Second-Order Know-How Strategies

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Consider the following traffic situation

Assumption:
- The driver of car $d$ does not notice the stop sign.
Truck $b$ maintains the same speed
Truck $b$ slows down
Truck $b$ accelerates

To prevent a collision, both truck $b$ and car $a$ must accelerate.
Truck $b$ accelerates

To prevent a collision, both truck $b$ and car $a$ must accelerate.
Coalition \{a, b\} has a \textit{strategy} to prevent a collision.
Coalition $\{a, b\}$ has a *strategy* to prevent a collision.

Coalition $\{a, b\}$ does not know such a strategy exists.

Car $c$ knows what is the strategy of coalition $\{a, b\}$ to avoid a collision.
Second-order know-how strategies

\[ w_1 \models H_{\{a,b\}}^{\{c\}} \text{ ("avoid a collision") } \]

\[ w \models H_D^C \varphi: \text{ if coalition } C \text{ has distributed knowledge of how coalition } D \text{ can achieve outcome } \varphi \text{ from state } w. \]
Epistemic Transition System

Strategy profile \((s_a, s_b)\)

Domain of actions:

+: to accelerate

\(-: to slow down

0: to maintain speed

\[ w_4: \text{b-d rear-side collision} \]

\[ w_5: \text{a-b rear-end collision} \]

\[ w_6: \text{no collision} \]

\[ w_7: \text{b-d front-side collision} \]
Epistemic Transition System

- $w_1$: car d is at the spot X
- $w_2$: car d is not present at the scene

Strategy profile $(s_a, s_b)$

Domain of actions:
- $+$: to accelerate
- $-$: to slow down
- 0: to maintain speed

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Definition 1
A tuple \((W, \{\sim_a\}_{a \in \mathcal{A}}, \Delta, M, \pi)\) is an (epistemic) transition system, if

1. \(W\) is a set (of epistemic states),
2. \(\sim_a\) is an indistinguishability equivalence relation on set \(W\) for each \(a \in \mathcal{A}\),
3. \(\Delta\) is a nonempty set, called the domain of actions,
4. \(M \subseteq W \times \Delta^\mathcal{A} \times W\) is an aggregation mechanism,
5. \(\pi\) is a function from propositional variables to subsets of \(W\).

For any states \(w_1, w_2 \in W\) and any coalition \(C\), let \(w_1 \sim_C w_2\) if \(w_1 \sim_a w_2\) for each agent \(a \in C\).

A strategy profile of a coalition \(C\) is a tuple of values from \(\Delta\) indexed by set \(C\). A complete strategy profile is a strategy profile of the coalition \(\mathcal{A}\).
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For any states $w_1, w_2 \in W$ and any coalition $C$, let $w_1 \sim_C w_2$ if $w_1 \sim_a w_2$ for each agent $a \in C$.

A **strategy profile of a coalition** $C$ is a tuple of values from $\Delta$ indexed by set $C$. A **complete** strategy profile is a strategy profile of the coalition $\mathcal{A}$. 
Definition 2

Let $\Phi$ be the minimal set of formulae such that

1. $p \in \Phi$ for each propositional variable $p$,
2. $\neg \varphi, \varphi \rightarrow \psi \in \Phi$ for all formulae $\varphi, \psi \in \Phi$,
3. $K_C \varphi, H^D_C \varphi \in \Phi$ for each coalition $C$, each finite coalition $D$, and each formula $\varphi \in \Phi$. 
Definition 3
For any epistemic state $w \in W$ of a transition system $(W, \{\sim_a\}_{a \in A}, V, M, \pi)$ and any formula $\varphi \in \Phi$, let relation $w \models \varphi$ be defined as follows:

1. $w \models p$ if $w \in \pi(p)$, where $p$ is a propositional variable,
2. $w \models \neg \varphi$ if $w \not\models \varphi$,
3. $w \models \varphi \rightarrow \psi$ if $w \not\models \varphi$ or $w \models \psi$,
4. $w \models K_C \varphi$ if $w' \models \varphi$ for each $w' \in W$ such that $w \sim_C w'$,
5. $w \models H^D_C \varphi$ if there is a strategy profile $s \in V^D$ such that for any two states $w', u \in W$ and any complete strategy profile $s'$, if $w \sim_C w'$, $s =_D s'$, and $(w', s', u) \in M$, then $u \models \varphi$. 
Axioms

1. Truth: $K_C\varphi \rightarrow \varphi$,

2. Negative Introspection: $\neg K_C\varphi \rightarrow K_C\neg K_C\varphi$,

3. Distributivity: $K_C(\varphi \rightarrow \psi) \rightarrow (K_C\varphi \rightarrow K_C\psi)$,

4. Monotonicity: $K_C\varphi \rightarrow K_D\varphi$, if $C \subseteq D$, 

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5. Cooperation: $H_{C_1}^{D_1} (\varphi \rightarrow \psi) \rightarrow (H_{C_2}^{D_2} \varphi \rightarrow H_{C_1 \cup C_2}^{D_1 \cup D_2} \psi)$, where $D_1 \cap D_2 = \emptyset$. 
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6. Strategic Introspection: $H^{D}_{C} \varphi \rightarrow K_C H^{D}_{C} \varphi$, 

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6. Strategic Introspection: $H^D_C \varphi \rightarrow K_C H^D_C \varphi$,

7. Empty Coalition: $K_\emptyset \varphi \rightarrow H_\emptyset \varphi$. 
Axioms

1. Truth: \( K_C \varphi \rightarrow \varphi \),
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3. Distributivity: \( K_C (\varphi \rightarrow \psi) \rightarrow (K_C \varphi \rightarrow K_C \psi) \),
4. Monotonicity: \( K_C \varphi \rightarrow K_D \varphi \), if \( C \subseteq D \),
5. Cooperation: \( H_{D_1}^{D_2} C_1 (\varphi \rightarrow \psi) \rightarrow (H_{C_2}^{D_2} \varphi \rightarrow H_{C_1 \cup C_2}^{D_1 \cup D_2} \psi) \), where \( D_1 \cap D_2 = \emptyset \).
6. Strategic Introspection: \( H_C^{D} \varphi \rightarrow K_C H_C^{D} \varphi \),
7. Empty Coalition: \( K_\emptyset \varphi \rightarrow H_\emptyset \varphi \).
8. Knowledge of Unavoidability: \( K_A H_B^\emptyset \varphi \rightarrow H_A^\emptyset \varphi \).
Inference Rules

1. Necessitation: $\frac{\varphi}{K_C \varphi}$

2. Strategic Necessitation: $\frac{\varphi}{H_D C \varphi}$

3. Modus Ponens: $\frac{\varphi, \varphi \rightarrow \psi}{\psi}$
Conclusion

AAMAS (International Conference on Autonomous Agents and Multiagent Systems) 2018

- A language for modeling an interplay between the distributed knowledge modality and the second-order coalition know-how modality.
- A sound and complete logical framework.