Modeling and Control Alternatives for Robots in Dynamic Cooperation

A. De Luca R. Mattone

Dipartimento di Informatica e Sistemistica
Università di Roma "La Sapienza"
Via Eudossiana 18, 00184 Roma, Italy

Abstract

A general control-oriented formalism has been introduced in [1] to describe robot-environment interaction in the case of a single robot in contact with a possibly dynamic environment. Two possibilities were obtained in the design of hybrid motion-force control laws depending on whether motion or force is explicitly controlled along properly defined dynamic directions. An extension of this formalism for modeling and controlling cooperating robots in the manipulation of a payload is presented. Several contact arrangements and environmental constraints on the object can be considered, mixing the presence of kinematic constraints and dynamic interactions. The modeling approach leads to the characterization of complementary directions in the task space: those where only end-effector velocities are admissible, those where only reaction forces may exist, and those in which energy can be transferred between each robot and the payload. Two classes of model-based hybrid force-motion controllers are then designed, similarly to the single robot case. We show that the classical approach of controlling payload motion and internal forces for cooperating robots is recovered as one special case. Moreover, within this framework a new control alternative naturally arises, namely the possibility of controlling both internal forces and active forces producing motion. Numerical simulation results are reported for power grasp and hard finger point contacts and with the two alternative control laws.

1. Introduction

Modeling and control of robotic systems in strict cooperation tasks has become a main area of theoretical investigation and applied research. A typical cooperative system is constituted by two or more robots rigidly grasping and manipulating a single object in the free space. Specific kinematic modeling issues deal mainly with the analysis of contact constraints [2] and the characterization of mobility and differential kinematics [3]. Duality between kinematics and statics of cooperating robots has been developed in [4], where the concept of internal forces was also introduced. Classical approaches to dynamic modeling of cooperating robots have been developed by many authors [2],[5–8]. Dynamic equations are written separately for each robot, while Newton-Euler equations are used for the carried payload. Generalized contact forces couple the robot and the object equations, but are typically eliminated through algebraic manipulations. With a Lagrangian approach, the dynamic model of the overall robotic system is obtained from the global mechanical energy [8,9], together with the holonomic constraints of the contact. A full characterization of the grasping kinematics and dynamics can be found in [9].

Several control strategies have been proposed for multiple robots handling a single object, both at a kinetostatic level or taking into account the system dynamics. In most cases, dynamic control is achieved through a model-based law that follows from a similar scheme developed for a single robot in constrained motion: *impedance control* [10] is based on a dynamic impedance behavior assumed for the controlled position variables, while *hybrid control* separately handles object motion and contact forces [5,6].

In the case of a single robot in contact with an environment for which a dynamic description is available, De Luca and Manes [1],[11,12] have introduced a convenient formalism for describing the robot-environment interaction, that allows a generalization of hybrid force-motion control. Based on energy transfer arguments, one can define and explicitly characterize three sets of generalized task directions: *i)* directions where motion is not allowed, due to the presence of rigid kinematic constraints; *ii)* free directions where no contact forces or torques may arise; *iii) dynamic directions* where *either* force *or* motion can be independently assigned, being these quantities related by an energy transfer balance. The description is based on a parametrization of all relevant quantities, from which suitable complementarity relationships among the above sets of directions follow. In particular, the introduction of dynamic directions results in two alternatives in the design of hybrid controllers, correspond-

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ing to the choice of controlling force or, respectively, motion along the dynamic directions.

In this paper we show how to extend this formalism to the case of multiple cooperating robots, where the carried payload is seen from each robot as a coupling dynamic environment. An overall treatment of the interactions is needed in order to select dynamic components of the contact forces which contribute to the object motion. In this context, internal forces in the cooperation are equivalent to reaction forces in the case of a single robot in compliant motion. The proposed formalism allows to recover other already known approaches, such as those in [2,6,13,14], giving further flexibility to include more general cases, such as the presence of other kinematic constraints imposed by the rest of the environment on the commonly held object.

The paper is organized as follows. First, the kinematic and dynamic modeling approach for cooperating robots is presented in general. Two representative case studies are then worked out, where two robots commonly hold an object with a power grasp or, respectively, with hard finger point contact. Decoupling and linearizing hybrid controllers are then derived, directly controlling either object motion and internal forces or the overall set of (active and internal) contact forces. Numerical simulation results are reported for the different contacts and control laws, using two planar 3-dof arms.

2. Cooperating Robots-Environment Model

The modeling approach proposed in [1] and suitable for the design of hybrid motion-force control laws, can be extended to the case of m cooperating robots manipulating a rigid object with d degrees of freedom which represents the external environment.

Consider, without loss of generality, $m = 2$ robots with $n_i \geq 6$ joints, $i = 1, 2$, handling a rigid object with $d \leq 6$ dof's of motion. Each manipulator configuration is identified by the joint variables vector $\mathbf{q}_i \in I\!R^{n_i}$, while its end-effector pose by a vector $\mathbf{p}_i$ that collects the position, $\mathbf{r}_i \in I\!R^3$, and a minimal representation of the orientation, $\mathbf{o}_i \in I\!R^3$ (e.g. Euler angles). The end-effector velocity $\mathbf{v}_i = (\dot{\mathbf{r}}_i, \omega_i) \in I\!R^6$, is related to the pose time derivative $\dot{\mathbf{p}}_i$ by a matrix transformation depending on $\mathbf{o}_i$. The end-effector kinematics is described *from the i-th robot side* by

$$\mathbf{p}_i = \mathbf{k}_i(\mathbf{q}_i), \qquad \mathbf{v}_i = \mathbf{J}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i, \tag{1}$$

where $\mathbf{J}_i(\mathbf{q}_i)$ is the standard (geometric) Jacobian of the ith robot.

