from polyhedron L consists in construction of strict separating hyperplane $\pi(c, a) = \{x \in R^n : \langle c, x \rangle = a\}$, $c \neq \mathbf{0}$ such that

$\inf\limits_{z\in L} \langle c, z\rangle \geq 0 = \langle c, \mathbf{0}\rangle$. The following theorem gives the rule of the construction of this type hyperplanes.

Theorem 1 . *(theorem on strict separability (see the paper [4], p.22))*
If $\mathbf{0} \notin L$, *then for all* $y \in \Omega^L$, $y \neq \mathbf{0}$ *a hyperplane* $\langle y, z\rangle = \|y\|^2$ *strict separates the zero point of space* R^n *from the polyhedron* L.

In [4] it has been proved that if $\Omega^L = \left\{\mathbf{0}\right\}$, then $\mathbf{0} \in L$, i.e. a zero point of the space can not be separated from the polyhedron L. If the hyperplane $\pi(c, a)$, for $a > 0$ strict separates the zero point of the space from the polyhedron L, then $y = tc/\|c\|^2 \in \Omega^L$, where $t = \min\limits_{i\in I} \langle c, z_i\rangle$. This means that to each separating hyperplane with the positive constant corresponds the normal vector from the set Ω^L.

The following theorems justify the interconnection between the solutions of the considered in this abstract problems.

Theorem 2 . *For* y *to be the solution of the variational inequality*

$$\langle y, z - y\rangle \geq 0 \quad \forall z \in L. \tag{1}$$

it is necessary and sufficient to have $y \in \Omega^L$.

Theorem 3 . *The variational inequality (1) has a unique solution* $y = \mathbf{0}$ *if and only if* $\Omega^L = \left\{\mathbf{0}\right\}$.

In [4] it has been proposed to reduce the problem of finding $y \in \Omega^L$ to two dual to each other quadratic programming problems, which can be solved by well-known quadratic programming tools.

Finally, we conclude that interconnection of the learned problems are provided by the rule of construction of the set Ω^L.

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DC MODELS FOR CLASSIFICATION PROBLEMS

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The SVM (Support Vector Machine) approach, together with the introduction of kernel transformation, has revealed a powerful tool for addressing classification problems in many application fields. SVM is based on the detection of an optimal separating hyperplane for two discrete point-sets in the sample space. Of course the goodness of the results of any classification tool depends on the geometric configuration of the data. Consequently, to give the practitioner of a specific application the opportunity of choosing the most appropriate one from among a wide spectrum of possible tools, several other classes of separation surfaces have been tested in view of designing effective classifiers. Polyhedral, ellipsoidal and spherical separation algorithms are in the literature. In addition, several classification techniques employing also information coming from unclassified data (the so called transductive approach) have been devised. In both such areas the optimisation problem to be solved in order to design the classifier can be put in the DC (Difference of Convex) form. In the talk we survey a number of such methods, focussing on their formulation. We report also the results of some numerical experiments, where the DC–Algorithm, based on partial linearization of the objective function, has been used.

HOW THE AUGMENTED LAGRANGIAN ALGORITHM DEALS WITH AN INFEASIBLE CONVEX QUADRATIC OPTIMIZATION PROBLEM

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The behavior of the augmented Lagrangian algorithm on an infeasible convex quadratic optimization problem is analyzed. It is shown that the algorithm finds a point that satisfies the shifted constraints with the smallest possible shift in the sense of the Euclidean norm and that it minimizes the objective on the corresponding shifted constrained set. The convergence to such a point is realized at a global linear rate, which depends explicitely on the augmentation parameter. This suggests us a rule for determining the augmentation parameter that aims at controlling the speed of convergence of the shifted constraint norm to zero; this rule has the advantage of generating bounded augmentation parameters even when the problem is infeasible. As a by-product, the algorithm computes the smallest translation in the Euclidean norm that makes the constraints feasible. Implications on an SQP algorithm using the AL algorithm for solving its osculating quadratic problems are discussed.

