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Stabilization of Underactuated Robots: Theory and Experiments for a Planar 2R Manipulator

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Abstract

We outline a general approach for the stabilization of robots with passive joints, an interesting example of mechanical systems that may not be controllable in the first approximation. The proposed method is based on a recently introduced iterative steering paradigm, which prescribes the repeated application of a contracting open-loop control law. In order to compute efficiently such a law, the dynamic equations of the robot are put in a suitable form via partial feedback linearization and approximate nilpotentization. The design procedure is illustrated for a 2R robot moving in the horizontal plane with a single actuator at the base. Experimental results are presented for a laboratory prototype.

1 Introduction

Underactuated robotic systems (i.e., with less control inputs than generalized coordinates) are attracting quite a large interest, consistently with the *minimalistic* trend in the field [1]. Mechanisms that can perform complex tasks with a small number of actuators and/or sensors are desirable in view of their reduced cost and weight. On the other hand, innovative approaches are often required in order to synthesize effective control strategies.

In general, underactuated mechanical systems may be controllable via either kinematic or dynamic coupling. Typical examples of the first class are provided by first-order nonholonomic systems, such as wheeled mobile robots and dextrous robotic hands (e.g., see [2] and the references therein). The equations of these systems are nonlinear and driftless when generalized velocities are taken as control inputs. As a consequence, smooth time-invariant stabilization is not possible [3]. The feedback stabilization problem for such systems has been solved using either time-varying [4] and/or discontinuous feedback [5, 6].

The second class includes, among others, overhead cranes [7], manipulators with flexible elements [8] and gymnast robots, e.g., the Acrobot [9]. The corresponding system equations are again nonlinear but a drift term accounting for gravitational or elastic forces is now present. Therefore, all the above systems are smoothly—in particular, linearly—stabilizable.

However, some mechanisms that are controllable via dynamic coupling inherit the limitations of kinematic nonholonomic systems, and in particular the lack of smooth stabilizability. This situation arises whenever the drift term tends to zero when the generalized velocities do. An example is provided by underactuated manipulators (i.e., with some passive joints) in the absence of gravity [10]. The same is true for redundant manipulators driven only through end-effector generalized forces [11]. The aforementioned control techniques for first-order nonholonomic systems cannot be applied in these cases, on account of the presence of a nontrivial drift.

In particular, we address here the stabilization problem for underactuated manipulators in the absence of gravity (e.g., moving in the horizontal plane). Control methods for special instances of this class have been presented in [12], based on a Poincaré map analysis, and in [13], through a suitable trajectory synthesis. Our solution relies on the following general scheme: devise an open-loop control which can steer the system state closer to the desired equilibrium point in finite time, and apply it in an iterative fashion (i.e., from the state attained at the end of the previous iteration). Under appropriate hypotheses, such a strategy provides robust exponential stabilization for a wide class of controllable systems [14]. To simplify the computation of the open-loop control, one can approximate the system equations by a nilpotent form [15], which can be easily integrated and, at the same time, preserves the controllability properties of the original system. Approximate nilpotentization has been used for nonholonomic motion planning in [16].

The paper is organized as follows. In the next section, we outline the main steps of our approach to the control of underactuated manipulators, which includes a partial feedback linearization, a nilpotent approximation and an iterative stabilization procedure. In Sect. 3, we apply the proposed approach to a 2R planar robot equipped with a single actuator at the base and present simulation as well as experimental results.

2 The control problem

Consider a manipulator with n joints, of which only m are actuated. Denote by $q \in I\!R^n$ the joint coordinates vector, and by $\tau \in I\!R^m$ the vector of generalized forces.

2.1 Partial feedback linearization

Partition vector q as (q_a, q_b), being $q_a \in I\!R^m$ the *active* joints and $q_b \in I\!R^{n-m}$ the *passive* joints. Following the Lagrangian approach, the dynamic model of the system can be written as

$$\begin{bmatrix} B_{aa} & B_{ab} \\ B_{ab}^T & B_{bb} \end{bmatrix} \begin{bmatrix} \ddot{q}_a \\ \ddot{q}_b \end{bmatrix} + \begin{bmatrix} h_a \\ h_b \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix},$$

with the corresponding partitions of the $n \times n$ inertia matrix $B(q)$ and of the n-vector $h(q, \dot{q})$, which collects centrifugal, Coriolis and possibly gravitational terms. Note that the last $n - m$ equations directly provide a second-order differential constraint which is satisfied by the robot during its motion.