The object configuration is identified by a set of generalized coordinates $\mathbf{s}_D \in I\!R^d$ that are needed to describe the *environment dynamics*. Depending on the type of contact between each robot and the object, an additional set $\mathbf{s}_{K_i} \in I\!R^{k_i}$ of *purely kinematic* variables is in general necessary to completely describe the pose of the ith end-effector *from the environment side*. Then,

$$\mathbf{p}_i = \mathbf{\Gamma}_i(\mathbf{s}_{K_i}, \mathbf{s}_D), \quad \mathbf{v}_i = \mathbf{T}_i(\mathbf{s}_{K_i}, \mathbf{s}_D) \begin{bmatrix} \dot{\mathbf{s}}_{K_i} \\ \dot{\mathbf{s}}_D \end{bmatrix}. \tag{2}$$

Matrix $\mathbf{T}_i$ is partitioned as

$$\mathbf{T}_i(\mathbf{s}_{K_i}, \mathbf{s}_D) = \begin{bmatrix} \mathbf{T}_{K_i}(\mathbf{s}_{K_i}, \mathbf{s}_D) & \mathbf{T}_{D_i}(\mathbf{s}_{K_i}, \mathbf{s}_D) \end{bmatrix}, \tag{3}$$

expliciting the contributions due to *kinematic* and *dynamic* degrees of freedom in the robot-environment interaction. This matrix is assumed to be full rank, at least in a region of interest for task execution.

Equations (1) and (2), written for each robot separately, can be rewritten for whole robotic system as

$$\mathbf{p} = \mathbf{k}(\mathbf{q}), \qquad \mathbf{v} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \tag{4a}$$

and

$$\mathbf{p} = \mathbf{\Gamma}(\mathbf{s}), \qquad \mathbf{v} = \mathbf{T}(\mathbf{s})\dot{\mathbf{s}}, \tag{4b}$$

where $\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2)$, $\mathbf{s} = (\mathbf{s}_{K_1}, \mathbf{s}_{K_2}, \mathbf{s}_D)$, $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2)$, $\mathbf{v} = (\mathbf{v}_1, \mathbf{v}_2)$, $\mathbf{k} = (\mathbf{k}_1(\mathbf{q}_1), \mathbf{k}_2(\mathbf{q}_2))$, $\mathbf{\Gamma} = (\mathbf{\Gamma}_1(\mathbf{s}), \mathbf{\Gamma}_2(\mathbf{s}))$, and

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} \mathbf{J}_1(\mathbf{q}_1) & \mathbf{O} \\ \mathbf{O} & \mathbf{J}_2(\mathbf{q}_2) \end{bmatrix},$$

$$\mathbf{T}(\mathbf{s}) = \begin{bmatrix} \mathbf{T}_{K_1} & \mathbf{O} & \mathbf{T}_{D_1} \\ \mathbf{O} & \mathbf{T}_{K_2} & \mathbf{T}_{D_2} \end{bmatrix} = \begin{bmatrix} \mathbf{T}_K(\mathbf{s}) \; \mathbf{T}_D(\mathbf{s}) \end{bmatrix}. \tag{5}$$

The exchanged forces between the ith end-effector and the object are collected in a 6-dimensional vector $\mathbf{F}_i$ of forces and torques, with $\mathbf{F} = (\mathbf{F}_1, \mathbf{F}_2)$. Similarly to (4), contact forces can be parameterized as $\mathbf{F} = \mathbf{Y}(\mathbf{s})\lambda$. The columns of the matrix $\mathbf{Y}(\mathbf{s})$ are generalized directions used as a basis for the vector space $span[\mathbf{Y}(\mathbf{s})]$ of admissible contact forces. Since work cannot be performed on kinematic degrees of freedom, matrix $\mathbf{Y}$ must be such that $\mathbf{Y}^{\mathrm{T}}\mathbf{T}_K = \mathbf{0}$. Moreover, $\mathbf{Y}$ is assumed full rank just as $\mathbf{T}$, yielding a vector of parameters $\lambda \in I\!R^{12-k}$, with $k = k_1 + k_2$. At each $\mathbf{s}$, the vector space $span[\mathbf{Y}(\mathbf{s})]$ can be decomposed into two subspaces. The subspace of static reaction force directions, $span[\mathbf{Y}_R(\mathbf{s})]$, is defined through $\mathbf{T}^{\mathrm{T}}\mathbf{Y}_R = \mathbf{0}$ and is of dimension $12-e$, with $e = k + d$. Its complement, $span[\mathbf{Y}_A(\mathbf{s})]$, is the d-dimensional subspace of active force directions responsible for object motion. Any matrix $\mathbf{Y}_A$ used as a basis will be such that $\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A$ is nonsingular. Following the decomposition of $\mathbf{Y}(\mathbf{s})$, a partition is induced on $\lambda = (\lambda_A, \lambda_R)$. Note that we

3-dimensional vector which contains a minimal representation of the orientation defined by matrix $\mathbf{R}$, and ${}^{O}\mathbf{K}(\varepsilon_O)$ is the transformation between the time derivative of vector ε_O and the angular velocity of the object (expressed in the object frame).