QUADRATIC CORRECTIONS IN CUTTING PLANE MODELS

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We present a bundle type method for minimization of a real convex function of several variables, not necessarily differentiable. The main feature of our approach is to model the objective function as the pointwise maximum of certain quadratic pieces, whose number depends on the cardinality of the bundle. Our model doesnt support from below the objective function. In fact such property is ensured only at the "bundle" points. The approach gives rise to two possible variants, the proximal and the trust region one, respectively. Convergence to an optimal solution is proved and numerical results are reported.

TOWARDS SECOND ORDER IMPLEMENTATIONS OF THE SPECTRAL BUNDLE METHOD

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The usefulness of semidefinite optimization is increasingly recognized in ever wider areas of mathematics and entails increasing demand for efficient and hopefully reliable numerical solvers. The spectral bundle method is a nonsmooth first order solver developed for semidefinite optimization over large sparse matrices. In theory it is well known through the work of Oustry how to combine this first order approach with the second order approach of Overton, but no tool offering this functionality is available and it is not clear how to best approach this in a large scale setting. In this presentation we report on work in progress towards including second order functionality in a practical implementation of the spectral bundle method within the ConicBundle callable library and present some first numerical results.

A SOLUTION ALGORITHM FOR GRAPH-CONVEX QUASI-VARIATIONAL INEQUALITIES

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We introduce the class of graph-convex Quasi-Variational Inequalities and propose a solution algorithm for this class of problems based on a proximal-regularization approach. To date there exists extremely few algorithms for the solution of QVIs and our proposal seems to enlarge considerably the spectrum of solvable QVIs. We analyze the convergence properties of the new algorithm and also report some preliminary numerical results.

ON PROXIMAL-LIKE METHODS FOR EQUILIBRIUM PROGRAMMING

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We consider the classical equilibrium problem, defined by Blum and Oettli: Given a nonempty, closed and convex set $K \subset R^n$ and a bivariate function $f : K \times K \to R$, the problem consists in finding some $x^* \in K$ such that

$$f(x^*, x) \geq 0 \quad \forall\, x \in K.$$

Two proximal-like methods for the stable solution of such problems have been introduced by Flam and Antipin (Math. Programming 78, 1997). They established convergence of the generated sequences under the assumption that the feasible set is compact and that the gradient of the regularizing functional is Lipschitz continuous.

Our first aim is to allow also unbounded feasible sets. This gives rise to other questions like well-definedness, since solvability of the generated auxiliary problems cannot be deduced any longer by means of compactness. Further, also a modification of the stopping criterion of Flam and Antipin is necessary since, in general, their criterion gets useless when K is unbounded. In addition, we also allow a relaxed use of ε-subgradients.

Finally, we also discuss the use of zone-coercive regularizing functionals under some additional assumption on the convex (but non-polyhedral) set K. This permits to treat the auxiliary problems as unconstrained and thus structurally much simpler ones.

BLACK BOX METHOD IN CROSS-VALIDATION FOR SUPPORT VECTOR MACHINES

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In this work we present a suitable black box approach for the problem of cross validation in Support Vector Machine to define the best parameter for the model. We will present several applications of this approach. The most important one is the prediction of the levels of the ozone pollutant in the urban area of Rome. The black box method will be also compared with the matrix method normaly suggested in litterature not only on the ozone dataset, but also on other more classical dataset provided by the letterature. The comparison will be both from the point of view of precision, and from the point of view of execution time. The results suggest that the black box approach seems to obtain slightly better results than the matrix method, but in much less time.

NUMERICAL METHODS FOR LCPS THAT ARISE IN AMERICAN OPTIONS PRICING

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In the Black-Scholes-Merton model, the price of an American option solves a system of partial diff erential variational inequalities. When these inequalities are discretized, one obtains a linear complementarity problem that must be solved at each time step. In this talk we present an algorithm for the solution of these types of linear complementarity problems that is significantly faster than the methods currently used in practice. The new algorithm is a two-phase method that combines the active-set identification properties of the projected SOR iteration with the second-order acceleration of a (recursive) reduced-space phase. We show how to design the algorithm so that it exploits the structure of the linear complementarity problems arising in these financial models and present numerical results that show the effectiveness of our approach.

