THE MINIMUM-TIME CRASHING PROBLEM FOR THE DUBINS' CAR

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Abstract: This paper determines the time-optimal trajectories bringing a car-like robot, whose driving velocity is constrained to be strictly positive (Dubins' car), in collision with the obstacles in its workspace. Both the robot and the obstacles are assumed to have polygonal shape. Based on these trajectories, a distance function is defined and computed that takes into account the nonholonomic constraints and captures the non-symmetric nature of the system.

Keywords: Dubins' car, shortest paths, Pontryagin's maximum principle.

1. INTRODUCTION

A wheeled mobile robot whose driving velocity is constrained to assume a constant positive value is known in the literature as Dubins' car since the result achieved by Dubins (Dubins, 1957) in 1957 characterizes the structure of the shortest paths between any two given configurations for this system.

After Dubins work, this system has been extensively studied in the context of motion planning and control of nonholonomic systems both for the theoretical challenges that it provides and for its practical relevance in modeling the kinematics of road vehicles, aircrafts cruising at constant altitude, sea vessels and, more recently, needle motion inside tissues (Alterovitz *et al.*, 2005).

The difficulty of planning for this system derives from the pure rolling constraint of the wheels which prevents the robot to move instantaneously towards certain directions. As a consequence, not all the paths in the configuration space are feasible for this robot. For this reason, the Euclidean metric is not appropriate for determining the distance to obstacles in the robot environment. In a previous work (Vendittelli *et al.*, 1999) we have defined and showed how to compute a distance between the robot and the obstacles in its environment which takes into account the nonholonomic constraints. In that work we considered a point-wise robot, i.e., we did not take into account the shape of the robot, hence computing the distance between one point on the robot body (the reference point) and the boundaries of the obstacles. In this paper, we define and show how to compute the distance between the robot and the obstacles which takes into account both the nonholonomic constraints and the shape of the robot. In other words, we compute the distance between the boundary of the robot body and the boundaries of the obstacles, both assumed to be polygonal. Our work relies on Dubins' results but adopt the optimal control approach proposed in (Sussmann and Tang, 1991; Boissonnat *et al.*, 1992). In particular, the study of transversality conditions allows selecting a sufficient family of time-optimal trajectories whose length will determine the distance to the obstacles. The paper is organized as follows. In sec. 2 we summarize the basic result on time-optimal-trajectories for the Dubins' car. Due to the constant velocity constraint, these trajectories minimize also the path length. This is exactly the property that we exploit in defining the distance function in sec. 3 where we also give the procedure for its computation. Since the distance function is based on the computation of the length of the shortest paths belonging to a sufficient family, in sec. 4 we show how to reduce the number of paths in this family to lower the distance computation complexity. In sec. 5 we propose some simulation results showing the aspect of the level curves relatives to different robot shapes.

2. DUBINS' SHORTEST PATHS

The dynamics of the Dubins' car considered in this paper is described by the control system

$$\dot{\xi} = f(\xi(t), u(t)) = g_1(\xi(t)) + g_2(\xi(t))u(t), \quad (1)$$

where $\xi(t) = (x_r(t), y_r(t), \theta(t))^T$ is the configuration of the vehicle at time t, and is given by the position (x_r, y_r) of a reference point P on the robot body and by the orientation θ of the vehicle w.r.t. the abscissa axis of a fixed reference frame, while $g_1(\xi(t)) = (\cos\theta(t), \sin\theta(t), 0)^T$, $g_2(\xi(t)) = (0, 0, 1)^T$ are the input vector fields. The driving velocity has been set, w.l.o.g., equal to 1 and the steering velocity $u(t)$ is such that $|u(t)| \leq 1$. In his pioneering work (Dubins, 1957), Dubins determined the curves of minimal length between any two points in the plane with assigned initial and final tangent and subject to curvature constraints. The family of paths obtained by Dubins are time-optimal solutions of system (1). Later, Sussmann and Tang (Sussmann and Tang, 1991) and Boissonnat *et al.* (Boissonnat *et al.*, 1992) obtained the same result by adopting an optimal control point of view. Based on these results, we determined (Vendittelli *et al.*, 1999) the shortest paths between any given robot configuration and the obstacles populating its workspace. We defined the distance between a point-wise robot and the obstacles in the environment as the length of the shortest path between the robot and the obstacles. Due to the one-to-one correspondence between the paths in the configuration space and the corresponding paths in the plane, this represents also a distance in configuration space. In the present paper, we remove the hypothesis of point robot and take into account the shape of the robot in determining the distance to the obstacles. Prior to describe our research, we recall the main result on which our work is based.

