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Brief paper

Relaxed fault detection and isolation: An application to a nonlinear case study☆

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Abstract

Given a number of possibly concurrent faults (and disturbances) that may affect a nonlinear dynamic system, it may not be possible to solve the standard fault detection and isolation (FDI) problem, i.e., to detect and isolate each single fault from all other, possibly concurrent faults and disturbances, due to the violation of the available necessary conditions of geometric nature. Motivated by a robotic application where this negative situation structurally occurs, we propose some relaxed formulations of the FDI problem and show how necessary and sufficient conditions for their solution can be derived from those available for standard FDI. The design of a hybrid residual generator follows directly from the fulfillment of the corresponding solvability conditions. In the considered nonlinear case study, a robotic system affected by possible actuator and/or force sensor faults, we detail the application of these results and present experimental tests for validation.

Keywords: Fault detection and isolation; Fault sets; Nonlinear systems; Geometric conditions; Robot manipulators

1. Introduction

The capability of detecting the occurrence of faults and isolating each of them from other possible faults and from disturbances (the standard fault detection and isolation (FDI) problem) is of basic importance in the automatic operation of complex dynamic plants (Frank, 1990). The case of systems modeled by nonlinear dynamics affine in the fault inputs has recently received special attention (De Persis & Isidori, 2001; Hammouri, Kinnaert, & El Yaagoubi, 1999; Zhang, Polycarpou, & Parisini, 2002). In particular, in De Persis and Isidori (2001) the geometric approach of Massoumnia (1986) has been extended to nonlinear systems, leading to necessary conditions for detection and isolation of single or multiple concurrent faults. When the stated necessary conditions are satisfied, under some additional technical assumptions, this approach enables the design of residual generators that solve the standard FDI problem. Nonetheless, these conditions may fail to be satisfied in many cases of practical interest, e.g., when the number of considered potential faults affecting the system exceeds the dimension of the state space. The possible violation of the conditions for standard FDI has been only implicitly considered in the literature, and only few alternative formulations of the diagnostic problem have been proposed. For linear time-invariant systems (see, e.g., Frank & Ding, 1994) and, more recently, for linear periodic systems (Zhang, Ding, Wang, & Zhou, 2005), discrimination of faults from disturbances (fault detection) has been performed by minimizing a suitable influence ratio index. In other papers, the assumption of possible fault concurrency is implicitly relaxed (see, e.g., Larson, Parker Jr., & Clark, 2002; Simani, Fantuzzi, & Beghelli, 2000; Zhang et al., 2002), but its consequences on the conditions for FDI are not analyzed.

Motivated by a nonlinear FDI problem in robot manipulators that violates the necessary conditions (De Persis & Isidori, 2001) for the solvability of the standard FDI problem (Section 2), we address in this paper the case when not every single fault can be detected and isolated from all other faults and disturbances. Correspondingly, we formulate in Section 3 a more general class of FDI problems, where the focus of the isolation

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issue is moved from *single* to suitable *sets* of faults. In particular, different ways to relax the original formulation of the FDI problem are proposed and, for each of them, we show how solvability conditions can be logically derived from those available for standard FDI. The fulfillment of the resulting weaker conditions automatically leads to the design of a hybrid diagnostic system for the corresponding relaxed problem, constituted by the cascade of a bank of residual generators and of a suitable combinatorial isolation logics. The introduced concepts and results are illustrated in Section 4 through experiments on a robot manipulator affected by possible faults of the actuators and/or of the components of an end-effector force sensor.

2. Necessary conditions for standard FDI

We consider the general nonlinear system

$$\dot{x} = g_0(x) + \sum_{k=1}^{m} g_k(x)u_k + \sum_{i=1}^{s} l_i(x) f_i + \sum_{j=1}^{d} n_j(x)w_j,$$
$$y = h(x), \tag{1}$$

with state $x \in \mathbb{R}^n$, inputs u_k, $k = 1, \ldots, m$ (controls), f_i, $i = 1, \ldots, s$ (faults), and w_j, $j = 1, \ldots, d$ (disturbances), measured output $y \in \mathbb{R}^q$, and where $g_0(x), g_1(x), \ldots, g_m(x)$, $l_1(x), \ldots, l_s(x), n_1(x), \ldots, n_d(x)$ are smooth vector fields and $h(x)$ is a smooth mapping. For system (1), affected by the possibly concurrent faults $f_1, \ldots, f_s$, the standard FDI problem can be concisely formulated as that of designing a bank of s *filters* (referred to as *residual generators*), having as inputs the u_k's, $k = 1, \ldots, m$ and y of Eq. (1), and with outputs $r_1, \ldots, r_s$ (*residuals*), such that, for each $\kappa = 1, \ldots, s$, residual r_κ is affected by f_κ, is decoupled from the other faults f_i ($i \neq \kappa$) and the disturbances w_j's, and asymptotically converges to zero whenever $f_\kappa \equiv 0$.