Choosing the generalized forces τ as

$$\tau = \left(B_{aa} - B_{ab}B_{bb}^{-1}B_{ab}^T\right) u + h_a - B_{ab}B_{bb}^{-1}h_b, \quad (1)$$

with $u \in I\!R^m$ an auxiliary input vector, one obtains

$$\begin{aligned} \ddot{q}_a &= u, & (2)\\ \ddot{q}_b &= -B_{bb}^{-1}h_b - B_{bb}^{-1}B_{ab}^T u = f_b(q,\dot{q}) + G_b(q)u. & (3) \end{aligned}$$

It is easy to verify that, in the absence of gravity, the linear approximation of system (2–3) around any equilibrium point is not controllable. Besides, due to the presence of the drift term $f_b(q, \dot{q})$, the accessibility property—which may be tested via the Lie algebra rank condition [17]—would not imply controllability. Therefore, the only systematic way to check controllability is to apply the sufficient conditions for small-time local controllability (STLC) given in [18], and subsequently refined in [19]. In [11, Prop. 3] a STLC test is given for systems in the form (2–3); however, being based on sufficient conditions, such test may not be conclusive. In general, controllability must be established constructively.

2.2 Approximate nilpotentization

Nilpotent approximations [15] of control systems are an example of higher-order approximation that prove useful when linearization does not preserve the original controllability properties. In particular, in [16] a systematic approximate nilpotentization procedure is proposed, which can be applied to any system of the form

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i, \quad x \in I\!R^n, \quad (4)$$

provided that the accessibility property is satisfied.

The nilpotentization procedure is based on the existence of a suitable set of *privileged* coordinates z, locally defined around any point x^0 where the system is accessible. With the system in these coordinates, the nilpotent approximation is obtained by expanding the components of the system vector fields in Taylor series and truncating them at a proper order. As a consequence, the approximating vector fields $\hat{f}, \hat{g}_1, \ldots, \hat{g}_m$ are polynomial. Moreover, they generate a nilpotent Lie algebra which is full rank around x^0, so that also the approximating system is locally accessible.

In particular, the i-th component ($i = 1, \ldots, n$) of the vector fields $\hat{f}, \hat{g}_1, \ldots, \hat{g}_m$ depends at most on $z_1, \ldots, z_{i-1}$. Hence, the approximating polynomial system has the *triangular* form

$$\begin{aligned} \dot{z}_i &= \hat{f}_i + \sum_{j=1}^{m} \hat{g}_{j_i} u_j, \qquad i = 1, \ldots, \nu, & (5)\\ \dot{z}_k &= \hat{f}_k(z_1, \ldots, z_{k-1}) + \sum_{j=1}^{m} \hat{g}_{j_k}(z_1, \ldots, z_{k-1}) u_j, & \\ & \qquad k = \nu + 1, \ldots, n, & (6) \end{aligned}$$

being ν the dimension of span$\{f, g_1, \ldots, g_m\}$ at x^0, and $\hat{f}_i, \hat{g}_{1_i}, \ldots, \hat{g}_{m_i}$ constant values, for $i = 1, \ldots, \nu$.

It can be proven that, if the original system (4) is partially decoupled and linearized, the decoupled dynamics (e.g., eq. (2)) is exactly recovered by the nilpotent approximation (5–6). This suggests to perform the partial feedback linearization of Sect. 2.1 before proceeding with the nilpotentization.

2.3 Stabilization

We now address the problem of determining a feedback controller that transfers the system from an initial equilibrium point $x^0 = (q^0, 0) = (q_a^0, q_b^0, 0)$ to a desired equilibrium point $x^d = (q^d, 0) = (q_a^d, q_b^d, 0)$. As shown in [10], a limitation to be taken into account is that underactuated manipulators moving in the absence of gravity are not smoothly stabilizable via time-invariant feedback—a consequence of Brockett's theorem [3]. Therefore, one must resort to time-varying and/or discontinuous feedback. However, while systematic approaches to the design of such control laws exist for driftless systems—e.g., see [4, 5, 20]—the case of systems with drift has received much less attention; existing work includes [21].