2.1 Shortest paths without obstacles

This section summarizes the results presented in (Boissonnat *et al.*, 1992; Bui *et al.*, 1994; Sussmann and Tang, 1991) and leading to a synthesis of the shortest paths for a Dubins robot. In accordance with the notation proposed in (Bui *et al.*, 1994), we will use C and S to denote, respectively, an arc of circle of minimal radius and a straight line segment. To specify the direction of motion along the path, letters L and R will denote, respectively, a counter-clockwise or clockwise sense of rotation of the direction vector $\overrightarrow{v}$, while S will mean motion along a straight segment. Subscripts are positive real numbers giving the length of each elementary path composing an optimal path and they will be referred to as *path parameters* (a, b, e). For example, a path of type CSC may be specified as $L_aS_bR_e$, that is: forward left turn of length a, straight motion of length b and forward right turn of length e. The starting configuration of each path is assumed, w.l.o.g., to be the origin $(0, 0, 0)^T$.

Considering system (1), we want to minimize the time to travel from $\xi(t_i)$ to $\xi(t_f)$. For system (1), this is equivalent to minimize the path length. The Hamiltonian (Pontryagin *et al.*, 1962) is

$$H = \langle\psi, f\rangle = \psi_1 \cos\theta + \psi_2 \sin\theta + \psi_3 u = \phi_1 + \phi_2 u$$

where the costate ψ satisfies the adjoint equation

$$\dot{\psi} = \frac{\partial H}{\partial \xi}(\psi, \xi, u) = -\psi \left[\frac{\mathrm{d}g_1}{\mathrm{d}\xi} + u\frac{\mathrm{d}g_2}{\mathrm{d}\xi}\right] \quad (2)$$

for almost all t, and $\phi_1 = \langle\psi, g_1\rangle$, $\phi_2 = \langle\psi, g_2\rangle$ represent the switching functions. If a constraint on the final state $\chi(\xi_f) = 0$ of dimension σ_f is present, it is possible to derive a set of transversality conditions $\psi_f = M^T\zeta$, where $M = \partial\chi/\partial\xi_f$ is a $\sigma_f \times 3$ matrix and ζ is an auxiliary vector of dimension σ_f.

Results from (Sussmann and Tang, 1991; Boissonnat *et al.*, 1992) allow to restrict the search of optimal paths for the Dubins car to a sufficient set of paths consisting in concatenations of arcs of circle of minimum radius (C) or straight line segments (S). The straight line segments and the points of inflection lie on a line $\mathcal{D}_0$ defined by the equation:

$$\mathcal{D}_0 : \; \psi_3 = \psi_1 y_r - \psi_2 x_r + \psi_3(t_0) = 0 \quad (3)$$

where ψ_1 and ψ_2 are constants and the ratio ψ_2/ψ_1 gives the direction of $\mathcal{D}_0$.

The sufficient set of optimal paths can be partitioned into 2 families

$$\begin{array}{lll} (I) & C_aC_bC_e & 0 \leq a \leq b, \pi < b < 2\pi, 0 \leq e \leq b, \min\{a, e\} < b - \pi \\ (II) & C_aS_eC_b & 0 \leq a < 2\pi, 0 \leq b \leq 2\pi, e \geq 0, \end{array} \quad (4)$$

where

- family (I) includes 2 types: LRL and RLR;
- family (II) includes 4 types: LSL, LSR, RSL and RSR.

These paths induce a partition of the configuration space into a finite number of cells, each corresponding to a path type optimal for linking a point in the cell to the origin (Bui *et al.*, 1994). Every Dubins path maps smoothly the parameter space into the configuration space (Moutarlier *et al.*, 1996), i.e., for each path p_i one can define a function $W_i : \mathbb{R}^3 \longrightarrow \mathcal{C}$ associating the final configuration in $\mathcal{C}$ to the parameters (a, b, e)

$$\begin{pmatrix} x_r \\ y_r \\ \theta \end{pmatrix} = W_i(a, b, e) = \begin{pmatrix} X_i(a, b, e) \\ Y_i(a, b, e) \\ \Theta_i(a, b, e) \end{pmatrix} \quad (5)$$

where X_i, Y_i, Θ_i are smooth. This property will be used in the following computations.