Necessary conditions for the solvability of the stated problem have been proven in De Persis and Isidori (2001). Let $L_\kappa = \{l_1, \ldots, l_{\kappa-1}, l_{\kappa+1}, \ldots, l_s, n_1, \ldots, n_d\}$ be the set of all fault and disturbance vector fields, except l_κ. A solution to the standard FDI problem exists only if

$$\text{span}\{l_\kappa\} \not\subseteq \mathscr{U}(L_\kappa), \quad \kappa = 1, \ldots, s, \tag{2}$$

where $\mathscr{U}(L_\kappa)$ is a suitable distribution that can be computed from L_κ and from the system vector fields $g_0, g_1, \ldots, g_m$, through a recursive algorithm (see De Persis and Isidori, 2001 for details). For the case study considered in Section 4, where the state is fully available ($h(x) = x$) and the distribution span $\{L_\kappa\}$ is involutive, it is $\mathscr{U}(L_\kappa) = \text{span}\{L_\kappa\}$, while in general one has

$$\mathscr{U}(L_\kappa) \supseteq \text{span}\{L_\kappa\}. \tag{3}$$

Taking into account (3), it is clear that condition (2) cannot be verified for more than n faults at the same time (in the best case, i.e., when no disturbances affect the system), being n the dimension of the state space. Thus, the maximum number of possibly concurrent faults that can be detected and isolated is certainly less than or equal to n. Moreover, for nonlinear mechanical systems it can be shown that this maximum number is further reduced to the number N of generalized coordinates, being $n = 2N$ (Mattone & De Luca, 2004).

Under suitable technical assumptions (see De Persis & Isidori, 2001, Proposition 5 & Assumption II), the necessary condition (2) is also sufficient (and constructive) for the solution of the stated FDI problem. This holds, e.g., when the state is completely measurable, as in our robot application case. In order to simplify the statement of most results, we assume in the following that these technical assumptions are verified.[1]

3. Relaxed FDI problems

For the nonlinear system (1), the violation of condition (2) for some κ implies that not every fault can be detected and isolated from *all* other possibly *concurrent* fault inputs and disturbances. We relax here the original FDI problem formulation in several ways by moving the focus of the isolation issue from *single* to suitable *sets* of faults. Clearly, fault isolation only makes sense when fault detection is feasible, i.e., when all faults $f_1, \ldots, f_s$ can be at least discriminated from disturbances. Therefore, we make here the following assumption.

Assumption 1. All faults $f_1, \ldots, f_s$ in system (1) are *detectable*, i.e., it holds

$$\text{span}\{l_\kappa\} \not\subseteq \mathscr{U}(\{n_1, \ldots, n_d\}), \quad \kappa = 1, \ldots, s. \tag{4}$$

3.1. Fault set detection and isolation

In some applications, e.g., when the same recovery procedure applies to different faults, it might be sufficient to recognize the occurrence of one or more faults in a given set, without distinguishing among the different faults therein. This is what we define as the *fault set detection and isolation* (FSDI) problem.

Problem 2 (*FSDI*). In system (1), let $S = \{f_{k_1}, \ldots, f_{k_v}\} \subseteq \{f_1, \ldots, f_s\}$. Find, if possible, a dynamic system whose output r is affected by *each* fault in S, is *not* affected by any other fault $f_i \notin S$ or disturbance w_j, $j = 1, \ldots, d$, and asymptotically converges to zero whenever all fault inputs in S are zero. If the stated problem is solvable for S, we call S an *FDI-set* and say that it is detected and isolated by residual r.

The following result can be readily established for the solvability of FSDI problem.

Proposition 3. *Let* $S = \{f_{k_1}, \ldots, f_{k_v}\}$ *and* $L = \{l_i, i \notin \{k_1, \ldots, k_v\}, n_1, \ldots, n_d\}$ *in system* (1). *The FSDI problem is solvable for* S (*i.e.,* S *is an FDI-set*) *if and only if*

$$\text{span}\{l_k\} \not\subseteq \mathscr{U}(L), \quad \forall k \in \{k_1, \ldots, k_v\}. \tag{5}$$

Proof. The necessity of (5) directly follows from the general necessary condition (2). As regards sufficiency, fulfillment of

[1] If sufficiency of condition (2) does not hold, all conditions stated in the rest of the paper are only necessary.

condition (5) guarantees that for each $k \in \{k_1, \ldots, k_v\}$ a residual r_k can be found,[2] that is affected by f_k and not affected by any fault or disturbance out of S, so that the set S is detected and isolated, e.g., by the residual $r = r_{k_1}^2 + \cdots + r_{k_v}^2$. □

In order to determine for which sets the FSDI problem turns out to be solvable, we give next a structural characterization of FDI-sets.[3] For this, the following definition is needed.

Definition 4 (*Minimal FDI-set*). In system (1), let $S = \{f_{k_1}, \ldots, f_{k_v}\} \subseteq \{f_1, \ldots, f_s\}$. We say that S is a *minimal FDI-set*, if (i) it is an FDI-set, and (ii) does not strictly include any other FDI-set.

For system (1), there is a finite number N_r of minimal FDI-sets, which constitute a basis for determining all possible FDI-sets. In fact, the following result can be readily established.

Corollary 5. *For system* (1), *any FDI-set S can be written as*

$$S = \bigcup_{k=1}^{N_r} \alpha_k S_k^{\min}, \quad \alpha_k \in \{0, 1\}, \tag{6}$$

where $S_k^{\min}$, $k = 1, \ldots, N_r$, *are all associated minimal FDI-sets.*

For the actual computation of all minimal FDI-sets, a recursive algorithm having the structure of a tree exploration can be devised (Mattone & De Luca, 2005). In general, minimal FDI-sets have different cardinalities and may have non-empty intersections. Furthermore, under Assumption 1, each fault f_κ is contained in at least one minimal FDI-set. In particular, if the standard FDI problem is solvable for f_κ, then the only minimal FDI-set containing f_κ is the set $\{f_\kappa\}$ itself. Due to structure (6), it follows that the FDI-set S is detected and isolated by the diagnostic signal $r = \sum_{k=1}^{N_r} \alpha_k r_k^2$, where r_k detects and isolates $S_k^{\min}$. Thus, any set of residuals $\mathscr{R}^{\min} = \{r_1, \ldots, r_{N_r}\}$, such that r_k detects and isolates $S_k^{\min}$, provides all available FDI information about the system. This information is completely summarized by the *residual matrix*, i.e., a binary matrix RM whose entry $RM(i, k)$ is '1' if fault f_i is in the minimal FDI-set $S_k^{\min}$, or, equivalently, if f_i affects the corresponding residual r_k. In the following, we refer to the set $\mathscr{R}^{\min} = \{r_1, \ldots, r_{N_r}\}$ as a *residual basis* for system (1).