Our method for stabilizing an underactuated robot in the partially linearized form (2–3) prescribes the execution of two phases:

I. Drive in finite time T_1 the active joint variables q_a to their desired values q_a^d. At the end of this phase it will be $q_a(T_1) = q_a^d$ and $\dot{q}_a(T_1) = 0$. Correspondingly, $q_b(T_1) = q_b^I$ and $\dot{q}_b(T_1) = \dot{q}_b^I$, being in general $q_b^I \neq q_b^d$ and $\dot{q}_b^I \neq 0$.

II. Obtain asymptotic convergence of the passive joint variables q_b to their desired values q_b^d while guaranteeing that q_a returns to q_a^d.

The first phase, which we shall refer to as *alignment*, can be performed in feedback using a standard terminal controller (e.g., see [22]) for the decoupled chains of double integrators represented by eq. (2).

For the second phase, called *contraction*, we adopt the *iterative state steering* approach [14]. The main tool is a *contracting* open-loop control, that steers the system closer to the desired equilibrium x^d in a finite time T. If such a control can be computed, its iterated application (i.e., from the state attained at the end of the previous iteration) renders x^d exponentially stable, provided that T is bounded and that the open-loop control is continuous with respect to the initial conditions. Moreover, non-persistent perturbations are rejected, while ultimate boundedness of the error is guaranteed in the presence of persistent perturbations. The overall control is given by a time-varying law whose expression depends on a sampled feedback action.

In order to apply the above technique, we must compute for system (2–3) a contracting open-loop control law. One way to achieve contraction is to perform a *cyclic* motion of duration T_2 on the q_a variables (i.e., a motion such that $q_a^{II} = q_a(T_1 + T_2) = q_a(T_1)$ and $\dot{q}_a^{II} = \dot{q}_a(T_1 + T_2) = 0$) while giving a final position $q_b^{II} = q_b(T_1 + T_2)$ for the passive joints that is closer to q_b^d than the initial condition q_b^I, with final velocity $\dot{q}_b^{II}$ smaller in norm than $\dot{q}_b^I$. If such cycle can be produced by a control law that is continuous with respect to the initial conditions, the position $q_b(t)$ of the passive joints is guaranteed to converge over the iterations to its desired value q_b^d for all t, i.e., the state trajectory is arbitrarily bounded. Therefore, $(q^d, 0)$ is Lyapunov stable.

The search for a suitable control u may be conveniently performed within a parameterized class of inputs. In some cases (e.g., when the system can be put in second-order triangular or Čaplygin form [11]), the computation of the parameters identifying u in the chosen class can be directly performed by forward integration of the passive joints equation (3). In general, however, one can resort to the nilpotent approximation (5–6) of the dynamic equations, which is polynomial and hence always integrable.

3 Application to a planar 2R robot

Consider the planar robot of Fig. 1, having two revolute joints and a single actuator at the base. We assume that neither gravity nor friction is present at the joints. The same mechanism was considered also by Suzuki *et al.* [12].

After the partial feedback linearization of Sect. 2.1, and with the state vector $x = (q_1, q_2, \dot{q}_1, \dot{q}_2) \in I\!R^4$, the dynamic model of the robot becomes

$$\dot{x} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ 0 \\ -K s_2 \dot{q}_1^2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 - K c_2 \end{bmatrix} u = f(x) + g(x)u, \quad (7)$$

with $s_2 = \sin q_2$, $c_2 = \cos q_2$, and $K > 0$ a constant depending on geometric and inertial properties of the robot. Since the vector fields $\{g, [f,g], [g,[f,g]], [f,[g,[f,g]]]\}$ span $I\!R^4$ at any x such that $q_2 \neq k\pi/2$, $k = 0, 1 \ldots$, the system is accessible. However, one may verify that the sufficient condition given in [11] for STLC is not satisfied.

