

“E. MAJORANA” FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE
INTERNATIONAL SCHOOL OF MATHEMATICS “G. STAMPACCHIA”

59TH WORKSHOP

**Nonlinear optimization:
a bridge from theory to applications**

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ABSTRACTS

of the

INVITED LECTURES

A Fokker-Planck Strategy for the Optimal Control of Stochastic Processes

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An efficient framework for the optimal control of probability density functions (PDF) of multidimensional stochastic processes and piecewise deterministic processes is presented. This framework is based on Kolmogorov-Fokker-Planck-type equations that govern the time evolution of the PDF of stochastic processes and piecewise deterministic processes. In this approach the control objectives are formulated in terms of the PDF to follow a given trajectory and to reach a desired terminal configuration. The corresponding optimal control problems are formulated as a sequence of open-loop optimality systems in a nonlinear model predictive control strategy. Theoretical results concerning the forward and the optimal control problems are provided. In the case of stochastic (Ito) processes, the Fokker-Planck equation is of parabolic type and it is shown that under appropriate assumptions the open-loop bilinear control function is unique. In the case of piecewise deterministic processes (PDP), the Fokker-Planck equation consists of a first-order hyperbolic system. Discretization schemes are discussed that guarantee positivity and conservativeness of the forward solution. The proposed control framework is validated with multidimensional biological, quantum mechanical, and financial models.

This work is in collaboration with Mario Annunziato (U. Salerno) and it is supported in part by the EU Marie Curie International Training Network Multi-ITN STRIKE Projekt ‘Novel Methods in Computational Finance’ and in part by the ESF OPTPDE Programme.

Constructive optimality conditions in nonsmooth optimization

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In solving optimization problems, necessary and sufficient optimality conditions play an outstanding role. They allow, first of all, to check whether a point under study satisfies the conditions, and, secondly, if it does not, to find a “better” point.

For the class of directionally differentiable functions in \mathbb{R}^n , a necessary condition for an unconstrained minimum requires for the directional derivative to be non-negative in all directions. This condition becomes efficient for special classes of directionally differentiable functions. For example, in the case of convex and max-type functions, the necessary condition for a minimum takes the form $0_n \in C$ where $C \subset \mathbb{R}^n$ is a convex compact set. The problem of verifying this condition is reduced to that of finding the point of C which is the nearest to the origin. If the origin does not belong to C , we easily find the steepest descent direction, and are able to construct a numerical method.

For the classical Chebyshev approximation problem (the problem of approximating a function $f(t) : G \rightarrow \mathbb{R}$, where $G \subset \mathbb{R}$, by an n -degree polynomial $P(t)$), the condition for a minimum takes the so-called alternance form: for a polynomial $P^*(t)$ to be a solution to the Chebyshev approximation problem, a collection of $n + 2$ points $\{t_i \mid t_i \in G\}$ should exist at which the difference $P^*(t) - f(t)$ attains its maximal absolute value with alternating signs. This condition can easily be verified, and if it does not hold, one can find a “better” polynomial.

In the present report, based on results recently obtained jointly by V.N. Malozemov and the author, it will be demonstrated that the alternance form of the necessary conditions is valid not only for Chebyshev approximation problems, but also in the general case of directionally differentiable functions. In some cases it may be more convenient to use the conditions in form $0_n \in C$, in other ones – the condition in the alternance form.

Both unconstrained optimization problems and constrained ones are discussed.

The work is supported by the Russian Foundation for Basic Research (RFFI) under Grant No 12-01-00752.

On preconditioning in large-scale optimization: updating techniques for sequences of linear systems

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A strong impulse to the dramatic development undergone by large-scale optimization in the last decades has come from advancements in numerical linear algebra. A significant step forward has been made by applying suitably preconditioned iterative linear solvers instead of direct ones, which often require too much memory and time on large problems. Despite very effective preconditioners have been devised, building the preconditioners may still account for a large part of the overall optimization process, thus motivating the interest towards preconditioning techniques achieving a better tradeoff between effectiveness and computational cost. In this lecture we discuss some techniques aimed at building preconditioners for sequences of linear systems through low-cost updates of a “seed” preconditioner. We focus on sequences arising in trust region and overestimation methods for unconstrained nonlinear optimization and on sequences arising in interior point methods for quadratic programming. Theoretical as well as numerical results are provided, showing the efficiency of such techniques.

This lecture is based on recent work carried out in collaboration with S. Bellavia, V. De Simone and B. Morini, and is partially supported by INdAM-GNCS.

Complex Variational Inequalities and Applications

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We introduce and study variational inequalities in the complex domain, along with some technical tools useful in their study. We then extend to the complex domain some recent developments in the field of the distributed solution of (generalized) Nash equilibrium problems. In order to illustrate our techniques we consider some new MIMO games over vector Gaussian Interference Channels, modeling some distributed resource allocation problems in MIMO CR systems and femtocells. These games are examples of Nash equilibrium problems that can not be handled by current methodologies.