Each robot can independently exert any force and torque at the contact, so that the object can be *moved* and *squeezed* along and about every cartesian direction. Hence, λ_A and λ_R are both 6-dimensional. Given the matrix $\mathbf{T}$ from (13b), a feasible choice of $\mathbf{Y}_R$ satisfying $\mathbf{T}^{\mathrm{T}}\mathbf{Y}_R = \mathbf{0}$ is

$$\mathbf{Y}_R(\mathbf{s}) = \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{O}_{3\times3} \\ -{}^{B}\mathbf{R}_O(\mathbf{s})\mathbf{S}({}^{O}\mathbf{r}_{c_1}){}^{B}\mathbf{R}_O^{\mathrm{T}}(\mathbf{s}) & \mathbf{I}_{3\times3} \\ -\mathbf{I}_{3\times3} & \mathbf{O}_{3\times3} \\ {}^{B}\mathbf{R}_O(\mathbf{s})\mathbf{S}({}^{O}\mathbf{r}_{c_2}){}^{B}\mathbf{R}_O^{\mathrm{T}}(\mathbf{s}) & -\mathbf{I}_{3\times3} \end{bmatrix}, \tag{14}$$

where $\mathbf{S}(\mathbf{r})$ is the standard skew-symmetric matrix generated with the components of vector $\mathbf{r}$. Indeed, this is just one possible choice. A completion of $\mathbf{Y}$ with the maximal set of linear independent columns of $\mathbf{Y}_A$ can be chosen as

$$\mathbf{Y}_A(\mathbf{s}) = \begin{bmatrix} \frac{1}{2}\mathbf{I}_{3\times3} & \mathbf{O}_{3\times3} \\ -\frac{1}{2}{}^{B}\mathbf{R}_O(\mathbf{s})\mathbf{S}({}^{O}\mathbf{r}_{c_1}){}^{B}\mathbf{R}_O^{\mathrm{T}}(\mathbf{s}) & \frac{1}{2}\mathbf{I}_{3\times3} \\ \frac{1}{2}\mathbf{I}_{3\times3} & \mathbf{O}_{3\times3} \\ -\frac{1}{2}{}^{B}\mathbf{R}_O(\mathbf{s})\mathbf{S}({}^{O}\mathbf{r}_{c_2}){}^{B}\mathbf{R}_O^{\mathrm{T}}(\mathbf{s}) & \frac{1}{2}\mathbf{I}_{3\times3} \end{bmatrix} \tag{15}$$

which is such that $\mathbf{T}\mathbf{Y}_A$ is nonsingular. This choice is identical to the *non-squeezing pseudoinverse* $\mathbf{G}_{\Delta}^{+}$ defined in [14], starting from the grasp matrix $\mathbf{G}$ [9] of the considered robot-object interaction. In particular, it implies that the dynamic load is equally distributed between the two robots. With the choices (14) and (15), the parameter vector λ_A is the net generalized force acting on the object, while λ_R parametrizes the contact forces causing no object motion. In this case, the dynamic modeling can be worked out up to the form (10) that fits the known dynamic description of cooperating robot systems in [2],[5–8],[13]. Indeed, the dynamic format (11) is a perfectly valid and *new* alternative in which the object dynamics is treated as a dependent quantity, in place of contact forces.

3.2 Hard finger contact

Consider now the two robots holding the object with a point contact (see Fig. 1). We suppose the presence of infinite friction, so that robots can exert contact forces on the object in every direction and contact points do not slip on the object surface during task execution. Further, assume that the rigid object has one principal axis of inertia which coincides with the line h joining the two contact points. Since each end-effector behaves as a 'hard finger', i.e., it cannot exert any torque at the contact, no angular momentum variation can occur about the direction h. Thus, the angular velocity about this direction is constant and assumed zero at the initial instant. For the purpose of the cooperation task, only five parameters are sufficient to describe the object dynamics ($d = 5$).

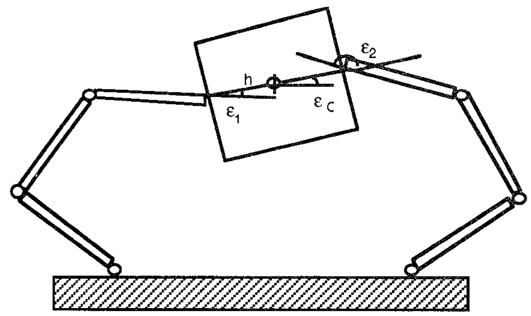

Fig. 1 – Hard finger contact

Choosing an object frame with the X axis along the direction h, a possible choice for $\mathbf{s}_D$ is

$$\mathbf{s}_D = (\mathbf{p}_O, \theta_{OZ}, \phi_{OY}), \tag{16}$$

where θ_{OZ} and ϕ_{OY} are the first two angles of a ZYX-Euler triple. Both end-effector orientations are not constrained by the contact type. Let $\mathbf{s}_{K_1} = \varepsilon_1$ and $\mathbf{s}_{K_2} = \varepsilon_2$ be two sets of Euler angles used to represent the absolute orientation of the two robot end effectors. Then, $k_1 = k_2 = 3$ and we have

$$\mathbf{p}_1 = \begin{bmatrix} \mathbf{p}_O - r_{c_1}\bar{\mathbf{x}}(\theta_{OZ}, \phi_{OY}) \\ \varepsilon_1 \end{bmatrix}, \quad \mathbf{p}_2 = \begin{bmatrix} \mathbf{p}_O + r_{c_2}\bar{\mathbf{x}}(\theta_{OZ}, \phi_{OY}) \\ \varepsilon_2 \end{bmatrix}, \tag{17a}$$

and

$$\begin{aligned} \mathbf{v}_1 &= \begin{bmatrix} \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} \\ {}^{B}\mathbf{R}_{R_1}(\varepsilon_1)\,{}^{R_1}\mathbf{K}(\varepsilon_1) & \mathbf{O}_{3\times3} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{s}}_{K_1} \\ \dot{\mathbf{s}}_{K_2} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{3\times3} & -r_{c1}\frac{\partial\bar{\mathbf{x}}(\theta_{OZ},\phi_{OY})}{\partial(\theta_{OZ},\phi_{OY})} \\ \multicolumn{2}{c}{\mathbf{O}_{3\times5}} \end{bmatrix} \dot{\mathbf{s}}_D, \\ \mathbf{v}_2 &= \begin{bmatrix} \mathbf{O}_{3\times3} & \mathbf{O}_{3\times3} \\ \mathbf{O}_{3\times3} & {}^{B}\mathbf{R}_{R_2}(\varepsilon_2)\,{}^{R_2}\mathbf{K}(\varepsilon_2) \end{bmatrix} \begin{bmatrix} \dot{\mathbf{s}}_{K_1} \\ \dot{\mathbf{s}}_{K_2} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{3\times3} & r_{c2}\frac{\partial\bar{\mathbf{x}}(\theta_{OZ},\phi_{OY})}{\partial(\theta_{OZ},\phi_{OY})} \\ \multicolumn{2}{c}{\mathbf{O}_{3\times5}} \end{bmatrix} \dot{\mathbf{s}}_D, \end{aligned} \tag{17b}$$