Assume now that we wish to steer the 2R robot from $q^0 = (q_1^0, q_2^0)$ to $q^d = (q_1^d, q_2^d)$, with initial and final zero velocity. We apply the stabilization strategy proposed in Sect. 2.3, with $q_a = q_1$ and $q_b = q_2$. In order to devise a contracting open-loop controller to be applied iteratively after of the alignment phase, we need to compute the nilpotent approximation of the system at states such that $\dot{q}_1^I = 0$ and $\dot{q}_2^I \neq 0$.

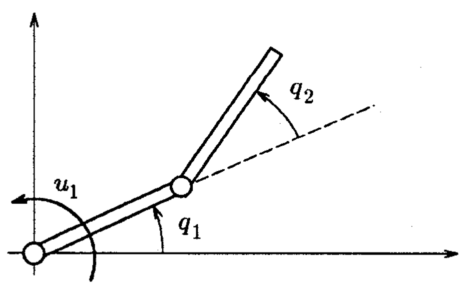

Figure 1: A 2R planar robot with a single actuator at the base

3.1 Nilpotent approximation

The nilpotent approximation technique of [16] has been applied to eq. (7), using the vector fields $\{f, g, [f,g], [g,[f,g]]\}$ to span $\mathbb{R}^4$ at the points of interest (see [23] for details). The change of coordinates $x = x(z)$ required to transform the system in privileged coordinates is

$$\begin{aligned} q_1 &= q_1^I - z_3, \\ q_2 &= q_2^I + \dot{q}_2^I z_1 + \beta z_3, \\ \dot{q}_1 &= z_2, \\ \dot{q}_2 &= \dot{q}_2^I - \beta z_2 + \gamma z_3 - \delta z_4 + \gamma z_1 z_2, \end{aligned} \tag{8}$$

with $\beta = 1 + K \cos q_2^I$, $\gamma = K \dot{q}_2^I \sin q_2^I$, $\delta = K^2 \sin 2q_2^I$, and the vector fields of the nilpotent approximation (5–6) are

$$\hat{f} = \begin{bmatrix} 1 \\ 0 \\ -z_2 \\ \dfrac{1}{2Kc_2^I} z_2^2 \end{bmatrix}, \quad \hat{g} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\dfrac{(\dot{q}_2^I)^2}{4Ks_2^I} z_1^2 - \dfrac{\beta}{2Kc_2^I} z_3 \end{bmatrix}. \tag{9}$$

As expected, the dynamics of q_1 and $\dot{q}_1$ (which correspond to z_3 and z_2, respectively) is exactly recovered, thanks to the partial feedback linearization. Instead, the use of the nilpotent dynamics for q_2 and $\dot{q}_2$ will induce an approximation error whose magnitude, however, can be made arbitrarily small by reducing T_2. Therefore, by enforcing sufficient contraction on the approximate system, one can guarantee that the contraction property is also preserved for the original one.

3.2 Control design

The above nilpotent approximation is now used to compute a contracting control law u. To simplify the notation, we reset time so that $t = 0$ at the beginning of the contraction phase. For any u cyclic on $(q_1, \dot{q}_1)$ with period T_2, eqs. (8) give

$$\begin{aligned} \dot{q}_1^{II} = \dot{q}_1^I = 0 \quad &\Longrightarrow \quad z_2(T_2) = 0, \\ q_1^{II} = q_1^I \quad &\Longrightarrow \quad z_3(T_2) = 0. \end{aligned}$$

Hence,

$$\Delta q_2 = q_2^{II} - q_2^I = \dot{q}_2^I z_1(T_2) = \dot{q}_2^I T_2, \tag{10}$$

since $z_1(t) = t$ from eq. (9). This shows that the variation Δq_2 of the passive joint position along the cycle does not depend on the particular control input, but only on its period and on the initial velocity $\dot{q}_2^I$. As for the passive joint velocity, we have

$$\Delta \dot{q}_2 = \dot{q}_2^{II} - \dot{q}_2^I = -\delta z_4(T_2).$$

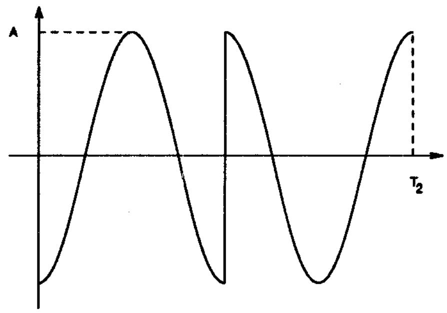

Figure 2: The profile of the cyclic open-loop control u in each iteration of the contraction phase