Canonical Duality Theory: a Bridge from Mathematical Optimization to Real-World Applications

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Canonical duality theory was developed originally from Gao and Strang's work on nonconvex mechanics in 1989. This potentially useful theory is composed mainly of (1) a canonical dual transformation methodology, which can be used to model complex systems within a unified framework; (2) a complementary-dual principle, which provides a correct way to formulate dual problem without duality gap; and (3) a triality theory, which can be used to identify both global and local extrema, and to develop effective primal-dual algorithms for solving a wide class of nonconvex/nonsmooth/discrete variational/optimization problems in real-world applications. Beginning with the oriental philosophy of I-Ching and some fundamental concepts in science, such as objectivity, Lagrangian, geometrical and physical nonlinearities, etc, the speaker will show how the canonical duality was naturally developed, why this theory is potentially powerful for studying complex systems, and what kind challenging problems can be or can't be solved. For continuous systems governed by general nonconvex/nonsmooth objective functions, he will show that the canonical dual problems possess very nice mathematical properties which can be solved easily by well-developed nonlinear programming techniques. For problems in discrete systems, he will show the integer constraints can be naturally relaxed by the canonical dual transformation; therefore, the NP-complete quadratic integer programming problems are actually identical to a continuous unconstrained Lipschitzian global optimization problem which can be solved via deterministic methods. Based on the canonical duality theory, a new powerful primal-dual algorithm will be presented which can be used for solving a large class of challenging problems in global optimization and analysis. Applications will be illustrated by certain well-known NP-Hard problems, including maxcut problem, sensor localization, and neural network optimization etc. He will show that in complex systems, the global minimizer may not be the best solution. Finally, some common misunderstandings, open problems, and challenges on the canonical duality theory will be addressed. This talk should bring some fundamentally new insights into complex systems theory, nonlinear optimization and computational science.

References

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Nonsmooth Optimization Techniques in Practical Optimization Problems

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We describe some applications of nonsmooth optimization techniques such as subgradient, bundle and coordinate search methods for dealing with several problems of the integer programming type and/or graph optimization. The application areas are the logistic chain management and the network design. In particular we focus on the so called cross-docking model for freight distribution, on the order consolidation model for replenishment, on the network design with minimum number of splitting nodes and on some classes of generalized assignment problems arising in container handling

On the Linear Convergence of the Alternating Direction Method of Multipliers

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We analyze the convergence rate of the alternating direction method of multipliers (ADMM) for minimizing the sum of two or more nonsmooth convex separable functions subject to linear constraints. Previous analysis of the ADMM typically assumes that the objective function is the sum of only two convex functions defined on two separable blocks of variables even though the algorithm works well in numerical experiments for three or more blocks. Moreover, there has been no rate of convergence analysis for the ADMM without strong convexity. In this work, we establish the global linear convergence of the ADMM for minimizing the sum of any number of convex separable functions. This result settles a key question regarding the convergence of the ADMM when the number of blocks is more than two or if the strong convexity is absent. It also implies the linear convergence of the ADMM for several contemporary applications including LASSO, Group LASSO and Sparse Group LASSO without any strong convexity assumption. Our proof is based on estimating the distance from a dual feasible solution to the optimal dual solution set by the norm of a certain proximal residual.

Experiences of Interactive Multiobjective Optimization and Some Engineering Applications

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Over the years, many methods have been developed for multiobjective optimization. Typically, they aim at supporting a decision maker in finding the best compromise solution in problems where several conflicting objective functions are to be optimized simultaneously. Because the compromise solutions, so-called Pareto optimal solutions, cannot be ordered without additional information, the solution process requires preference information from a decision maker in some form or another.

In this talk, classes of various methods developed for nonlinear multiobjective optimization are briefly outlined and then the main attention is paid to interactive methods. In interactive methods, a solution pattern is formed and repeated several times, and in each iteration further information about the decision maker's preferences is inquired. In this way, the decision maker can learn about the nature of the problem and about the interdependencies among the objective functions involved. (S)he can also adjust one's preferences while learning and concentrate on such solutions that seem most promising.

Examples of interactive methods to be discussed are Pareto Navigator, PAINT and NAUTILUS, which starts from the worst possible objective function values and proceeds towards the most preferred Pareto optimal solution without the need of trading off between the objectives. Finally, the NIMBUS method is introduced which has been most widely applied in various application fields.

At the end, some industrial engineering and design applications are considered. For example, some computational challenges of solving complex simulation-based design problems are discussed. High computational cost and black-box type functions are examples of such challenges that set restrictions and requirements on the optimization methods to be used. Finally, experiences of solving some applications with the methods described are presented.