3.2. Candidate fault set detection and isolation

Assume that a faulty situation is caused by the (yet unknown) set of active faults $S_a = \{f_{a_1}, \ldots, f_{a_v}\}$, and that we want to determine a fault diagnosis based on the observation of a given set $\mathscr{R} = \{r_1, \ldots, r_\mu\}$ of μ diagnostic signals, respectively, associated to the FDI-sets $S_1, \ldots, S_\mu$. If a residual $r_k \in \mathscr{R}$ turns out to be affected, we can conclude that one or more faults of the associated FDI-set S_k are in S_a. On the other hand, if a residual r_j is unaffected, we can exclude[4] the occurrence of any of the faults in the associated FDI-set S_j. The obtained set of *fault candidates* associated to the set S_a of active faults through the available set $\mathscr{R}$ of diagnostic signals is denoted as $\mathscr{C}^{\mathscr{R}}(S_a)$. This set will certainly include S_a, provided that each of the faults $f_1, \ldots, f_s$ belongs to at least one of the FDI-sets $S_1, \ldots, S_\mu$. This reasoning suggests the introduction of a further class of fault sets that extends the notion of FDI-set.

Definition 6 (*Exonerated FDI-set*). We define as an exonerated FDI-set for system (1) any fault set S that can be written in the form

$$S = \left\{ \bigcup_{k=1}^{\mu} \alpha_k S_k \right\} \setminus \left\{ \bigcup_{k=1}^{\mu} \bar{\alpha}_k S_k \right\}, \quad \alpha_k \in \{0, 1\}, \tag{7}$$

being $\bar{\alpha}_k = 1 - \alpha_k$, and $\{S_1, \ldots, S_\mu\}$ a collection of FDI-sets for system (1) verifying $\bigcup_{k=1}^{\mu} S_k = \{f_1, \ldots, f_s\}$.

The set of fault candidates $\mathscr{C}^{\mathscr{R}}(S_a)$ will then have form (7) of an exonerated FDI-set, where S_k is the FDI-set associated to the residual $r_k \in \mathscr{R}$ and $\alpha_k = 1$ if $S_k \cap S_a \neq \emptyset$. The set $\mathscr{C}^{\mathscr{R}}(S_a)$ will in general only strictly include S_a, while it is desired that $\mathscr{C}^{\mathscr{R}}(S_a)$ approximates as close as possible the set S_a. It is then natural to formulate the following *candidate fault set detection and isolation* (CFSDI) problem.

Problem 7 (*CFSDI*). For system (1), design a set of residuals $\mathscr{R} = \{r_1, \ldots, r_\mu\}$, respectively associated to the FDI-sets $S_1, \ldots, S_\mu$, such that, for each (possibly empty) set of active faults S_a, the corresponding candidate fault set

$$\mathscr{C}^{\mathscr{R}}(S_a) = \left\{ \bigcup_{S_k \cap S_a \neq \emptyset} S_k \right\} \setminus \left\{ \bigcup_{S_j \cap S_a = \emptyset} S_j \right\}, \tag{8}$$

is the smallest exonerated FDI-set that includes S_a.

The following result can be readily established for the solution of CFSDI problem (the proof based on set algebra is omitted).

Proposition 8. For system (1), the CFSDI is solved by any residual basis $\mathscr{R}^{\min} = \{r_1, \ldots, r_{N_r}\}$, where r_k detects and isolates the minimal FDI-set $S_k^{\min}$, $k = 1, \ldots, N_r$. For each set S_a of active faults, the corresponding (minimal) candidate fault set (8) is denoted by $\mathscr{C}(S_a)$.

Summarizing, the diagnostic system that provides, for any set of active faults S_a, the (minimal) candidate fault set $\mathscr{C}(S_a)$ has the *hybrid* structure given in Fig. 1. It is constituted by a bank of dynamic filters providing a residual basis $\mathscr{R}^{\min} = \{r_1, \ldots, r_{N_r}\}$,

[2] Residual r_k can be designed by solving the standard FDI problem for f_k, after removing from the formulation all faults in S except f_k.

[3] Under Assumption 1, at least one FDI-set always exists, i.e., the trivial set of all faults $\{f_1, \ldots, f_s\}$.

[4] This is also known as the *exoneration assumption* within the Artificial Intelligence community (see, e.g., Cordier et al., 2004; Fagarasan, Ploix, & Gentil, 2004).