3. DISTANCE FUNCTION

The aim of this work is to find the length d of the shortest path bringing any point on the boundary of a Dubins robot of polygonal shape in contact with any point on the boundary of any polygonal obstacle in physical space. The searched path will be optimal for linking the robot starting configuration to the configuration in contact with one of the obstacles in the environment; it will, therefore, belong to one of the families of Dubins optimal paths (4). For this reason, the search will be restricted to these families. The one-to-one correspondence between the paths in the configuration space and the corresponding paths in the plane allows us to choose d as the distance between the robot and the obstacles. Note that, due to the non-symmetric nature of system (1) (Sussmann and Tang, 1991), the length of the shortest paths induces a quasi-metric because the symmetry property $d(x, y) = d(y, x)$ is not verified. Symmetry can always be recovered by defining the distance $d^\star = \max\{d(x, y), d(y, x)\}$. The use of a non-symmetric distance function, however, would be more appropriate in planning collision free motion for the Dubins car since it naturally captures the non-symmetric nature of this system. The implication of the presence of discontinuities in the distance function is, however, beyond the scope of the present paper.

In a polygonal environment, the distance computation problem can be partitioned (Moutarlier *et al.*, 1996) into the three subproblems of bringing in contact (see fig. 1):

(i) one vertex q_i of the robot with one vertex o_j of the obstacle;
(ii) one vertex q_i of the robot with the line v_j supporting one edge $o_j o_{j+1}$ of the obstacle;
(iii) the line w_i supporting one edge $q_i q_{i+1}$ of the robot with one vertex of the obstacle o_j.

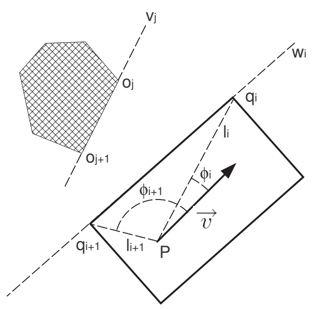


Fig. 1. Partitioning the distance computation.

If one is able to solve these subtasks, the shortest path to an obstacle is found by iterating the three steps over all the robot/obstacle vertex/edge combinations and by choosing the minimum among the obtained path lengths. The three problems (i), (ii) and (iii), can be associated to the following three functions:

1. $L^{VV}: \quad \mathbb{R}^4 \to \mathbb{R}$, $\quad (q_i, o_j) \to L^{VV}(q_i, o_j)$
2. $L^{VE}: \quad \mathbb{R}^4 \to \mathbb{R}$, $\quad (q_i, v_j) \to L^{VE}(q_i, v_j)$
3. $L^{EV}: \quad \mathbb{R}^4 \to \mathbb{R}$, $\quad (w_i, o_j) \to L^{EV}(w_i, o_j)$

where L^{VV}, L^{VE}, and L^{EV} will be defined in the following sections. The distance d is defined as:

$$d() : \mathbb{R}^8 \to \mathbb{R}$$
$$d() = \min\{\min_{i,j} L^{VV}, \min_{i,j} L^{VE}, \min_{i,j} L^{EV}\}. \tag{6}$$

In the next sections we will detail the procedure adopted to solve each specific subproblem. The idea is to associate a *contact manifold* to each subproblem, expressing the constraint that the final state of the robot should be such that the corresponding type of contact (i.e., VV, VE or EV) will occur between the robot and the obstacle. In addition, a transversality condition will express the constraint that the contact should occur in minimum time. These constraints will provide a system of three nonlinear equations in the three unknowns a, b and e.

In sec. 4, the family (I) will be proved to be never optimal for the considered problem, thus the search for the optimal path can be limited to the family (II). In the following sections, we will denote by p a specific Dubins path and by OP the set of the optimal paths belonging to the family (II). Due to lack of space, the proofs will be limited to the VV case only. The remaining cases can be proved following the same line of reasoning.