VARIATIONAL INEQUALITY APPROACH TO ECONOMIC EQUILIBRIUM PROBLEM WITH APPLICATION TO PARETO OPTIMIZATION

Zdzisław Naniewicz
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Variational inequality approach to economic equilibrium problem in a reflexive Banach space will be presented. The existence result for the generalization of the well known Arrow-Debreu model will be established on the basis of the theorey of multivalued pseudo-monotone mappings. Neither the existence of interior points in the commodity cone nor any kind of MasColell ω-properness are required. Some applications to multiobjective optimization problem will be shown by applying fixed point methods.

TRAINING MAX-MARGIN SEQUENCE MODELS WITH RELAXED SLACK VARIABLES

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Sequence models are widely used in many applications such as named entity recognition, online optical character recognition, part-of-speech tagging, etc.. In this work, we propose a new approach to train the max-margin based sequence model by relaxing the slack variables. With the canonical feature mapping definition, we solve the relaxed problem by training a multi-classification Support Vector Machine. Compared with the state-of-the-art solutions for sequence data, our new method has the following advantages: 1) the discriminative information at low level granularity in the sequence can be explored by using different slack variables for different adjacent mircolabel pairs; 2) the new algorithm has less computational complexity than the algorithms of training sequence models directly; 3) nonlinear feature space can be easily explored by employing kernels. Experimental results on the task of named entity recognition and handwritten letter recognition with the public data sets illustrate the effectiveness of our new method.

OPTIMAL CONTROL FOR INTEGRODIFFERENTIAL HEMIVARIATIONAL INEQUALITIES

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We consider a class of optimal control problems for abstract evolution hemivariational inequalities. The latter are hyperbolic partial differential equations which appear in the study of frictional contact problems for viscoelastic materials with long memory term. In our model a dynamic equation of motion is considered with the viscoelastic constitutive relationship of the Kelvin-Voigt type and the contact is bilateral. The term responsible for memory of the body is given in the integral form of a linear continuous operator. The multivalued boundary condition comes from the nonconvex superpotential and it can be written in the form of the Clarke subdifferential for a locally Lipschitz function.

The aim of this paper is twofold. First we review recent results on the existence and uniqueness of weak solutions to dynamic hemivariational inequalities with long memory. These results are based on the solvability of associated nonlinear evolution inclusions with pseudomonotone multivalued operators considered on Sobolev spaces of vector valued functions. Then we consider the Bolza distributed parameter control problems. We deliver results on the continuous dependence of the solution to hemivariational inequality on the control parameter. We provide conditions that guarantee the existence of optimal solutions to control problems by applying the direct method of the calculus of variations. Finally, we provide a mechanical model and examples of nonsmooth and nonconvex superpotentials to which the theory applies.

LEARNING THE KERNEL

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Recently, kernel methods such as the support vector machine have been very successful in diverse application areas such as bioinformatics and computer vision. One key element of this success is the nonlinear feature mapping defined by the kernel function, which captures the application specific

similarity between objects. Multiple kernel learning (MKL) has emerged as a useful tool when the specific kernel is unknown, or when there are various sources of information to combine. MKL searches over a convex combination of kernels for the one most suited to the task. In this work, we cast MKL as an oracle based optimization framework, relating several recent proposals for optimization. We propose the analytic center method and show that this results in very desirable convergence behaviour. Finally, we discuss several limitations of the MKL framework and several open problems regarding learning the kernel.

GAP FUNCTIONS AND PENALIZATION FOR SOLVING EQUILIBRIUM PROBLEMS WITH NONLINEAR CONSTRAINTS

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An equilibrium problem (EP) can be reformulated as an optimization problem through so called gap functions and therefore it can be solved devising descent algorithms. However, these gap functions are not easy to evaluate unless the constraints set has a polyhedral structure. We first introduce a new gap function for EPs, using linear approximations to the constraint functions, and we show some properties about the continuity, the differentiability and the stationary points of this function. Next, we add a penalty term to the gap function in order to take into account the linearization of the constraints, and we analyse the relationships between appropriate descent directions and the penalty parameter. Finally, we propose a solution method based on this penalized gap function for solving EPs with nonlinear constraints and we prove that it is globally convergent under suitable assumptions.