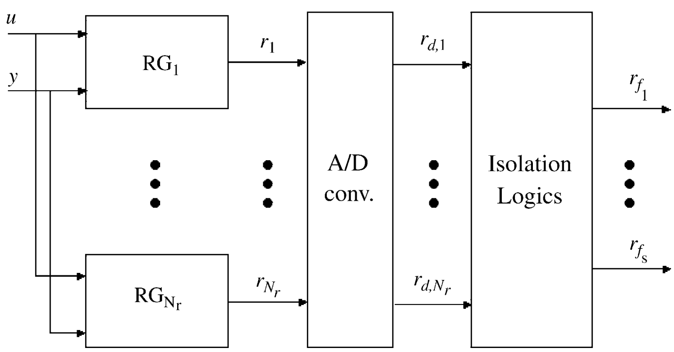

Fig. 1. Structure of a hybrid residual generator providing the candidate fault set $\mathscr{C}(S_{\mathrm{a}})$ corresponding to any set S_{a} of active faults.

an 'analog-to-digital' conversion block, providing a boolean version[5] $\mathscr{R}^d = \{r_1^d, \ldots, r_{N_r}^d\}$ of the basis $\mathscr{R}^{\min}$, and a combinatorial logics applied to $\mathscr{R}^d$. In particular, each output r_{f_i}, $i = 1, \ldots, s$, of this *isolation logics* is active (signalizing that $f_i \in \mathscr{C}(S_{\mathrm{a}})$) when all minimal FDI-sets including f_i have the corresponding residual affected, i.e.,

$$r_{f_i} = \bigwedge_{f_i \in S_k^{\min}} r_k^d, \tag{9}$$

where the symbol $\bigwedge$ indicates the logical AND operator.

As mentioned above, $\mathscr{C}(S_{\mathrm{a}})$ does not coincide in general with the set S_{a} of active faults, but it only satisfies $\mathscr{C}(S_{\mathrm{a}}) \supseteq S_{\mathrm{a}}$. Then, it is natural to ask in which cases $\mathscr{C}(S_{\mathrm{a}}) \equiv S_{\mathrm{a}}$ or, equivalently, what are the conditions for S_{a} to be an exonerated FDI-set. The following result holds.

Proposition 9. *Let* $S_{\mathrm{a}} = \{f_{a_1}, \ldots, f_{a_v}\}$ *be a set of currently active faults, and let* $L_{\mathrm{a}} = \{l_{a_1}, \ldots, l_{a_v}, n_1, \ldots, n_d\}$. *The candidate fault set* $\mathscr{C}(S_{\mathrm{a}})$ *coincides with* S_{a}, *i.e.*, S_{a} *is an exonerated FDI-set, if and only if*

$$\forall i : f_i \notin S_{\mathrm{a}}, \quad \mathrm{span}\{l_i\} \nsubseteq \mathscr{U}(L_{\mathrm{a}}). \tag{10}$$

Proof. Violation of (10) implies that, for some 'inactive' fault f_i, all minimal FDI-sets including f_i also include some active fault, so that f_i necessarily results to be in the fault candidate set. On the other hand, fulfillment of condition (10) guarantees that, for each inactive fault f_i, at least a minimal FDI-set exists, that includes f_i and does not include any active fault. Thus, all inactive faults can be excluded from the fault candidate set and sufficiency holds. □

Note that, when minimal FDI-sets $S_1^{\min}, \ldots, S_{N_r}^{\min}$ replace the generic FDI sets $S_1, \ldots, S_\mu$ in Definition (7) of exonerated FDI-sets, this clearly extends structure (6) of FDI-sets. Then, the geometric condition (10) for exonerated FDI-sets in form (7) is equivalent to condition (5) for FDI-sets in form (6).

[5] In the simplest case, residual $r_i^d \in \{0, 1\}$, $i = 1, \ldots, s$, is the result of the logical comparison $|r_i| > T_i$, being $T_i > 0$ a suitable threshold value that takes into account the overall level of noise affecting the analogica residual r_i.

3.3. Non-concurrent fault detection and isolation

When it can be assumed that the set S_{a} of active faults is always constituted by at most one element (*non-concurrency* of faults), the fulfillment of condition (10) can be verified a priori for all possible sets of active faults $S_{\mathrm{a}} = \{f_k\}$, $k = 1, \ldots, s$, and guarantees that the candidate set provided by the FDI scheme of Fig. 1 always coincides with the current active fault. The *non-concurrent fault detection and isolation* (N-CFDI) problem can be regarded as an alternative way for relaxing the standard FDI problem.

Problem 10 (*N-CFDI*). Assume that, at any time instant, at most one of the s faults $f_1, \ldots, f_s$ can affect system (1) (non-concurrency). Find, if possible, a residual generator that is able to detect and isolate any single fault f_i, $i = 1, \ldots, s$, from the other faults f_k, $k \neq i$, and from the disturbances $w_1, \ldots, w_d$.

The following result immediately follows from the application of Proposition 9 to the case of S_{a} constituted by just one fault input.

Corollary 11. *For each* $k = 1, \ldots, s$, *let* $L_k = \{l_k, n_1, \ldots, n_d\}$. *The N-CFDI problem is solvable for system* (1) *if and only if*

$$span\{l_i\} \nsubseteq \mathscr{U}(L_k), \quad \forall i, k, \ i \neq k. \tag{11}$$

Condition (11) implies that, for each couple of faults f_i and f_k, there always exists a minimal FDI-set that includes f_i but not f_k, so that the corresponding rows of the residual matrix RM are certainly different (there is a column with a '1' at row i and a '0' at row k) and non-zero. Being the N-CFDI problem a special instance of CFSDI, the same hybrid residual generator of Fig. 1 can be used for its solution.

