

# Roman Games II

Thursday April the 26th, 2018  
DIAG Aula Magna, Via Ariosto 25, Roma

The Department of Computer, Control, and Management Engineering Antonio Ruberti (DIAG) hosts the second edition of Roman Games, an informal one-day workshop on Multiagent Optimization (game theory, optimization over networks, distributed computing, equilibrium problems and more). The aim of these meetings is to bring together people in the roman region and some guests working in Multiagent Optimization, but is open to contributions from anybody interested in this broad research area.

The meeting will take place on April the 26th, 2018 in the Aula Magna of DIAG, Via Ariosto 25, Roma. Participation is free.

This meeting has been organized by Francisco Facchinei (Università di Roma La Sapienza) and Roberto Lucchetti (Politecnico di Milano).

- 09.45 – 10.30: Roberto Cominetti (Universidad Adolfo Ibañez, Chile)  
Dynamic equilibria for network flows
- 10.30 – 11.15: Marco Scarsini (Luiss, Roma)  
Demand-Independent Optimal Tolls
- 11.15 – 11.45: Break
- 11.45 – 12.30: Lorenzo Lampariello (Università di Roma 3, Roma)  
Numerically tractable optimistic bilevel problems
- 12.30 – 13.15: Maurizio Falcone (Università di Roma La Sapienza, Roma)  
Some remarks on pursuit-evasion games with  $p$ -players
- 13.15 – 14.30: Lunch
- 14.30 – 15.15: Sylvain Sorin (Sorbonne Université, Paris)  
Nash equilibria and variational inequalities: on the graph of the equilibrium correspondence
- 15.15 – 16.30: Tristan Garrec (Université Toulouse 1 Capitole, Toulouse)  
On Search Games
- 16.30 – 17.15: Nicola Gatti (Politecnico di Milano, Milano)  
Leadership in Singleton Congestion Games

## **Dynamic equilibria for network flows**

Roberto Cominetti,  
Universidad Adolfo Ibañez, Chile

Dynamic routing games describe the temporal evolution of a continuous stream of players who seek to travel from origin to destination in the least possible time. Players are forward looking and anticipate the congestion they would experience upon arrival to any edge. Although the model has been around for decades, little is known on the existence of equilibria and even less on their characterization and computation. In this talk we review some recent progress in the case of fluid queueing networks where each edge is composed of a queue plus a constant delay link. We discuss the existence of dynamic equilibria, their characterization and computation, and the finite time convergence to a steady state.

## **Demand-Independent Optimal Tolls**

Marco Scarsini,  
Luiss, Roma

Wardrop equilibria in nonatomic congestion games are in general inefficient as they do not induce an optimal flow that minimizes the total travel time. Network tolls are a prominent and popular way to induce an optimum flow in equilibrium. The classical approach to find such tolls is marginal cost pricing which requires the exact knowledge of the demand on the network. In this paper, we investigate under which conditions demand-independent optimum tolls exist that induce the system optimum flow for any travel demand in the network. We give several characterizations for the existence of such tolls both in terms of the cost structure and the network structure of the game. Specifically we show that demand-independent optimum tolls exist if and only if the edge cost functions are shifted monomials as used by the Bureau of Public Roads. Moreover, non-negative demand-independent optimum tolls exist when the network is a directed acyclic multigraph. Finally, we show that any network with a single origin-destination pair admits demand-independent optimum tolls that, although not necessarily non-negative, satisfy a budget constraint.

## **Numerically tractable optimistic bilevel problems**

Lorenzo Lampariello,  
Università di Roma 3, Roma

We consider a class of optimistic bilevel problems. Specifically, we address bilevel problems in which the lower level objective function is fully convex. We show that this nontrivial class of mathematical programs is sufficiently broad to encompass significant real-world applications and proves to be numerically tractable. From this respect, we establish that the critical points for a relaxation of the original problem can be obtained addressing a suitable generalized Nash equilibrium problem. The latter game is proven to be convex and with a nonempty solution set. Leveraging this correspondence, we provide a provably convergent, easily implementable scheme to calculate critical points of the relaxed bilevel

program. As witnessed by some numerical experiments on an application in economics, this algorithm turns out to be numerically viable also for big dimensional problems.

### **Some remarks on pursuit-evasion games with $p$ -players**

Maurizio Falcone,

Università di Roma La Sapienza, Roma

The Dynamic Programming approach for 2-players pursuit evasion games leads to the characterization of the value function as the unique viscosity solution of the Hamilton-Jacobi-Isaacs equation. This approach has been investigated from the analytical point of view and some numerical schemes have been proposed. The main advantage of the DP approach is to provide a synthesis of feedback controls which allows to get the optimal trajectories for the two players starting from any initial condition in a space domain, moreover state constraints can be easily incorporated in the model. The drawback of this approach is that the Isaacs equation is set in a space of dimension  $2d$  where  $d$  is the dimension of the state space of each player. This makes the numerical solution unfeasible for  $d > 3$  due to the huge number of variables required to store the grid and the values. The situation is even more difficult for pursuit-evasion games with  $p$ -players where in principle one has to take into account  $p \times d$  variables to describe the global dynamics of the game. We will discuss how to get a sub-optimal strategy for the players dealing with a series of 2-players pursuit evasion games and we will show some numerical results for that problem.

### **Nash equilibria and variational inequalities: on the graph of the equilibrium correspondence**

Sylvain Sorin,

Sorbonne Université, Institut de Mathématiques de Jussieu-Paris Rive Gauche, Paris

We recall some classes of games where Nash equilibria are obtained as solutions of variational inequalities. We then extend to this framework some results of Kohlberg, Mertens, Demichelis, Germano, Govidan, Wilson on the equilibrium correspondence.

### **On Search Games**

Tristan Garrec,

Université Toulouse 1 Capitole, Toulouse

In a search game, two players, a searcher and a hider, act on a search space. The searcher typically intends to minimize the time needed to locate the mobile or immobile hider. The searcher chooses a trajectory in the search space and detects the hider when he is sufficiently close to him. We will present variants of search games for which we compute the value and optimal strategies of the players depending on the geometry of the search space.

### **Leadership in Singleton Congestion Games**

Nicola Gatti,

Politecnico di Milano, Milano

We study, for the first time to the best of our knowledge, Stackelberg games where the underlying structure is a congestion game. We recall that, while leadership in 2-player games has been widely investigated, only few results are known when the number of players is three or more. The intractability of finding a Stackelberg equilibrium (SE) in normal-form and polymatrix games is among them. In this paper, we focus on congestion games in which each player can choose a single resource (a.k.a. singleton congestion games) and a player acts as leader. We show that, without further assumptions, finding an SE when the followers break ties in favor of the leader is not in Poly-APX, unless  $P = NP$ . Instead, under the assumption that every player has access to the same resources and that the cost functions are monotonic, we show that an SE can be computed efficiently when the followers break ties either in favor or against the leader.