LAURENT SERIES FOR INVERSION OF LINEARLY PERTURBED BOUNDED LINEAR OPERATORS ON BANACH SPACE

Charles Pearce*, Phil. Howlett, Amie Albrecht
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We are motivated to find best approximate solutions to equations on Banach space. We begin with finding necessary and sufficient conditions for the existence of a Laurent series expansion with a finite-order pole at the origin for the invrse of a linearly perturbd bounded linear operator mapping one Banach space to another. In particular we show that the inversion defines linear projections that separate the Banach spaces into complementary subspaces. We present some applications.

USING OF CLUSTER ANALYSIS TECHNIQUE FOR THE SELECTION OF PATTERNS FOR OZONE BY NEURAL NET MODELS

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In the present work, we predicted ozone levels 24 hours in advance using neural networks (NN) in the urban area of Rome. The data used covered a period of one year, from January through December 2007 and come from a background monitoring station (Villa Ada monitoring station). The prediction of ozone 24 hours in advance is obtained by using the conventional meteorological variables (T, RH, WS and GSR), primary pollutants variables (CO, NO, NO2) and others variables related to the relationships between primary and secondary pollutants in order to take into account the effects

of sources emissions and the inclusion of photochemical reactions . The goal of our paper is the evaluation of different performances for the NN using different methodologies for the pattern choice during the training phase. The pattern choice was executed by two methodologies: the random pattern selection and the cluster techniques (K-means algorithm) at different percentage of input data. We utilise cluster methods exclusively to select the best and significant patterns, whereas these techniques usually are used to synthesise data in homogeneous groups whose members show a high degree of similarity among themselves but a high degree of dissimilarity with the members of other clusters. The simulations show different significant results both in terms of forecasting capabilities accuracy and in term of generalization capability to predict ozone concentrations 24 hours in advance. In term of NN performances, we consider different percentages of input patterns and we observe a rapid increase of performance after the 10range from 0.55 up to 0.71. The use of cluster analysis as pattern selection increases NN performances in a very significant way. The NN training obtained by use of 10of total data ranging from 0.70 to 0.78 in term of determination coefficient (R2). Our results are very encouraging and show that NN model performance is improved using cluster analysis respect to the conventional random pattern choice. In fact, simulations based on cluster analysis show that NN converges more rapidly and the accuracy of the prediction is very accurate.

Index TermsOzone, artificial neural networks, Data mining, cluster analysis, k-means

ON THE CONVERGENCE OF A JACOBI-TYPE ALGORITHM FOR SINGLY LINEARLY-CONSTRAINED PROBLEMS SUBJECT TO SIMPLE BOUNDS

G. Liuzzi, L. Palagi, **M. Piacentini***
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In this work we define a block decomposition Jacobi-type method for nonlinear optimization problems with one linear constraint and bound constraints on the variables. We prove convergence of the method to stationary points of the problem under quite general assumptions.

SPEEDP: A FAST METHOD FOR SOLVING THE SDP RELAXATION OF MAX CUT

L. Grippo, L.Palagi, M. Piacentini, **V. Piccialli***, G. Rinaldi, F. Rendl, A. Wiegele
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The max cut problem is one of the most important combinatorial optimization problems. Given an undirected graph $G = (V, E)$, with weights on the edges, the max cut problem consists in partitioning V into two sets so as to maximize the sum of the weights on the edges that have one end in each side of the partition. It can be formulated as

$$\max x^T C x \text{ such that } x \in \{-1, 1\}^n.$$

This problem is NP-hard and in 1995 Goemans and Williamson proposed a randomized 0.878 approximation algorithm based on the use of the following Semidefinite Programming (SDP) relaxation:

$$\max \langle C, X \rangle \text{ such that } \operatorname{diag}(X) = e,\ X \succeq 0. \tag{2}$$

Many efficient algorithms have been proposed for solving this problem, and many of them make use of a low rank reformulation. In this talk, we describe a low rank algorithm called SpeeDP that is based on a suitable unconstrained reformulation of problem (2), and show that it outperforms the other existing methods. Furthermore, we use this algorithm as a tool to build both an exact and a heuristic method for solving the max cut problem.