4. Case study: Faults in a robot manipulator

In this section we describe the application of the relaxed FDI techniques to a robotic case study, the Quanser planar robot in De Luca and Mattone (2004) having $N = 2$ rotational joints, the first driven by a DC motor and the second unactuated, and with possible contacts between the end-effector and the environment. This mechanical system can be modeled by Euler–Lagrange equations of the form

$$B(q)\ddot{q} + c(q, \dot{q}) + e(q) = \tau + J^{\mathrm{T}}(q)F + \sum_{i=1}^{s} \hat{l}_i(q) f_i, \tag{12}$$

where $q \in \mathbb{R}^2$ is the joint variable vector, $B(q)$ is the positive definite symmetric inertia matrix, $c(q, \dot{q})$ is the vector of centrifugal, Coriolis and friction terms, $e(q)$ collects the gravitational terms, $\tau \in \mathbb{R}^2$ is the vector of joint torques (directly performing work on q), $F \in \mathbb{R}^2$ is the vector of external forces acting on the robot end-effector and $J(q)$ is the associated Jacobian matrix (transforming joint to end-effector linear velocities). The expression of the inertia matrix $B(q)$, and of the dynamic terms $e(q)$ and $c(q,\dot{q})$ is

$$B(q)=\begin{bmatrix} a_1+2a_2c_2 & a_3+a_2c_2 \\ a_3+a_2c_2 & a_3 \end{bmatrix},\quad e(q)=\begin{bmatrix} a_4s_1+a_5s_{12} \\ a_5s_{12} \end{bmatrix},$$

$$c(q,\dot{q})=\begin{bmatrix} -a_2\dot{q}_2(\dot{q}_2+2\dot{q}_1)s_2+F_{v1}\dot{q}_1+F_{c1}\mathrm{sign}(\dot{q}_1) \\ a_2\dot{q}_1^2s_2+F_{v2}\dot{q}_2+F_{c2}\mathrm{sign}(\dot{q}_2) \end{bmatrix},$$

where a shorthand notation for sine/cosine has been used (e.g., $c_{12}=\cos(q_1+q_2)$) and $(q_1,q_2)=0$ is the asymptotically stable, free equilibrium configuration (arm stretched downward). The expressions and numerical values of the dynamic coefficients a_j $(j=1,\ldots,5)$ and of the viscous and Coulomb friction coefficients F_{vi} and F_{ci} $(i=1,2)$ can be found in De Luca and Mattone (2004). The two-dimensional vector of external forces $F=(F_X,F_Y)$ is naturally expressed in the frame attached to the force sensor, with F_X and F_Y axes on the motion plane of the robot, but with the F_X axis rotated by an angle α w.r.t. the second robot link. Accordingly, the Jacobian matrix in Eq. (12) takes the form

$$J(q)=\begin{bmatrix} \ell_1\sin(q_2+\alpha)+\ell_2\sin\alpha & \ell_2\sin\alpha \\ \ell_2\cos\alpha+\ell_1\cos(q_2+\alpha) & \ell_2\cos\alpha \end{bmatrix}, \tag{13}$$

where ℓ_1, ℓ_2 are the two link lengths of the robot.

The last term in Eq. (12) models the faults possibly affecting the robot; they appear in the mechanical system at the *acceleration level* through smooth vector fields $\hat{l}_i(q)$ that only depend on joint positions q. For the considered case study, in particular, the following faults have been considered:

- *Actuator faults.* The applied torque by the possibly failed actuators is $\tau=\tau_c+f_\tau$, where τ_c is the vector of commanded values for the joint torques. For this kind of fault, it is $\hat{l}_i(q)=E_i$, the ith column of the (2×2) identity matrix. Note that by f_τ we can capture any type and time profile of actuator faults (see Mattone & De Luca, 2004 for a complete list).
- *Force sensor faults.* These are defined as $f_F=F-F_m$, where F is the actual vector of external forces applied to the robot end-effector and F_m is the corresponding measure provided by the force sensor. Hence, the generic fault vector field is in this case $\hat{l}_i(q)=j_i^T(q)$, where $j_i(q)$ is the ith row of the Jacobian $J(q)$.

Considering the possible occurrence of all the above faults, and the absence of any further disturbance (unavoidable input and measurement noise are not included in the formulation, but will be considered in the processing of the analogical residuals), the robot model (12) can be rewritten in state-space form

$$\dot{x}=g_0(x)+\sum_{k=1}^{m}g_k(x)u_k+\sum_{i=1}^{s}l_i(x)f_i,\quad (m=s=4) \tag{14}$$

with state $x=(q,\dot{q})\in\mathbb{R}^4$ and input vector $u=(\tau_c,F_m)\in\mathbb{R}^4$ (the expressions of vector fields g_0, g_k, and l_i can be easily derived). Correspondingly, it is $f_{1,2}=f_{\tau_1,\tau_2}$ and $f_{3,4}=f_{F_X,F_Y}$. As for the system output, the full state is measurable (joint positions are measured by sliding-contact encoders, while joint velocities are obtained by numerical differentiation of joint positions), so that we can take $y=x\in\mathbb{R}^4$.

4.1. Test of relaxed FDI conditions

For a nonlinear mechanical system in form (14), the necessary (and sufficient, by virtue of the full state availability) condition (2) for the detection and isolation of possibly concurrent faults can be written as

$$\mathrm{span}\{l_\kappa\}\not\subseteq\mathrm{span}\{L_\kappa\},\quad \kappa=1,\ldots,s, \tag{15}$$

where L_κ is here the set of all fault vector fields except l_κ. As anticipated in Section 2, being $(s=4>2=N)$, condition (15) is automatically violated for any l_κ, $\kappa=1,\ldots,4$, without the need of performing computations. Thus, none of the faults possibly affecting the robot manipulator can be exactly isolated from *all* other, possibly concurrent, faults.