where $\bar{\mathbf{x}}$ is the unit vector of the X-axis of the object frame, r_{c_i} is the distance between the origin of object can always normalize the parametrization so that the components of λ physically correspond to a generalized force (a force or a torque). In particular, the components of λ_R represent the so-called *internal forces* in the grasping, while λ_A are the net active forces producing motion of the object. We call in the sequel *dynamic directions* those in the span of the columns of either $\mathbf{T}_D$ or $\mathbf{Y}_A$.

Summarizing, the kinematic and static descriptions are

$$\mathbf{v} = \mathbf{T}(\mathbf{s})\dot{\mathbf{s}} = \mathbf{T}_K(\mathbf{s})\dot{\mathbf{s}}_K + \mathbf{T}_D(\mathbf{s})\dot{\mathbf{s}}_D, \quad (6a)$$

$$\mathbf{F} = \mathbf{Y}(\mathbf{s})\lambda = \mathbf{Y}_A(\mathbf{s})\lambda_A + \mathbf{Y}_R(\mathbf{s})\lambda_R, \quad (6b)$$

with

$$\begin{aligned} &\mathbf{T}_{(12\times e)} = \left[\mathbf{T}_{K\,(12\times k)}\ \mathbf{T}_{D\,(12\times d)}\right], \\ &\mathbf{Y}_{[12\times(12-k)]} = \left[\mathbf{Y}_{A\,(12\times d)}\ \mathbf{Y}_{R\,(12\times(12-e))}\right], \\ &\mathbf{T}^{\mathrm{T}}\mathbf{Y}_R = \mathbf{0}_{e\times(12-e)}, \quad \mathbf{Y}^{\mathrm{T}}\mathbf{T}_K = \mathbf{0}_{(12-k)\times k}, \\ &\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A : \ d\times d \text{ nonsingular matrix.} \end{aligned} \quad (7)$$

Following the Lagrangian approach, the dynamic model of the overall robotic system can be written in the usual form as

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{u} - \mathbf{J}^{\mathrm{T}}(\mathbf{q})\mathbf{F}, \quad (8a)$$

$$\mathbf{B}_O(\mathbf{s}_D)\ddot{\mathbf{s}}_D + \mathbf{n}_O(\mathbf{s}_D,\dot{\mathbf{s}}_D) = \mathbf{T}_D^{\mathrm{T}}(\mathbf{s})\mathbf{F}, \quad (8b)$$

where the force $\mathbf{F}$ acts from the robots to the object, $\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2)$ collects the joint input forces/torques of the two robots, and

$$\mathbf{B}(\mathbf{q}) = \begin{bmatrix} \mathbf{B}_1(\mathbf{q}_1) & \mathbf{O} \\ \mathbf{O} & \mathbf{B}_2(\mathbf{q}_2) \end{bmatrix}, \quad \mathbf{n}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} \mathbf{n}_1(\mathbf{q}_1,\dot{\mathbf{q}}_1) \\ \mathbf{n}_2(\mathbf{q}_2,\dot{\mathbf{q}}_2) \end{bmatrix}.$$

The symmetric and positive definite inertia matrices $\mathbf{B}(\mathbf{q})$ of the robots and $\mathbf{B}_O(\mathbf{s}_D)$ of the object have dimensions $n\times n$ and $d\times d$, respectively. The couplings between the dynamics of the robots and of the object are given by the constraints on the end-effectors pose obtained equating (4a) or, respectively, (4b)

$$\mathbf{k}(\mathbf{q}) = \Gamma(\mathbf{s}), \qquad \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{T}(\mathbf{s})\dot{\mathbf{s}}, \quad (9)$$

i.e. in their algebraic and differential forms. Equations (9) can be used to eliminate the explicit appearance of the force vector $\mathbf{F}$ in (8). This can be achieved in two different ways, depending on the handling of the dynamic directions. In the first case, the input $\mathbf{u}$ affects directly the dynamic accelerations, that cause in turn active forces λ_A. In the second, λ_A are explicited as directly depending on $\mathbf{u}$, while accelerations $\ddot{\mathbf{s}}_D$ are seen as a consequence. The resulting alternative dynamic relationships can be written as (see also [1])

$$\mathbf{Q}(\mathbf{q},\mathbf{s}) \begin{bmatrix} \ddot{\mathbf{s}}_D \\ \lambda_R \\ \ddot{\mathbf{s}}_K \end{bmatrix} = \mathbf{m}(\mathbf{q},\dot{\mathbf{q}},\mathbf{s},\dot{\mathbf{s}}) + \mathbf{J}\mathbf{B}^{-1}\mathbf{u}, \quad (10a)$$

$$\lambda_A = (\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A)^{-1}\mathbf{n}_O + (\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A)^{-1}\mathbf{B}_O\ddot{\mathbf{s}}_D. \quad (10b)$$

or, respectively,

$$\widehat{\mathbf{Q}}(\mathbf{q},\mathbf{s}) \begin{bmatrix} \lambda_A \\ \lambda_R \\ \ddot{\mathbf{s}}_K \end{bmatrix} = \widehat{\mathbf{m}}(\mathbf{q},\dot{\mathbf{q}},\mathbf{s},\dot{\mathbf{s}}) + \mathbf{J}\mathbf{B}^{-1}\mathbf{u}, \quad (11a)$$

$$\ddot{\mathbf{s}}_D = -\mathbf{B}_O^{-1}\mathbf{n}_O + \mathbf{B}_O^{-1}\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A\lambda_A, \quad (11b)$$

Both matrices $\mathbf{Q}$ and $\widehat{\mathbf{Q}}$ are invertible, under the assumed hypotheses, and depend on the robots and object inertia matrices, $\mathbf{B}$ and $\mathbf{B}_O$, and on the Jacobians $\mathbf{J}$ and $\mathbf{T}$, while vectors $\mathbf{m}$ and $\widehat{\mathbf{m}}$ depend also on their time derivatives (see [1,12]).