Simple computations yield

$$\Delta \dot{q}_2 = K^2 s_2^I c_2^I \int_0^{T_2} z_2^2(t)\, dt - K c_2^I (\dot{q}_2^I)^2 \int_0^{T_2} z_3(t)\, dt. \tag{11}$$

The sign of the first term in the above expression does not depend on the choice of the specific cyclic input, but only on q_2^I, while the second term is $o((\dot{q}_2^I)^2)$. Hence, it appears that the STLC property does not hold for the approximating system at equilibrium points. Nevertheless, the system is controllable, as will be shown constructively.

At this point, we choose a class of cyclic control inputs as

$$u(t) = \begin{cases} -A \cos 4\pi t/T_2, & t \in [0, T_2/2), \\ A \cos 4\pi (t - T_2/2)/T_2, & t \in [T_2/2, T_2], \end{cases} \tag{12}$$

with duration T_2 and amplitude A (see Fig. 2). From eqs. (9) and (12) we get

$$\int_0^{T_2} z_3(t)\, dt = -\int_0^{T_2} \int_0^{\sigma} \int_0^{\rho} u(t)\, dt\, d\rho\, d\sigma = 0$$

and

$$\int_0^{T_2} z_2^2(t)\, dt = \int_0^{T_2} \left(\int_0^{\sigma} u(\rho)\, d\rho \right)^2 d\sigma = \frac{T_2^3}{8\pi^2} A^2,$$

so that eq. (11) implies

$$\Delta \dot{q}_2 = \frac{T_2^3 K^2 s_2^I c_2^I}{8\pi^2} A^2. \tag{13}$$

This shows that, at each iteration, we can obtain only $\Delta \dot{q}_2$ of the same sign of $\sin 2q_2^I$, i.e., positive for q_2^I in the first and third quadrant and negative in the second and the fourth (see Fig. 3).

In order to meet the iterative steering paradigm, we must guarantee that the error contracts, i.e.,

$$\begin{aligned} |q_2^d - q_2^{II}| &\leq \eta_1 |q_2^d - q_2^I|, \qquad (14) \\ |\dot{q}_2^{II}| &\leq \eta_2 |\dot{q}_2^I|, \qquad (15) \end{aligned}$$

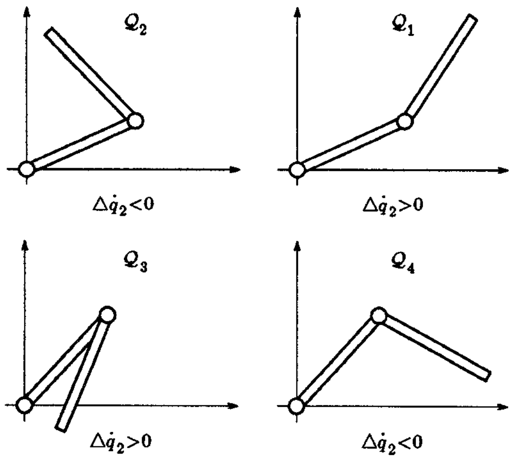

Figure 3: The sign of the achievable velocity variation $\Delta\dot{q}_2$ depends on the second link posture

with η_1, $\eta_2 \in [0,1)$. In view of eqs. (10) and (13), the above conditions can be directly satisfied only in particular situations—again, a consequence of the lack of STLC.

For example, assume that q_2^d belongs to the first quadrant Q_1 and

$$q_2^I \in Q_1, \quad q_2^I > q_2^d, \quad \dot{q}_2^I < 0. \tag{16}$$

Using eqs. (10) and (13), it may be easily verified that one way to satisfy the error contraction conditions (14–15) is to use the open-loop control (12) with

$$T_2 = (1+\eta_1)\frac{q_2^I - q_2^d}{|\dot{q}_2^I|}, \tag{17}$$

$$A = \frac{4\pi}{KT_2}\sqrt{\frac{|\dot{q}_2^I|(1-\eta_2)}{T_2 \sin 2q_2^I}}. \tag{18}$$

Continuity of the resulting control law u with respect to the initial conditions is guaranteed under assumption (16). Moreover, boundedness of T_2 is ensured by letting $\eta_1 \leq \eta_2$, so that the fraction in eq. (17) admits a finite limit as $\dot{q}_2^I$ tends to zero. These two properties imply that the contraction phase produces exponential convergence to the desired equilibrium point $(q_2^d, 0)$.