3.1 Vertex-Vertex distance

Let $p_k \in OP$, $k = 1 \ldots 4$ be an optimal path of the sufficient family (II) and (a_k, b_k, e_k) the associated path parameters; we define the function

$$VV_{p_k}: \quad \mathbb{R}^4 \to \mathbb{R}^3, \quad (q_i, o_j) \to (a_k, b_k, e_k) \tag{7}$$

where $VV_{p_k}(q_i, o_j)$ is the map which solves the problem (i) using the specific path p_k.

The function $L^{VV}(q_i, o_j)$ is then defined as:

$$L^{VV}(q_i, o_j) = \min_{p_k \in OP} L_{p_k}(VV_{p_k}(q_i, o_j)), \tag{8}$$

where L_{p_k} is the length of the path p_k in terms of the three parameters (a_k, b_k, e_k).
Remarks: When solving for each p_k, three scenarios may arise:

- the solution does not exist, i.e., at least one path parameter is complex;
- the solution exists but it is not valid, i.e., at least one path parameter is outside its range of validity;

- a valid solution exists.

In the first two cases p_k is discarded.

With reference to fig. 1, let P be the robot reference point, (l_i, ϕ_i) be the pair representing the length of the segment $\overline{Pq_i}$ and the angle between the vectors $\overrightarrow{Pq_i}$ and $\overrightarrow{v}$. The cartesian coordinates (q_{i_x}, q_{i_y}) of the robot vertex are given by:

$$\begin{cases} x_r + l_i \cos(\theta + \phi_i) &= q_{i_x} \\ y_r + l_i \sin(\theta + \phi_i) &= q_{i_y}. \end{cases} \tag{9}$$

Denoted by (o_{j_x}, o_{j_y}) the cartesian coordinates of the target vertex o_j of the obstacle, the 1-dimensional contact manifold is defined by

$$C_{VV}^{ij}(x_r, y_r, \theta) = \{(x_r, y_r, \theta) | \, q_i \equiv o_j\} \tag{10}$$

and can be represented by the following equation

$$\chi_{VV}^{ij}(x_r, y_r, \theta) = \begin{pmatrix} x_r - o_{j_x} + l_i \cos(\theta + \phi_i) \\ y_r - o_{j_y} + l_i \sin(\theta + \phi_i) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{11}$$

Equation (11) represents a vertical helix centered on o_j (fig. 2) and will be used for finding the solution path, i.e., for determining the three path parameters (a, b, e). An additional constraint is necessary to make the problem "square" (3 parameters and 3 equations) and it will be derived from transversality conditions, as shown below.

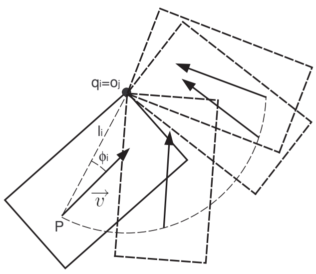


Fig. 2. *VV* contact.

Lemma 1. A necessary condition for a path in OP to be optimal for the problem (i) is that the line $\mathcal{D}_0$ passes through the point o_j.

Proof: Let $\xi_f = (x_r(t_f), y_r(t_f), \theta(t_f))$ be the final robot configuration. The constraint on the robot final state $\chi_{VV}^{ij}(\xi_f) = 0$ can be used to derive the transversality condition $\psi_f = M^T \zeta$, where:

$$M = \frac{\partial \chi_{VV}^{ij}(\xi_f)}{\partial \xi_f} = \begin{pmatrix} 1 & 0 & -l_i \sin(\theta_f + \phi_i) \\ 0 & 1 & l_i \cos(\theta_f + \phi_i) \end{pmatrix}$$

and $\zeta = (\zeta_1 \quad \zeta_2)^T$. We then get the system

$$\begin{cases} \psi_1 &= \zeta_1 \\ \psi_2 &= \zeta_2 \\ \psi_3(t_f) &= -l_i \sin(\theta_f + \phi_i)\zeta_1 + l_i \cos(\theta_f + \phi_i)\zeta_2 \end{cases}$$

from which, by substituting ψ_1 and ψ_2 in the third equation, we obtain

$$\psi_3(t_f) = -l_i \sin(\theta_f + \phi_i)\psi_1 + l_i \cos(\theta_f + \phi_i)\psi_2.$$