The first step for applying the results of Section 3 consists in the computation of all minimal FDI-sets $S_i^{\min}$, and thus of the system residual matrix. When $\alpha\notin\{0,\pi\}$, the following $N_r=4$ minimal FDI-sets are found: $S_1^{\min}=\{f_1,f_3,f_4\}$, $S_2^{\min}=\{f_2,f_3,f_4\}$, $S_3^{\min}=\{f_1,f_2,f_3\}$, and $S_4^{\min}=\{f_1,f_2,f_4\}$, corresponding to the residual matrix of Table 1. Note that the computation of the residual matrix RM did not require the actual design of residual generators. All FDI-sets are given by these minimal FDI-sets, plus the trivial FDI-set $\{f_1,\ldots,f_4\}$ (see Eq. (6)).

Concerning the CFSDI problem, all possible exonerated FDI-sets are easily computed, according to Eq. (7), as: $S_1^{\mathrm{exo}}=\{f_1\}$, $S_2^{\mathrm{exo}}=\{f_2\}$, $S_3^{\mathrm{exo}}=\{f_3\}$, $S_4^{\mathrm{exo}}=\{f_4\}$, and $S_5^{\mathrm{exo}}=\{f_1,f_2,f_3,f_4\}$. These sets show that, for the considered robotic system, any set S_a with concurrent active faults will necessarily result in the 'trivial' set of fault candidates $\mathscr{C}(S_a)=\{f_1,f_2,f_3,f_4\}$, i.e., the only exonerated FDI-set that includes more than one fault input. In other words, concurrent faults in system (14) can be detected but not isolated.

The N-CFDI problem can instead be successfully solved, according to Corollary 11. In particular, Table 1 shows that the implementation of the whole residual basis $\mathscr{R}^{\min}=\{r_1,\ldots,r_4\}$ is not required in the solution, since columns 1–3 of the residual matrix have already different and non-zero rows (i.e., residuals r_1, r_2, and r_3 allow isolation of all non-concurrent faults). Nonetheless, the inclusion of residual r_4 guarantees that all rows of the residual matrix differ by two elements and allows to recognize also the violation of the non-concurrency assumption (when all residuals are excited), thus increasing the overall robustness of the FDI system.

Table 1
Residual matrix RM associated to system (14) (faults vs. minimal FDI-sets/residuals)

	$S_1^{\min}/r_1$	$S_2^{\min}/r_2$	$S_3^{\min}/r_3$	$S_4^{\min}/r_4$
$f_1=f_{\tau_1}$	1	0	1	1
$f_2=f_{\tau_2}$	0	1	1	1
$f_3=f_{F_X}$	1	1	1	0
$f_4=f_{F_Y}$	1	1	0	1

4.2. Design of a residual basis

Residual generators for systems in form (14) can be essentially designed as nonlinear observers, with the residual corresponding to the observation error (asymptotically converging to zero in the absence of faults). Following a geometric approach, the basic ingredient is a suitable change of coordinates that induces a system decomposition into subsystems that are affected/not affected by the desired subsets of fault inputs. The detailed design procedure is described in Mattone and De Luca (2004) for the whole class of Euler–Lagrange mechanical systems. Hereafter, we just provide the dynamic expression of an observer/residual generator for each residual r_i $(i = 1, \ldots, 4)$ in the basis $\mathscr{R}^{\min}$. All these expressions are computable, being based on the available signals τ_c (commanded) and q, $\dot{q}$, F_m (measured).

For residuals r_1 and r_2, by setting $p' = B(q)\dot{q}$, the corresponding residual generator is

$$\dot{\xi}' = \tilde{g}'_0(q, \dot{q}) + \tau_c + J^T(q)F_m + K'(p' - \xi),$$
$$r' = [r_1 \ \ r_2]^T = K'(p' - \xi'), \tag{16}$$

with $\xi' \in \mathbb{R}^2$, diagonal matrix $K' > 0$, and $\tilde{g}'_0(q, \dot{q}) = \dot{B}(q)\dot{q} - c(q, \dot{q}) - e(q)$. The dynamics of r' satisfies $\dot{r}' = -K'r' + K' f_\tau + K' J^T(q) f_F$, so that each of the two residuals r_1, r_2 is affected by just one of the two actuator faults $f_{\tau_{1,2}}$, and by both force sensor faults f_{F_X} and f_{F_Y}, in accordance with the residual matrix of Table 1.

For residuals r_3 and r_4, by setting $p'' = J^{T\#}(q)B(q)\dot{q}$, where $J^{T\#}(q)$ is the adjoint of matrix $J^T(q)$ divided by the factor $\ell_1\ell_2$, the corresponding residual generator is

$$\dot{\xi}'' = \tilde{g}''_0(q, \dot{q}) + J^{T\#}(q)\tau_c + s_2 F_m + K''(p'' - \xi'')$$
$$r'' = [r_3 \ \ r_4]^T = K''(p'' - \xi''), \tag{17}$$

with $\xi'' \in \mathbb{R}^2$, diagonal matrix $K'' > 0$, and $\tilde{g}''_0(q, \dot{q}) = J^{T\#}(q)(\dot{B}(q)\dot{q} - c(q, \dot{q}) - e(q)) + \dot{J}^{T\#}(q)B(q)\dot{q}$. Correspondingly, it holds $\dot{r}'' = -K''r'' + K''s_2 f_F + K'' J^{T\#}(q) f_\tau$, i.e., each of the two residuals r_3 and r_4 is affected by just one of the two force sensor faults f_{F_X, F_Y}, and by both actuator faults $f_{\tau_{1,2}}$, which corresponds to the residual matrix of Table 1.