If any of the conditions in eq. (16) does not hold, it is not possible to satisfy continuatively both eqs. (14–15) while approaching the desired configuration. Therefore, it is necessary to attain a modified initial condition $(q_2^I, \dot{q}_2^I)$ that satisfies eq. (16) *before* switching to the contraction phase. This *transition* phase can be executed in finite time as follows: if the initial velocity of the second joint is negative, keep it constant until q_2 enters Q_1, else keep it constant until q_2 enters Q_2 or Q_4, where $\dot{q}_2$ can be made negative.

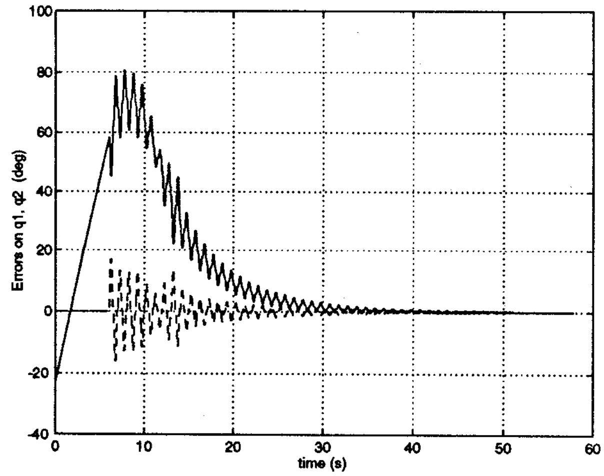

Figure 4: Simulation results: Errors on q_1 (dashed) and q_2 (solid)

Note that in order to keep $\dot{q}_2$ constant one simply sets $u = 0$ in eq. (7), resulting also in zero motion for the first joint.

Similarly, one may devise simple transition and contraction phases for the other cases $q_2^d \in Q_2$, Q_3 or Q_4. As a result, the convergence domain of the proposed control strategy can be made global.

3.3 Simulation results

To illustrate the performance of the proposed method, we present below a simulation for a 2R robot with $K = 0.5$. We assume that, at the end of the alignment phase, it is $q_2^I = 22.5°$ and $\dot{q}_2^I = 13.2°/s$, while the desired configuration of the passive joint is $q_2^d = 45°$.

Being $q_2^d \in Q_1$ but $\dot{q}_2^I > 0$, the control strategy of Sect. 3.2 prescribes the execution of a transition phase, in which $\dot{q}_2$ is kept constant until q_2 enters Q_2, where $\dot{q}_2$ can be made negative. When q_2 returns in Q_1, the contraction phase takes over. By properly tuning the contraction factors η_1 and η_2, it has been possible to use a constant duration $T_2 = 1$ s for all iterations.

The time history of the errors on both joint positions during transition and contraction is reported in Fig. 4. Note the constant velocity of the second joint during the transition phase and the exponential convergence rate during the contraction phase. The long time needed to complete the reconfiguration is due to the fact that motion of the passive joint is not damped by friction in the simulated model.

3.4 Experimental results

We have applied the proposed stabilization method to the FLEXARM, a lightweight 2R planar manipulator available in our laboratory [24]. The second link, which is flexible, has been stiffened for our purposes by appropriately bonding the forearm. As a nominal model after partial feedback linearization, we have used eq. (7) with $K = 0.4643$.

It should be emphasized that the accuracy of the resulting model is quite poor, due to unmodeled dynamic effects such as dry and viscous friction on both joints, the residual elasticity of the second link, and the presence of a bound on the first joint torque (to avoid saturation of the actuator). Besides, no direct measure is available for the joint velocity, which is reconstructed by numeric filtering.

The following remarks are in order with reference to the implementation of the method.

- To avoid chattering, the alignment phase was performed by using a simple PD control law on the first joint position. Although the convergence is only asymptotic, the error can be made arbitrarily small in finite time.