3. Modeling Applications

3.1 Power grasp

Consider two robots with $n_1 = n_2 = 6$, tightly grasping an object at a fixed point so that no relative motion is admissible at the contact. The object is a rigid body free in space, so that $d = 6$. The most natural way to choose vector $\mathbf{s}_D$ is

$$\mathbf{s}_D = (\mathbf{p}_O, \varepsilon_O), \quad (12)$$

being $\mathbf{p}_O$ the absolute position of the object center of mass and ε_O a minimal representation of the orientation of a frame attached to it. Because of the power grasp, the dynamic variables of the object uniquely determine the pose of the two robot end-effector, so that $k_1 = k_2 = 0$, $\mathbf{s} = \mathbf{s}_D$, and $\mathbf{T} = \mathbf{T}_D$. Hence, for $i = 1, 2$,

$$\mathbf{p}_i = \begin{bmatrix} \mathbf{p}_O + {}^B\mathbf{R}_O(\varepsilon_O)\,{}^O\mathbf{r}_{c_i} \\ \varepsilon\left({}^B\mathbf{R}_O(\varepsilon_O)\,{}^O\mathbf{R}_{R_i}\right) \end{bmatrix} = \mathbf{\Gamma}_i(\mathbf{s}) \quad (13a)$$

and

$$\mathbf{v}_i = \begin{bmatrix} \mathbf{I}_{3\times 3} & \frac{\partial}{\partial \varepsilon_O}\left[{}^B\mathbf{R}_O(\varepsilon_O)\,{}^O\mathbf{r}_{c_i}\right] \\ \mathbf{O}_{3\times 3} & {}^B\mathbf{R}_O(\varepsilon_O)\,{}^O\mathbf{K}(\varepsilon_O) \end{bmatrix} \dot{\mathbf{s}} = \mathbf{T}_i(\mathbf{s})\dot{\mathbf{s}}. \quad (13b)$$

In (13), ${}^B\mathbf{R}_O(\varepsilon_O)$ is the rotation matrix from the object to a base frame, ${}^O\mathbf{R}_{R_i}$ defines the *constant* relative orientation between the object frame and the ith robot arm tip frame, ${}^O\mathbf{r}_{c_i}$ is a *constant* vector locating the ith contact point in the object frame, $\varepsilon(\mathbf{R})$ is the frame and the ith contact point along the X axis, and ${}^{R_i}\mathbf{K}(\varepsilon_i)$ is the transformation between $\dot{\varepsilon}_i$ and the angular velocity of the ith end-effector (expressed in the end-effector frame).

Following the general rules, the single vector in the kernel of matrix $\mathbf{T}$ defined by (17b) is

$$\mathbf{Y}_R = \begin{bmatrix} \bar{\mathbf{x}} \\ \mathbf{0}_{3\times 1} \\ -\bar{\mathbf{x}} \\ \mathbf{0}_{3\times 1} \end{bmatrix}, \tag{18}$$

and the only contact force in the cooperating system which cause no object motion is directed along h. The one-dimensional parameter $\lambda_R \geq 0$ is physically a force. For the parametrization of the active forces, taking into account the unilateral nature of the contact constraint, we can choose

$$\mathbf{Y}_A(\mathbf{s}, \lambda_A) = \begin{bmatrix} \mathbf{Y}_{F_1} \\ \mathbf{O}_{3\times 5} \\ \mathbf{Y}_{F_2} \\ \mathbf{O}_{3\times 5} \end{bmatrix}, \tag{19a}$$

with the two 3×5 matrices

$$\begin{aligned} \mathbf{Y}_{F_1} &= \frac{1}{2}\big[\,(1 + \mathrm{sign}(\hat{\mathbf{x}}^{\mathrm{T}}\lambda_A))\bar{\mathbf{x}}\,\hat{\mathbf{x}}^{\mathrm{T}}\,\big] + \frac{r_{c2}}{r}\big[\,\bar{\mathbf{y}}\,\hat{\mathbf{y}}^{\mathrm{T}} + \bar{\mathbf{z}}\,\hat{\mathbf{z}}^{\mathrm{T}}\,\big] \\ &\quad + \frac{1}{r}\big[\,\bar{\mathbf{y}}\,[\,\mathbf{0}_{1\times 4} \quad -1\,] + \bar{\mathbf{z}}\,[\,\mathbf{0}_{1\times 3} \quad 1 \quad 0\,]\,\big], \\ \mathbf{Y}_{F_2} &= \frac{1}{2}\big[\,(1 - \mathrm{sign}(\hat{\mathbf{x}}^{\mathrm{T}}\lambda_A)\bar{\mathbf{x}}\,\hat{\mathbf{x}}^{\mathrm{T}}\,\big] + \frac{r_{c1}}{r}\big[\,\bar{\mathbf{y}}\,\hat{\mathbf{y}}^{\mathrm{T}} + \bar{\mathbf{z}}\,\hat{\mathbf{z}}^{\mathrm{T}}\,\big] \\ &\quad + \frac{1}{r}\big[\,\bar{\mathbf{y}}\,[\,\mathbf{0}_{1\times 4} \quad 1\,] + \bar{\mathbf{z}}\,[\,\mathbf{0}_{1\times 3} \quad -1 \quad 0\,]\,\big]. \end{aligned} \tag{19b}$$

In (19b), $\bar{\mathbf{x}}$, $\bar{\mathbf{y}}$, and $\bar{\mathbf{z}}$ are the coordinate unit vectors of the object frame, $r = r_{c_1} + r_{c_2}$, and we use the notation $\hat{\mathbf{w}} = [\bar{\mathbf{w}}^{\mathrm{T}}\ \mathbf{0}_{1\times 2}]^{\mathrm{T}}$ with $\mathbf{w} = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$.