NEWTON-TYPE METHODS FOR THE LIFTED MATHEMATICAL PROGRAMS WITH COMPLEMENTARITY CONSTRAINTS

A. F. Izmailov, **A. L. Pogosyan***, and M. V. Solodov
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We consider a reformulation of mathematical programs with complementarity constraints, where by introducing an artificial variable the constraints are converted into equalities which are once but not twice differentiable. We show that the Lagrange optimality system of such a reformulation is semismooth and BD-regular at the solution under reasonable assumptions. Thus, fast local convergence can be obtained by applying the semismooth Newton method. Moreover, it turns out that the squared residual of the Lagrange system is continuously differentiable (even though the system itself is not), which opens the way for a natural globalization of the local algorithm. This globalization is organized by a descent method for the mentioned smooth merit function which based on the local algorithm. However, from the practical viewpoint, it seems more promising to use a non-smooth exact penalty function instead of the squared residual of the Lagrange system which leads to the semismooth sequential quadratic programming method. Preliminary numerical results for problems from MacMPEC test collection demonstrate that the approach is very promising.

NONLINEAR EQUILIBRIUM VS. LINEAR PROGRAMMING FOR LIMITED RECOURSES ALLOCATION

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We consider the Nonlinear Equilibrium (NE) as an alternative to Linear Programming (LP) approach for optimal recourse allocation. It was shown that under natural economic assumptions the NE exists and unique. Finding the NE is equivalent to solving a variation inequality (VI). For solving the VI a projected pseudo-gradient method was introduced, his global convergence with Q-linear rate was proven and its computational complexity was estimated. The method can be viewed as a natural pricing mechanism for establishing an Economic Equilibrium.

MULTI-TASK LEARNING: THEORY AND PRACTICE

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We discuss the problem of estimating a structured matrix with a large number of elements. A key motivation for this problem occurs in multi-task learning. In this case, the columns of the matrix correspond to the parameters of different regression or classification tasks, and there is structure due to relations between the tasks. We present a general method to learn the tasks' parameters as well as their structure. Our approach is based on solving a convex optimization problem, involving a data term and a penalty term. We highlight different types of penalty terms which are of practical and theoretical importance. They implement structural relations between the tasks and achieve a sparse representations of parameters. We address computational issues as well as the predictive performance of the method. Finally we discuss how these ideas can be extended to learn non-linear task functions by means of reproducing kernels.

DERIVATIVE-FREE METHODS FOR MIXED-INTEGER OPTIMIZATION

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We consider the problem of minimizing a continuously differentiable function of several variables subject to simple bound constraints where some of the variables are restricted to take integer values. We assume that the first order derivatives of the objective function can be neither calculated nor approximated explicitly. This class of mixed integer nonlinear optimization problems arises frequently in many industrial and scientific applications and this motivates the increasing interest in the study of derivative-free methods for their solution. We propose an algorithm convergent to points satisfying suitable stationarity conditions. The integer variables are tackled by using a local search-type approach. Numerical results are presented and sustain the proposed method.

A POTENTIAL-REDUCTION METHOD FOR THE SOLUTION OF GENERALIZED NASH EQUILIBRIUM PROBLEMS

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We consider the Generalized Nash Equilibrium Problem (GNEP), a generalization of the standard Nash equilibrium problem in which the feasible set of each player may depend on the variables of the other players. We propose a new solution algorithm based on the application of a potential-reduction method to the KKT conditions of the game. We study the convergence properties of the new algorithm, and show that they compare favorably to those of existing methods. We also report numerical results on a rather extensive set of test problems showing that the new method appears to perform well in practice.

ON SECOND-ORDER OPTIMALITY CONDITIONS FOR MULTIOBJECTIVE OPTIMIZATION

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In this work two second-order constraint qualifications for the multiobjective optimization problem are introduced. They come from first-order constraint qualifications originally devised for the single valued case. The first is based on the classical feasible arc constraint qualification and the second the constant rank constraint qualification was introduced by Janin in 1984. They are used to establish two second-order necessary conditions for the multiobjective optimization problem with general nonlinear constraints without any convexity assumption. The similarities and differences between the sets in charge for the semidefiniteness property in the scalar and the multiobjective cases are discussed.