An Exact Algorithm for Nonconvex Quadratic Integer Minimization using Ellipsoidal Relaxations

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Minimizing a quadratic function over integer variables is an NP-hard problem in general. This remains true even if the objective function is convex and all variables are unbounded or binary. In our talk, we present a branch-and-bound approach for minimizing a not necessarily convex quadratic function over integer variables. The algorithm is based on dual bounds computed as continuous minima of the objective function over appropriate ellipsoids both for convex and non-convex subproblems in the Branch-and-Bound tree. In spite of the nonconvexity, these minima can be easily characterized and computed quickly. Indeed the corresponding primal optimization problems are trust-region subproblems. We present several ideas that allow to accelerate the solution of the continuous relaxation

within a branch-and-bound scheme. In particular a suitable preprocessing phase allows to perform heaviest computation once for all at the first node of the branch-and-bound tree, so that computation at each node does require a minimal effort. The approach turns out to be particularly effective for problems with small variable domains, yielding a fast algorithm for ternary quadratic optimization. We report computational experiment by comparing with standard state-of-the-art algorithms. We present computational results also for variables in larger integer domains, up to order of ten integer values, to exploit potentiality of the approach. Extension to the linear equality constrained case will be discussed.

Joint work with Christoph Buchheim, Marianna De Santis and Mauro Piacentini

Recent Applications of Nash Equilibrium in Engineering Economics

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This presentation will discuss some recent applications of non-cooperative game problems in the areas of energy markets, capacity expansion, structured estimation, and signal processing for communication. Mathematical issues arising from these applied domains that necessitate the development of new analysis will be highlighted.

Reduction Methods for Variational Conditions

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We show how in many cases the solution of a finite-dimensional variational condition can be simplified by a process of reduction applied to a linearization of the problem after the key step of localization. Under a standard regularity condition (coherent orientation), one can often solve the linearized problem by solving an affine variational inequality in a space of considerably smaller dimension than that of the space in which the original problem was posed. In addition to explaining the procedure and its justification, we will present some examples showing how to apply it.

Parallel Optimization of Large-Scale Multi-Agent Systems

GESUALDO SCUTARI

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The advances in wireless technology necessitate the development of theory, models and tools to cope with new challenges posed by large-scale optimization problems over wireless networks. The classical optimization works under the premise that all problem data is available to some central entity. This centralized design does not apply to large networked (wireless) systems where typically each user (agent) in the network has access to its local information and can exchange only limited signaling with its neighbors. The lecture will cover recent developments of distributed computational models for optimal resource management in multiuser wireless networks, both deterministic and stochastic. Specific topics will include: i) Examine various distributed successive-convex-approximation-based techniques for nonconvex sum-utility optimization problems, subject to private and/or shared constraints; ii) Establish a unified analysis of convergence of (inexact) parallel best-response-based decomposition algorithms; iii) Apply the developed machinery to the design of heterogeneous multi antenna wireless networks; iv) Use stochastic optimization techniques to develop efficient methods for resource management in MIMO wireless interfering networks with limited channel state information.

The content of this lecture is based on a series of recent papers coauthored with (in alphabetic order): Alberth Alvarado, Francisco Facchinei, Daniel Palomar, Jong-Shi Pang, and Peiran Song.

Recent Advances in Optimization and Optimal Control of Stochastic Dynamics — Applications in Finance, Economics and Life Sciences

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This presentation introduces into some recent research achievements in continuous-time models of the financial sector and related fields, supported by modern optimization theory. Stochastic Optimal Control has an increasingly important role in science, economics and the sectors of environment and finance, and is extensively used in various applications. We present applications of Stochastic Hybrid models in biology, ecology, monetary systems and finance to account for regime switching dynamics. Stochastic models with a motion part and additionally a jump part are able to capture abrupt fluctuations that are a usual phenomenon in genetic and environmental networks and in financial markets. These kinds of models allow for more realistic investigation of portfolio optimization and utility maximization in financial markets and in genetic, metabolic and ecological interaction. The models comprise portfolio optimization with optimal investment and consumption strategies. Explicit consideration of risk aversion in an optimal investment and consumption problem allows for optimality conditions that are related to specific risk types in a market. A more general model for portfolio and gene-environment optimization is established afterwards. In related second study, we develop a new

theory of estimating Hurst parameter using conic multivariate adaptive regression splines (CMARS) method. Stochastic Differential Equations (SDEs) generated by fractional Brownian motion (fBm) with Hurst parameter, H , are widely used to represent noisy and real-world problems. The reason why fBm is preferred in modeling, to other Markov processes is its property of capturing the dependence structure of observations. It is, therefore, a more realistic model compared to Markov processes. The superiority of our approach to the others is, it not only estimates the Hurst parameter but also finds spline parameters of the stochastic process in an adaptive way. We examine the performance of our estimations using simulated test data. The presentation ends with a conclusion and an outlook to future studies.

An Augmented Lagrangian trust region method for equality constrained optimization

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In this talk, we present a trust region method for solving equality constrained optimization problems, which is motivated by the famous augmented Lagrangian function. It is different from standard augmented Lagrangian methods where the augmented Lagrangian function is minimized at each iteration. This method, for fixed Lagrange multiplier and penalty parameters, tries to minimize an approximate model of the augmented Lagrangian function to generate the next iterate. A condition is introduced to decide whether the Lagrange multiplier should be updated. We also suggest a new strategy for adjusting the penalty parameters. Global convergence of this method is established under mild conditions. Furthermore, we analyze the behavior of penalty parameters and figure out in which case they will be bounded from above. Numerical experiments on problems from the CUTer set collection reveal that our method is very promising.

This is a joint work with Dr. Xiao WANG