4.3. Experimental results

We report here experimental results for the N-CFDI problem. During normal operation, the first joint variable has to be regulated at the reference value $q_{1d} = 30°$ by a standard PID controller, with gains $K_P = 0.2$, $K_I = 1$, and $K_D = 0.02$. A loss of power of 50% (a multiplicative fault) is experienced by the first joint actuator between $t = 1.7$ s and $t = 2$ s, a total failure[6] of the second joint actuator occurs in the interval [2.5, 3] s (the commanded torque was $\tau_{c,2} = 0.05$ N m in this interval), while a bias of 1 and 0.7 N affects the X- and Y-force measures for $t \in [3.5, 4]$ and $t \in [4.5, 5]$ s, respectively (the actual contact forces were in both cases zero, i.e., the robot was in free motion). Therefore, non-concurrency holds for this fault scenario.

Fig. 2 shows the behavior of residuals $r_1, \ldots, r_4$ for $K' = K'' = \mathrm{diag}\{50, 50\}$, together with the fault time intervals. As expected, none of these signals is exactly zero (see also Fig. 3, where a detail of residual r_1 is reported), even in the absence

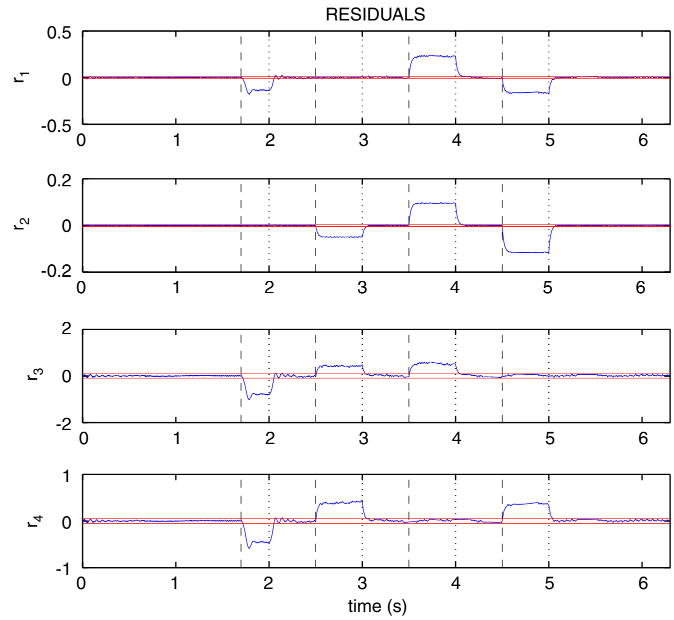

Fig. 2. Behavior of the residual basis $\mathscr{R}^{\min} = \{r_1, \ldots, r_4\}$ in the N-CFDI experiment on the $2R$ robot. Detection thresholds and fault time intervals are also indicated (fault occurrence: dashed; fault end: dotted).

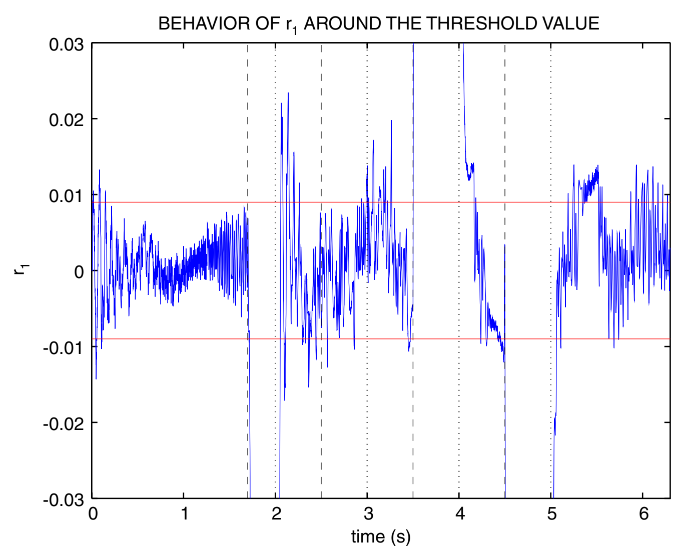

Fig. 3. Detail of residual r_1 in Fig. 2 around the threshold value.

[6] While several types of faults have been tested on the actuator at joint 1, only a total actuator failure could be emulated at the second (unactuated) joint, by requiring some $\tau_{c,2}$ that, of course, cannot be provided (thus, $f_{\tau,2} \equiv -\tau_{c,2}$).

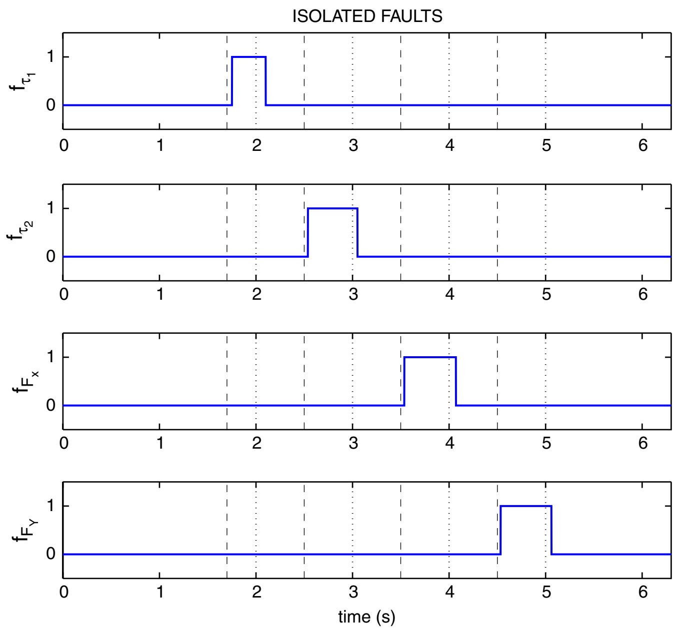

Fig. 4. Isolated fault intervals in the N-CFDI experiment on the $2R$ robot.