MULTI-BODY DESIGN CENTERING: AN APPROACH FOR SOLVING CUTTING AND PACKING PROBLEMS IN A CONTINUOUS FRAMEWORK

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Multi-body design centering (*MBDC*) is an extension to design centering and deals with the optimal embedding of several parametrized bodies (the *designs*) into another larger body (the *container*). In practice such problems arise as cutting and packing (*C&P*) problems. While there exist a large number of specialized methods for standard C&P problems, the number of available algorithms rapidly decreases as the structure of the designs and the container becomes more complicated. The set-valued formulated containment and non-overlapping constraints of a MBDC problem can be rewritten in terms of infinite ones getting a general semi-infinite problem (*GSIP*). Because of its bi-level structure it is possible to reformulate GSIP equivalently as a special Stackelberg game. In the talk we present an entropic regularization approach with inner and outer approximation property for solving such Stackelberg games in the case of general designs and submit proposals for modeling MBDC problems as nonlinear ones in the case of designs which are homeomorphic to the n-dimensional unit sphere. Furthermore, we show, how two application-relevant aspects, namely the generation of guillotine patterns and the regard of quality requirements, can be integrated in our framework. A natural goal in MBDC is the maximization of design volumes. However, when embedding two or more designs, it isn't enough to consider their total volume. We present an approach via ordered median functions which combines the design volumes in a flexible and manageable way. We illustrate the above mentioned modeling aspects as well as the theoretical and algorithmic issues by means

A VARIATIONAL-ANALYTIC APPROACH TO ELLIPTIC MATHEMATICAL PROGRAMS WITH EQUILIBRIUM CONSTRAINTS

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We define an elliptic mathematical program with equilibrium constraints or elliptic MPEC to be a mathematical program in which some functional $J(u, y)$ is minimized subject to the constraints $u \in \mathcal{U}$, where $\mathcal{U}$ is some Hilbert space containing the reflexive Banach space Y and which is densely embedded in Y^*, and $y \in S(u)$, where $S : Y^* \to Y$ represents the solution mapping of some perturbed elliptic variational inequality (in the classical sense of Stampacchia/Kinderlehrer) It is assumed throughout that J is Fréchet differentiable on $\mathcal{U} \times \mathcal{Y}$, but not necessarily on $Y^* \times Y$.

After providing a formula for the contingent derivative of the normal cone mapping associated with the variational inequality, a chain rule is applied leading to an explicit characterization of the contingent derivative of the lower-level solution mapping S. This leads to a formula providing a good upper approximation of the Fréchet normal cone to the feasible set of the elliptic MPEC. Next, dual stationarity/optimality conditions (similar to so-called S-stationarity conditions in the MPEC-literature) are derived for the general elliptic MPEC. These conditions involve only the classical critical cone and its polar cone and do not require the calculation of limiting variational objects.

The abstract results are then shown to yield the best known stationarity conditions associated the optimal control of the obstacle problem as derived by Mignot and Puel. By defining $Y := H_0^1(\Omega)$, with $\Omega \subseteq R^n$, $n \geq 1$, $\mathcal{U} := \mathcal{L}^{\in}(\otimes)$, and $Y^* := H^{-1}(\Omega)$, and assuming the gradients of the lower-level/state variables are bounded, new dual optimality conditions are derived for such elliptic MPECs. This problem class often arises in the study of elastoplastic materials. The needed tangent cone, normal cone, critical cone, and its polar are all explicitly calculated. In this sense, the new results lay the framework for future numerical studies.