It must be stressed that in this particular case $\mathbf{Y}_A$ depends *also* on λ_A, because each robot can only 'push' the object along direction h (in fact, the contact constraint is nonholonomic). The sign function would disappear, together with the dependence on λ_A, if the robot tips were 'glued' to the object surface. The 5-dimensional vector λ_A parametrize the *net* (i. e. that producing motion) generalized force acting on the object. In particular, its first three components are the net forces exerted on the object while the last two are the net torques about the Y and Z axes of the object frame. It is long but straightforward to show that this choice is a feasible one and leads to a nonsingular 5×5 matrix $\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A$.

4. Decoupling and Linearizing Control

A model-based hybrid force-motion control law is presented here, achieving exact input-output linearization and decoupling. A state-feedback nonlinear control law of this type can be derived for each set of controlled outputs, as implicitly defined in (10) or in (11). These two alternatives correspond to controlling the active forces *or* the object motion along the previously defined dynamic directions.

Choosing $(\mathbf{s}_D, \lambda_R, \mathbf{s}_K)$ as the system outputs, namely the whole set of motion parameters and the reaction force parameters, and applying

$$\mathbf{u} = (\mathbf{J}\mathbf{B}^{-1})^{*}\Big\{\mathbf{Q}\begin{bmatrix} \mathbf{a}_{D,r} \\ \lambda_{R,r} \\ \mathbf{a}_{K,r} \end{bmatrix} - \mathbf{m}\Big\}, \tag{20}$$

where $\mathbf{A}^*$ is any right inverse of $\mathbf{A}$, yields from (10) the closed-loop equations

$$\begin{aligned} \begin{bmatrix} \ddot{\mathbf{s}}_D \\ \lambda_R \\ \ddot{\mathbf{s}}_K \end{bmatrix} &= \begin{bmatrix} \mathbf{a}_{D,r} \\ \lambda_{R,r} \\ \mathbf{a}_{K,r} \end{bmatrix}, \\ \lambda_A &= (\mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A)^{-1}(\mathbf{n}_O + \mathbf{B}_O\mathbf{a}_{D,r}), \end{aligned} \tag{21}$$

that are decoupled and linearized in terms of the set of inputs $(\mathbf{a}_{D,r}, \lambda_{R,r}, \mathbf{a}_{K,r})$.

Alternatively, choosing $(\lambda_A, \lambda_R, \mathbf{s}_K)$ as the system outputs, namely the whole set of contact force parameters and the kinematic motion parameters, it follows from (11) that the input

$$\mathbf{u} = (\mathbf{J}\mathbf{B}^{-1})^{*}\Big\{\widehat{\mathbf{Q}}\begin{bmatrix} \lambda_{A,r} \\ \lambda_{R,r} \\ \mathbf{a}_{K,r} \end{bmatrix} - \widehat{\mathbf{m}}\Big\} \tag{22}$$

leads to the closed-loop system

$$\begin{aligned} \begin{bmatrix} \lambda_A \\ \lambda_R \\ \ddot{\mathbf{s}}_K \end{bmatrix} &= \begin{bmatrix} \lambda_{A,r} \\ \lambda_{R,r} \\ \mathbf{a}_{K,r} \end{bmatrix}, \\ \ddot{\mathbf{s}}_D &= -\mathbf{B}_O^{-1}(\mathbf{n}_O + \mathbf{T}_D^{\mathrm{T}}\mathbf{Y}_A\lambda_{A,r}), \end{aligned} \tag{23}$$

that is now decoupled and input-output linearized in terms of the new input set $(\lambda_{A,r}, \lambda_{R,r}, \mathbf{a}_{K,r})$.

As explained in detail in [1,12], the values $\mathbf{s}$, $\dot{\mathbf{s}}$, and λ to be used within (20) or (22) can be recovered from the actual sensor measurements of the robots.

The reference values to be plugged into (20) and (22) can be generated *on the linear side* of the problem using standard control techniques. We propose to apply nominal feedforward terms, with a proportional-derivative law for the motion parameters and a proportional-integral law for the force parameters, i.e.

$$\begin{aligned}\mathbf{a}_{K,r} &= \ddot{\mathbf{s}}_{K,d} + \mathbf{K}_{PK}(\mathbf{s}_{K,d} - \mathbf{s}_K) + \mathbf{K}_{VK}(\dot{\mathbf{s}}_{K,d} - \dot{\mathbf{s}}_K),\\ \lambda_{R,r} &= \lambda_{R,d} + \mathbf{K}_{FR}(\lambda_{R,d} - \lambda_R) + \mathbf{K}_{IR}\int(\lambda_{R,d} - \lambda_R)d\tau,\end{aligned} \tag{24}$$

together with

$$\mathbf{a}_{D,r} = \ddot{\mathbf{s}}_{D,d} + \mathbf{K}_{PD}(\mathbf{s}_{D,d} - \mathbf{s}_D) + \mathbf{K}_{VD}(\dot{\mathbf{s}}_{D,d} - \dot{\mathbf{s}}_D) \tag{25}$$

for (20), and together with

$$\lambda_{A,r} = \lambda_{A,d} + \mathbf{K}_{FA}(\lambda_{A,d} - \lambda_A) + \mathbf{K}_{IA}\int(\lambda_{A,d} - \lambda_A)d\tau, \tag{26}$$

for (22). The resulting control scheme allows to shape the dynamic behavior of the system in any desired way.