Using the definition (3) of ψ_3, we can get:

$$\psi_3(t_0) = -\psi_1 o_{j_y} + \psi_2 o_{j_x}.$$

Thus, the line $\mathcal{D}_0$ has equation:

$$\psi_3 = \psi_1(y_r - o_{j_y}) - \psi_2(x_r - o_{j_x}) = 0 \tag{12}$$

which implies our thesis. ⋄

Recalling from (5) that every path p_k is associated to a smooth map $(x_r, y_r, \theta)^T = W_k(a, b, e)$ and denoting by $\overline{Y}_k$ and $\overline{X}_k$ the final positions, computed via (5), associated to the subpath of p_k which brings the robot on the line $\mathcal{D}_0$ (where $\psi_3 = 0$) [1], we get from (11) and (12) a square system of equations for every VV_{p_k} directly projected into the path parameter space

$$\begin{cases} \chi_{VV}^{ij}(W_k(a, b, e)) = 0 \\ \psi_1 \cdot (\overline{Y}_k(a, b, e) - o_{j_y}) - \psi_2 \cdot (\overline{X}_k(a, b, e) - o_{j_x}) = 0. \end{cases} \tag{13}$$

The solution of (13) yields the optimal path.

3.2 Vertex-Edge distance

In this section we will show how to solve problem (ii). In this case, the contact manifold is 2-dimensional. Two additional constraints will be derived from transversality conditions. Since we assume the target edge of the obstacle to be an unbounded line, there could be solutions returning a landing point outside the edge boundaries; in this case the solution is discarded.

Analogously to the previous section, the function

$$\begin{array}{rcl} VE_{p_k}(): & \mathbb{R}^4 & \rightarrow \mathbb{R}^3 \\ & (q_i, v_j) & \rightarrow (a_k, b_k, e_k) \end{array}$$

describes the map which solves the problem (ii) for a given path $p_k \in OP$ returning the three path parameters and the landing point on the edge. The $L^{VE}()$ function can be expressed as:

$$L^{VE}(q_i, v_j) = \min_{p_k \in OP} L_{p_k}(VE_{p_k}(q_i, v_j)). \tag{14}$$

All the remarks stated for (8) hold in this case.

Let $y = m_j x + n_j$ be the equation of the target edge v_j; using (9), the contact manifold $C_{VE}^{ij}(x_r, y_r, \theta) = \{(x_r, y_r, \theta) | \, q_i \in v_j\}$ is described by:

$$\chi_{VE}^{ij}(x_r, y_r, \theta) = \\ y_r - m_j x_r - n_j - l_i m_j \cos(\theta + \phi_i) + l_i \sin(\theta + \phi_i) = 0 \tag{15}$$

Equation (15) represents a 2-dimensional surface whose projection on the plane xy for a given θ is a line parallel to v_j (fig. 3).

Lemma 2. If a path in OP is optimal for the problem (ii) then:

(1) the line $\mathcal{D}_0$ is perpendicular to the line v_j;
(2) the landing point lies at the intersection of $\mathcal{D}_0$ and v_j.

[1] For instance, if p_k is $L_a S_e R_b$, $\mathcal{D}_0$ is reached through the subpath L_a.

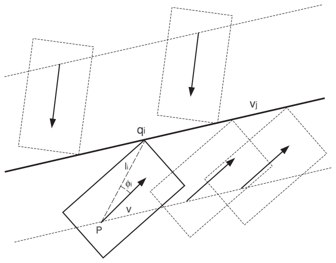


Fig. 3. VE contact.

Putting together the contact manifold constraint and the two transversality conditions we get a square system of equations for each path $p_k \in OP$.