CONVERGENCE OF DISTRIBUTED OPTIMAL CONTROL PROBLEMS GOVERNED BY ELLIPTIC VARIATIONAL INEQUALITIES

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Let u_g be the unique solution of an elliptic variational inequality (EVI) with source term g. We establish, in the general case, the error estimate between $u_3(\mu) = \mu u_{g_1} + (1-\mu)u_{g_2}$ (the convex combination of two solutions) and $u_4(\mu) = u_{\mu g_1+(1-\mu)g_2}$ (the solution corresponding to the convex combination of two data) for $\mu \in [0,1]$. We consider systems with a free boundary problem of obstacle type and a family of distributed optimal control problems governed by EVI over the internal energy g for each positive heat transfer coefficient α given on a part of the boundary of the domain. For a given cost functional and using some monotony property between $u_3(\mu)$ and $u_4(\mu)$ given in F. Mignot, J. Funct. Anal., 22 (1976), 130-185, we prove the strong convergence of the optimal controls and states associated to this family of distributed optimal control problems governed by EVI to a Dirichlet distributed optimal control problem, governed also by an EVI, when the parameter α goes to infinity. We obtain this convergence without using the adjoint state which is a great advantage with respect to the proof given in Gariboldi-Tarzia, Appl. Math. Optim., 47 (2003), 213-230, for optimal control problems governed by elliptic variational equalities (EVE).

When the systems are governed by EVE we can obtain the corresponding numerical analysis of the family of distributed optimal control problems by the finite element method. We can prove the convergence when the parameter of the finite element method h goes to zero and when the parameter heat transfer α goes to infinity obtaining a commutative diagram and also the corresponding error estimates.

The first part of this presentation is a joint paper with Mahdi Boukrouche (Univ. Saint Etienne, France).

NUMERICAL ANALYSIS OF NONLINEAR ELLIPTIC OPTIMAL CONTROL PROBLEMS OF SEMI-INFINITE TYPE

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We consider optimal control problems governed by a semilinear elliptic equation with differentiable nonlinear objective, finite-dimensional control space and (infinitely many) pointwise constraints on the state of the control system. For instance, the problem

$$\min \int_\Omega L(x, y(x), u)\, dx$$

subject to $u \in R^m$,

$$\begin{aligned} -\Delta y(x) + d(x, y(x), u) &= 0 \quad \text{in } \Omega \\ y(x) &= 0 \quad \text{on } \partial\Omega, \end{aligned}$$

$$u_a \le u \le u_b, \quad y(x) \le b \quad \text{for all } x \in \Omega,$$

belongs to this class. Problems of this type are important for applications, since often only finitely many parameters are to be optimized but the state has to obey bounds in the whole spatial domain. We consider the stability and error analysis for the numerical approximation of the problems and confirm them by numerical examples.

A REGULARIZATION APPROACH TO NONLINEAR VARIABLE SELECTION

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In this work, we are interested into the variable selection problem within the supervised learning framework, when the input-output dependence is described by a nonlinear model. The proposed method is based on a regularized least square estimator penalizing large values of the partial derivatives. The variational problem corresponding to this new functional cannot be solved using simple gradient descent methods. Towards this end we develop an iterative projection procedure based on a forward-backward splitting approach. Convergence of the method can be proved and the entire regularization path for the algorithm can be efficiently computed using a simple continuation method. The performance of the proposed algorithm is assessed on toy data as well as on a benchmark data set. The algorithm compares favorably to more standard ridge regression and .1 regularization schemes.

SEMIDEFINITE RELAXATIONS FOR NON-CONVEX QUADRATIC MIXED-INTEGER PROGRAMMING

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We present a semidefinite relaxation for non-convex quadratic mixed-integer optimization problems. This relaxation yields tight bounds and is computationally easy to solve for medium-sized instances, even if variables are unbounded in which case the problem contains an infinite number of constraints. These constraints are separated dynamically. We use this approach as the bounding routine in an SDP based branch-and-bound framework. To reduce the number of nodes in the branch-and-bound tree we use the technique of range-reduction.

Numerical experiments show that our algorithm performs well on various types of random instances.