5. Simulation Results

In the simulations, two symmetric and planar robots with three rotary joints have been considered, executing a 6-dimensional hybrid task defined on the horizontal plane. Each robot has equal links of length 0.1 m and uniformly distributed mass 1 kg. The object has a square section of side 0.1 m and a mass of 1 kg.

Both power grasp and hard finger contacts have been simulated, under the two control alternatives (20) and (22). In the power grasp, the three components of λ_R characterize the internal force along the line passing through the contact points, the internal force normal to this direction, and the internal torque around the axis normal to the plane of motion. The three components of $\mathbf{s}_D$ are the absolute x and y directions, and the orientation θ w.r.t. the x-axis. Similarly, the three components of λ_A are the active forces along the absolute x and y directions, and the torque around the absolute z-axis. In the hard finger contact, the scalar λ_R characterizes the (unique) internal force along the line h of Fig. 1. The components of $\mathbf{s}_K$ are the two orientation angles between the approach axis of each robot and the line h. The components of $\mathbf{s}_D$ and of λ_A are defined as in the power grasp case.

The performance of the two tracking controllers (20) and (22) has been evaluated with respect to the presence of an unknown disturbance force acting on the object, and of errors on the initial velocity of the object and on the initial contact forces. Desired motion and force trajectories, values of the control parameters, and force disturbances are reported in Appendix. The initial contact force and the initial object velocity are set equal to zero, independently of their desired initial values.

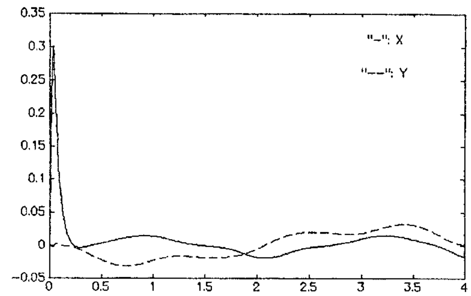

Fig. 2 – Power grasp: Error on x and y in $\mathbf{s}_D$ with (20)

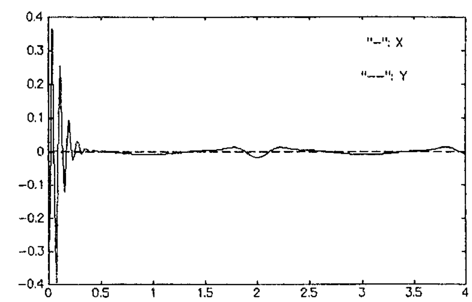

Fig. 3 – Power grasp: Error on x and y in λ_R with (20)

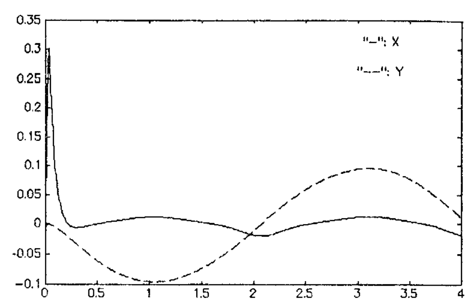

Fig. 4 – Hard finger: Error on x and y in $\mathbf{s}_D$ with (20)

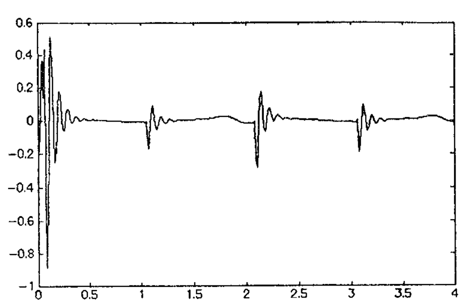

Fig. 5 – Hard finger: Error on λ_R with (20)

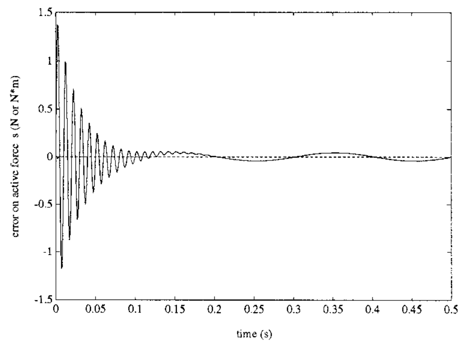

Fig. 6 – Power grasp: Error on λ_A with (22)

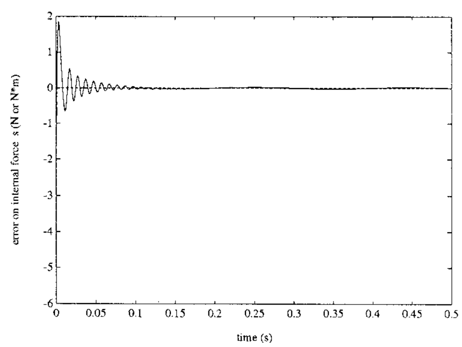

Fig. 7 – Power grasp: Error on λ_R with (22)

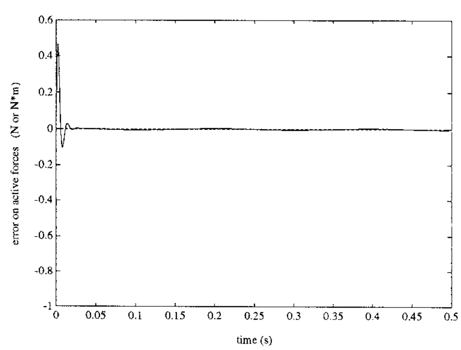

Fig. 8 – Hard finger: Error on λ_A with (22)

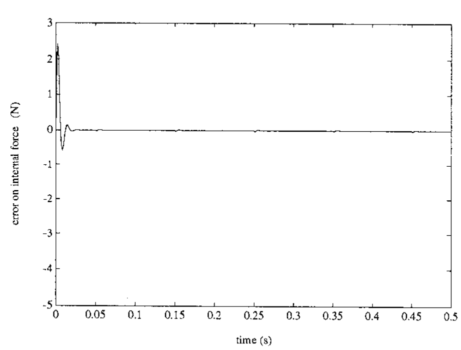

Fig. 9 – Hard finger: Error on λ_R with (22)

Figures 2–5 show the results on selected motion and force parameters with the nonlinear decoupling law (20). This is the conventional case, in which the object motion and the internal forces are controlled. In both contact situations, the larger position error along the absolute x direction at the motion start is due to an initial velocity error. On the other hand, the periodic peaks in the internal force error of Figs. 3 and 5 correspond to motion inversion and are more significant in the hard finger contact.