of faults, due to the presence of input and measurement disturbances affecting the system. These unmodeled disturbances mainly come from the PWM circuits driving the DC motor and from the numerical differentiation of joint positions, performed to get estimates of the joint velocities. In order to filter out this 'background noise' from the diagnostic signals and allow the reliable detection of faults, we have implemented a *dynamic thresholding* mechanism: the ith residual is treated as zero unless it has been over a fixed threshold T_i (displayed in Fig. 2 for each residual, and in Fig. 3 for r_1) for at least a given time T_{set}. When this happens, the occurrence of a fault is recognized. Similarly, after a fault diagnosis, residuals are considered 'unaffected' again (recognizing the fault end) only after they have been below their respective thresholds for a given time T_{reset}. At the cost of a small delay in the diagnosis, this mechanism allows the use of relatively low detection thresholds, corresponding to a good sensitivity to faults, without producing unacceptable false-alarm rates. This is confirmed by the reported experimental results. In fact, although the detection threshold is exceeded several times outside the fault intervals (see, e.g., Fig. 3), no false alarm is raised up by the FDI system, and all faults are correctly detected. Fig. 4 shows the final result of the FDI process for $T_{set} = 0.03$ s, $T_{reset} = 0.02$ s, and thresholds $(T_1, \ldots, T_4) = (0.009, 0.005, 0.09.0.05)$. The associated detection delays, in the order of 50 ms, are acceptable for a safe robot operation.

5. Conclusions

Motivated by a nonlinear robotic application where the necessary conditions for standard FDI are violated, we have introduced different relaxed formulations of fault detection and isolation problems where the focus of the isolation issue has been moved from single to suitable sets of faults of practical interest. Weaker necessary and sufficient conditions have been derived by logical reasoning from the conditions of geometric nature available for the standard FDI problem.

The resulting hybrid structure of the diagnostic system, cascading nonlinear residual generators with a combinatorial isolation logics, has been successfully tested for FDI of actuator and force sensor faults in a robot manipulator. The design steps have been implemented using the nonlinear dynamic model of the robot and the achieved performance was illustrated by experiments.

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References

Cordier, M.-O., Dague, P., Lévy, F., Montmain, J., Staroswiecki, M., & Travé-Massuyès, L. (2004). Conflicts versus analytical redundancy relations: a comparative analysis of the model based diagnosis approach from the artificial intelligence and automatic control perspectives. *IEEE Transactions on Systems Man, and Cybernetics—Part B: Cybernetics*, *34*(5), 2163–2177.

De Luca, A., & Mattone, R. (2004). An adapt- and- detect FDI system for robot manipulators. *Proceedings of 2004 IEEE International Conference on Robotics and Automation* (pp. 4975–4980). New Orleans, LA.

De Persis, C., & Isidori, A. (2001). A geometric approach to nonlinear fault detection and isolation. *IEEE Transactions on Automatic Control*, *46*(6), 853–865.

Fagarasan, I., Ploix, S., & Gentil, S. (2004). Causal fault detection and isolation based on a set-membership approach. *Automatica*, *40*, 2099–2110.

Frank, P. M. (1990). Diagnosis in dynamic systems using analytical and knowledge-based redundancy—a survey. *Automatica*, *26*, 459–474.

Frank, P. M., & Ding, S. X. (1994). Frequency domain approach to optimally robust residual generation and evaluation for model-based fault diagnosis. *Automatica*, *30*, 789–904.

Hammouri, H., Kinnaert, M., & El Yaagoubi, E. H. (1999). Observer-based approach to fault detection and isolation for nonlinear systems. *IEEE Transactions on Automatic Control*, *44*(10), 1879–1884.

Larson, E. C., Parker Jr., B. E., & Clark, B. R. (2002). Model-based sensor and actuator fault detection and isolation. *Proceedings of 2002 American Control Conference* (pp. 4215–4219), Anchorage, AK.

Massoumnia, M.-A. (1986). A geometric approach to the synthesis of failure detection filters. *IEEE Transactions on Automatic Control*, *31*(9), 839–846.

Mattone, R., & De Luca, A. (2004). *Fault detection and isolation in mechanical systems*, Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza", Tech. Rep. 10, June.

Mattone, R., & De Luca, A. (2005). Conditions for detecting and isolating sets of faults in nonlinear systems. *Proceedings of 44th IEEE Conference on Decision and Control/European Control Conference, Seville.*

Simani, S., Fantuzzi, C., & Beghelli, S. (2000). Diagnosis techniques for sensor faults of industrial processes. *IEEE Transactions on Control Systems Technology*, *8*(5), 848–855.

Zhang, P., Ding, S. X., Wang, G. Z., & Zhou, D. H. (2005). Fault detection of linear discrete-time periodic systems. *IEEE Transactions on Automatic Control*, *50*(2), 239–244.

Zhang, X., Polycarpou, M. M., & Parisini, T. (2002). A robust detection and isolation scheme for abrupt and incipient faults in nonlinear systems. *IEEE Transactions on Automatic Control*, *47*(4), 576–593.

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