3.3 *Edge-Vertex distance*

The approach adopted to solve problem (iii) is analogous to the method outlined in the last section: we assume an unbounded robot edge and we discard any solution yielding a landing point outside the edge boundaries $(q_i,\, q_{i+1})$. Defining:

$$EV_{p_k}() : \quad \mathbb{R}^4 \quad \to \mathbb{R}^3 \\ (w_i,\, o_j) \to (a_k,\, b_k,\, e_k)$$

as the map which solves problem (iii) for a specific path p_k, the $L^{EV}()$ function is:

$$L^{EV}(w_i,\, o_j) = \min_{p_k \in OP} L_{p_k}(EV_{p_k}(w_i,\, o_j)). \quad (16)$$

Let $(q_i,\, q_{i+1})$ be two adjacent robot vertices. With reference to fig. 4, the line w_i can be expressed as $y = m_i(\theta)x + n_i(x_r,\, y_r,\, \theta)$ using (9) and simple geometric arguments (the expressions of m_i and n_i are omitted for lack of space). Denoting by $o_j = (o_x,\, o_y)$ the target point, the contact manifold

$$C^{ij}_{EV}(x_r,\, y_r,\, \theta) = \{(x_r,\, y_r,\, \theta) |\; o_j \in w_i\} \quad (17)$$

represents a 2-dimensional surface whose projection on the xy plane is made of straight lines rotating at a fixed distance from o_j (fig. 4).

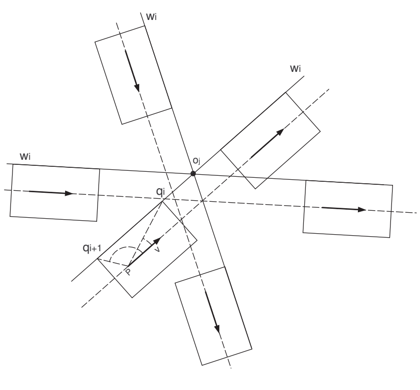


Fig. 4. EV contact.

Lemma 3. If a path in OP is optimal for (iii) then:

(1) the line $\mathcal{D}_0$ must be perpendicular to the edge w_i at the end of the path;
(2) the landing point lies at the intersection of $\mathcal{D}_0$ and w_i.

Contact constraint (17) and transversality conditions expressed by Lemma 3 provide the system of equations for each path $p_k \in OP$.

4. FAMILY REFINEMENT

In this section we will show that the family (I) of optimal paths is never optimal. The proof takes advantage of the continuity of the parameters $(a,\, b,\, e)$ w.r.t. the parameter l_i of the robot. Some preliminar results are needed:

Lemma 4. For any robot/obstacle vertex/edge q_i/o_j, w_i/v_j, we have:

(1) if $L^* = L^{VE}(q_i,\, v_j)$ is the optimal solution of problem (ii) and P^*_l the associated landing point, then $L^* = L^{VV}(q_i,\, P^*_l)$;
(2) if $L^* = L^{EV}(w_i,\, o_j)$ is the optimal solution of problem (iii) and P^*_l the associated landing point, then $L^* = L^{VV}(P^*_l,\, o_j)$.

The simple proof of this lemma is omitted for lack of space.

Lemma 4 allows to focus only on the L^{VV} distance, since the other two functions can always be reduced to this case. The solution of every instance $L^{VV}(q_i,\, o_j)$ is found by solving (13).

Lemma 5. ψ_1 and ψ_2 are smooth functions of the parameters $(a,\, b,\, e)$.

Proof:

- Family (I): let F_1 and F_2 be the two inflection points reached after the first and the second arc C, respectively. Their coordinates are smooth functions of $(a,\, b,\, e)$ since C and CC are both Dubins paths. Hence, $\psi_2/\psi_1 = (F_{y1} - F_{y2})/(F_{x1} - F_{x2})$;
- Family (II): the direction of $\mathcal{D}_0$ coincides with the constant robot orientation θ_s on the straight segment, hence $\psi_2/\psi_1 = \tan(\theta_s)$. $\diamond$

Thus, the equations in (13) are smooth w.r.t. the unknowns $(a,\, b,\, e)$ via the W_i maps and w.r.t. $(q_i,\, o_j)$ by construction of the contact manifold C^{ij}_{VV} and the transversality condition (12). The Dubins parameters appear either algebraically (when relative to straight segments) or inside trigonometric functions (when relative to arcs of circle); a classic change of variable, $\alpha = \tan(\beta/2)$, applied to each trigonometric function can transform these equations into a fully algebraic set whose solution is smooth w.r.t. the coefficients which, in turn, are smooth functions of the l_i.
From (Souères *et al.*, 1994) we know that when $l_i = 0$ (a point-wise robot) the optimal paths must satisfy the condition $\psi_3(t_f) = 0$, i.e., the car must stop exactly on the line $\mathcal{D}_0$. A path in family (I) can meet this condition only if it is of type $C_a C_b C_b$ as depicted in fig. 5.
The continuity of the three parameters $(a,\, b,\, e)$