- During the contraction phase, in view of the model inaccuracy, the first link was controlled via high-gain PD feedback in place of the partially linearizing feedback (1). The position reference signal is obtained by integrating twice the acceleration profile (12).

- Due to the various system perturbations, the first joint may not perform exactly a cyclic motion during the iterations of the contraction phase—a small displacement may occur. To prevent the first link to drift away from its desired position, each iteration actually consists of a re-alignment phase followed by a contraction phase.

Figures 5–6 show the results of a typical experiment. Here, the arm is required to move from $q_1^0 = 74°$, $q_2^0 = 91°$ to $q_1^d = 0°$, $q_2^d = 45°$. During each alignment phase, a PD control law on the first joint position was used with gains $K_P^I = 20$ and $K_D^I = 0.3$. Instead, we have set $K_P^{II} = 70$ and $K_D^{II} = 2$ for the contraction phases, whose period is always $T_2 = 1$ s.

The evolution of the joint errors and of the joint torque τ_1 are shown respectively in Fig. 5 and Fig. 6. For the sake of clarity, each contraction phase is marked in bold on the time axis. A comparison with Fig. 4 shows that, due to the presence of friction, stabilization of the robot is obtained in practice in a much smaller time. Note that τ_1 saturates during the first alignment phase, and that no transition phase is needed in this case.

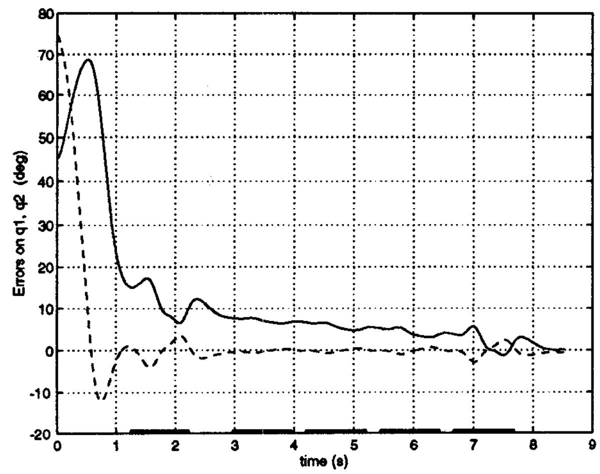

Figure 5: Experimental results: Errors on q_1 (dashed) and q_2 (solid)

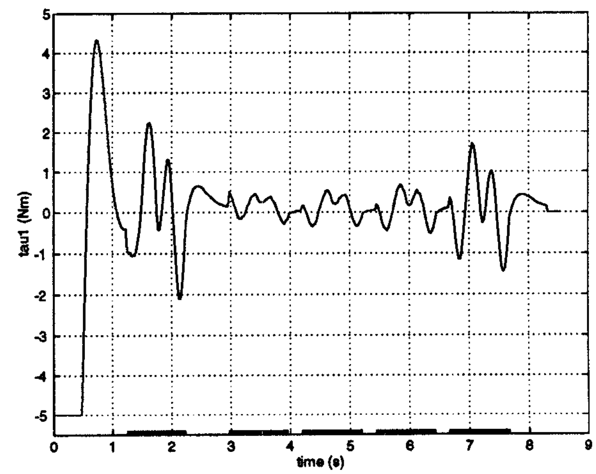

Figure 6: Experimental results: Torque τ_1

4 Conclusions

We have presented a solution method for the stabilization of underactuated manipulators. Such systems are not smoothly stabilizable in the absence of gravity. Moreover, the presence of a drift term in the dynamic equations complicates remarkably the control synthesis. The stabilization strategy consists of three phases, namely *(i)* alignment, in which the active joints are brought to their desired position, *(ii)* transition, where simple maneuvers are executed to obtain the correct initial condition for *(iii)* contraction, based on the iterative application of a suitable open-loop control designed on a nilpotent approximation of the system.

The proposed approach has been illustrated with reference to a planar 2R robot with a single actuator at the base. The presented simulation and experimental results show the satisfactory performance of the method. In principle, the method is applicable to most underactuated mechanical systems of interest in robotic applications.

Acknowledgments

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