ENCLOSING ELLIPSOIDS OF SEMI-ALGEBRAIC SETS AND ERROR BOUNDS IN POLYNOMIAL OPTIMIZATION

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In this talk, for a semi-algebraic set $\mathcal{F} \in R^m$, we propose an efficient SDP (SemiDefinite Programming) approach to compute an ellipsoid of the form

$$\mathcal{E}(\boldsymbol{z}, \rho) = \{\boldsymbol{v} \in R^m : (\boldsymbol{v} - \boldsymbol{z})^T \boldsymbol{M} (\boldsymbol{v} - \boldsymbol{z}) \leq \rho^2\},$$

which encloses $\mathcal{F}$ (*i.e.* $\mathcal{F} \subset \mathcal{E}(\boldsymbol{z}, \rho)$) with the smallest radius $\rho \geq 0$. In our approach, an $m \times m$ positive semidefinite matrix $\boldsymbol{M}$ is fixed, while the variables are the center $\boldsymbol{z} \in R^m$ and the radius $\rho \in R$.

The motivation of this talk comes from error bounding in polynomial optimization. Let us consider an polynomial optimization problem of the form

$$(POP) \quad \min f_0(\boldsymbol{x}) \text{ subject to } \boldsymbol{x} \in \boldsymbol{F}_0.$$

Here, f_0 is a real valued polynomial in $\boldsymbol{x} \in R^m$ and $\boldsymbol{F}_0$ is a semi-algebraic set in R^m. SDP relaxation method is known as an excellent method to estimate a tight lower bound of the above (POP) problem.

However, the solution $\widetilde{\boldsymbol{x}}$ derived by SDP relaxation is not always a feasible solution of $\boldsymbol{F}_0$. By applying some local method to $\widetilde{\boldsymbol{x}}$, we often obtain a feasible solution $\overline{\boldsymbol{x}} \in \boldsymbol{F}_0$. Therefore, we ensure that an optimal solution $\boldsymbol{x}^*$ of (POP) exists in the semi-algebraic set $\mathcal{F}$,

$$\mathcal{F} = \{\boldsymbol{x} \in \boldsymbol{F}_0 : f_0(\boldsymbol{x}) \leq f_0(\overline{\boldsymbol{x}})\}$$

Let us now fix $\boldsymbol{M}$ as the matrix whose elements are 0 except (i,i)th element is 1 for some $i = 1, \ldots, m$. Our approach enables to obtain $\boldsymbol{z}^*$ and ρ^*, the center and the radius of the enclosing ellipsoid with the smallest radius. As a result, we can bound x_i^* by $z_i^* - \rho^* \leq x_i^* \leq z_i^* + \rho^*$. If we choose the identity matrix as $\boldsymbol{M}$, we obtain the bound $||\boldsymbol{x}^* - \boldsymbol{z}^*|| \leq \rho^*$.

We show that computing an enclosing ellipsoid with the smallest radius can be reduced into an SDP. The basic idea of this reduction is a transforming the inner quadratic maximization problem of the following problem to a maximization problem with a linear objective function over a convex set in a higher dimensional lifting space.

$$\min_{\boldsymbol{z} \in R^m} \max_{\boldsymbol{v} \in \mathcal{F}} (\boldsymbol{v} - \boldsymbol{z})^T \boldsymbol{M} (\boldsymbol{v} - \boldsymbol{z})$$

In this talk, we discuss more details of this reduction and theoretical aspects. We also report some numerical results on polynomial optimization problems and sensor network location problems. The results verify that our approach is effective to evaluate error bounds of optimal solutions.

This talk is based on the following technical paper.

Masakazu Kojima and Makoto Yamashita, "Enclosing Ellipsoids and Elliptic Cylinders of Semialgebraic Sets and Their Application to Error Bounds in Polynomial Optimization", Research Report B-459, Dept. of Math. and Comp. Sciences, Tokyo Institute of Technology, Oh-Okayama, Meguro, Tokyo 152-8552, January 2010.

VECTOR VARIATIONAL-LIKE INEQUALITIES OF NONSMOOTH FUNCTIONS

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We study the relationships between strongly preinvex functions and strong invariant monotonicity of corresponding Clark-Rockafellar subdifferential and limiting subdifferential in the sense of Mordukhovich. We introduce two vector variational-like inequalities (VLI) and investigate the equivalence between their solutions for non-differentiable functions under strongly invariant pseudomonotonicity. Furthermore, some relationships between the solutions of (VLI) and of perturbed (VLI) are established.