Figures 6–9 show the results on all contact force parameters with the nonlinear decoupling law (22). The dominant transient error is in the first component of the plotted vector quantities. Since the overall motion of the object is not controlled in this mode, the constant disturbing force along the y direction induces a drift motion but no undesired internal forces. Note also that the cross coupling between the control loops is negligible.

6. Conclusions

A general control-oriented formalism has been introduced for the dynamic modeling and control of robots cooperating in the manipulation of a payload. For each cooperation task, we use a minimal parametrization of the variables needed to describe the object dynamics, possibly completed with kinematic parameters that characterize the grasp. Accordingly, contact forces between robots and object can be decomposed into active and internal ones, based on coordinate-independent energy arguments. Two different sets of dynamic equations have been derived, with interesting consequences in the design of model-based controllers for hybrid motion-force tasks. As a main result, beside the usual possibility of controlling object motion and internal forces, we have shown the feasibility of controlling in a decoupled way *all* the forces acting on the object. The formalism is general enough to accomodate for different types of interactions. The well-known case of power grasp of a rigid object has been recovered as a particular instance. We are currently working on some extensions, e.g. including extra degrees of freedom as in the case of a suspended oscillating payload [15], or taking into account external forces acting on the payload due to further constraints imposed by the environment.

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Appendix

Power grasp simulations

- $(\mathbf{s}_D,\lambda_R)$ controlled variables using (20)

$$\mathbf{s}_{Ddes} = \begin{bmatrix} 0.1+0.5\sin(\pi t) & 0.12-0.5\cos(\pi t) & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{aligned} \lambda_{Rdes} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, & \mathbf{F}_{dist} &= \begin{bmatrix} 0 & \sin(\pi t/2) & 0 \end{bmatrix}^{\mathrm{T}}, \\ \mathbf{K}_{PD} &= 625\cdot\mathbf{I}_{3\times3}, & \mathbf{K}_{VD} &= 50\cdot\mathbf{I}_{3\times3}, \\ \mathbf{K}_{FR} &= -0.5\cdot\mathbf{I}_{3\times3}, & \mathbf{K}_{IR} &= 50\cdot\mathbf{I}_{3\times3}. \end{aligned}$$

- (λ_A,λ_R) controlled variables using (22)

$$\lambda_{Ades} = \begin{bmatrix} \cos(10\pi t) & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{aligned} \lambda_{Rdes} &= \begin{bmatrix} 5 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, & \mathbf{F}_{dist} &= \begin{bmatrix} 0 & 0.1 & 0 \end{bmatrix}^{\mathrm{T}}, \\ \mathbf{K}_{FR} &= -0.5\cdot\mathbf{I}_{3\times3}, & \mathbf{K}_{FA} &= -0.5\cdot\mathbf{I}_{3\times3}, \\ \mathbf{K}_{IR} &= 50\cdot\mathbf{I}_{3\times3}, & \mathbf{K}_{IA} &= 50\cdot\mathbf{I}_{3\times3}. \end{aligned}$$

Hard finger contact simulations

- $(\mathbf{s}_D,\lambda_R,\mathbf{s}_K)$ controlled variables using (20)

$$\mathbf{s}_{Ddes} = \begin{bmatrix} 0.1+0.5\sin(\pi t) & 0.12-0.5\cos(\pi t) & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{aligned} \lambda_{Rdes} &= 1, & \mathbf{s}_{Kdes} &= \begin{bmatrix} 0 & \pi \end{bmatrix}^{\mathrm{T}}, \\ \mathbf{K}_{PK} &= 625\cdot\mathbf{I}_{2\times2}, & \mathbf{K}_{PD} &= 625\cdot\mathbf{I}_{3\times3}, \\ \mathbf{K}_{VK} &= 50\cdot\mathbf{I}_{2\times2}, & \mathbf{K}_{VD} &= 50\cdot\mathbf{I}_{3\times3}, \\ \mathbf{K}_{IR} &= 50, & \mathbf{K}_{FR} &= -0.5, \end{aligned}$$

$$\mathbf{F}_{dist} = \begin{bmatrix} 0 & \sin(\pi t/2) & 0 \end{bmatrix}^{\mathrm{T}}.$$

- $(\lambda_A,\lambda_R,\mathbf{s}_K)$ controlled variables using (22)

$$\lambda_{Ades} = \begin{bmatrix} \cos(10\pi t) & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\begin{aligned} \lambda_{Rdes} &= 5, & \mathbf{s}_{Kdes} &= \begin{bmatrix} 0 & \pi \end{bmatrix}^{\mathrm{T}}, \\ \mathbf{K}_{PK} &= 1225\cdot\mathbf{I}_{2\times2}, & \mathbf{K}_{FA} &= -0.5\cdot\mathbf{I}_{3\times3}, \\ \mathbf{K}_{VK} &= 625\cdot\mathbf{I}_{2\times2}, & \mathbf{K}_{IA} &= 50\cdot\mathbf{I}_{3\times3}, \\ \mathbf{K}_{IR} &= 50, & \mathbf{K}_{FR} &= -0.5, \end{aligned}$$

$$\mathbf{F}_{dist} = \begin{bmatrix} 0 & 0.1 & 0 \end{bmatrix}^{\mathrm{T}}.$$