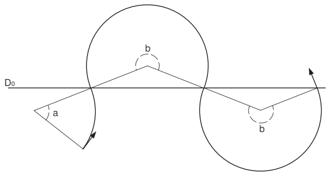


Fig. 5. A path $C_aC_bC_b$ which ends on the line $\mathcal{D}_0$

w.r.t. l_i implies that any path in familty (I) must converge continuously towards $C_aC_bC_b$ when $l_i \to 0$. In (Sussmann and Tang, 1991) the authors proved that a path $C_aC_bC_b$ is never optimal, i.e., it can be always replaced by a shorter path of another type. Thus, for continuity, we can conclude that family (I) is not optimal for solving problem (i), (ii) and (iii).

5. ISODISTANCE CURVES

Evaluating the distance (6) to the points in a region of the plane xy and by assigning the same grayscale level to those points which share the same distance, we can easily compute the isodistance curves. In order to obtain a wavefront effect we applied a modulus operation which reset the grayscale level when greater than a predetermined threshold.

In fig. 6 we report the case of a point-wise robot with $l_i = 0$; the discontinuous nature (Souères and Boissonnat, 1998) of the Dubins distance function clearly arise from the figure. Figure 7

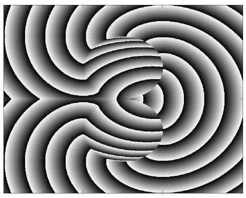
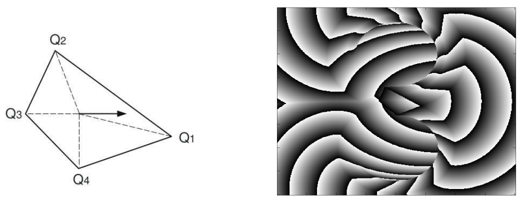


Fig. 6. Isodistance curves with $l = 0$

shows a robot of polygonal shape with the corresponding isodistance curves. The robot in fig.



(a) robot (b) isodistance curves

Fig. 7. Isodistance curves for a robot of polygonal shape.

7 is completely asymmetric and thus the relative curves are highly deformed. The coordinates of the vertices of robot are:

$$\begin{cases} Q_1 : l_{Q_1} = 0.4123 & \phi_{Q_1} = -0.2450 \\ Q_2 : l_{Q_2} = 0.3606 & \phi_{Q_2} = 2.1588 \\ Q_3 : l_{Q_3} = 0.3 & \phi_{Q_3} = \pi \\ Q_4 : l_{Q_4} = 0.3 & \phi_{Q_4} = 3\pi/2 \end{cases}$$

6. CONCLUSION

We have presented an analytical method to compute a distance to obstacles for a Dubins' car which takes into account the nonholonomic constraints and the non-symmetry of the system. By extending the Dubins' work, we computed the shortest path to a manifold (the C-obstacle) rather than to a point. In particular we reduce this problem to that of finding the solution of a set of algebraic equations by using optimal control arguments which, on the other hand, gave deeper understanding of the underlying structure of such shortest paths.

REFERENCES

Alterovitz, R., A. Lim, K. Goldberg, G. S. Chirikjian and A. M. Okamura (2005). Steering flexible needles under markov motion uncertainty. In: *Proc. IEEE Int. Conf. on Intelligent Robotics and Systems (IROS)*.

Boissonnat, J.D., A. Cerezo and J. Leblond (1992). Shortest paths of bounded curvature in the plane. In: *Proc. IEEE Int. Conf. on Robotics and Automation*. pp. 2315–2320.

Bui, X-N., P. Souères, J.-D. Boissonnat and J.-P. Laumond (1994). Shortest path synthesis for dubins nonholonomic robot. In: *Proc. IEEE Int. Conf. on Robotics and Automation*.

Dubins, L.E. (1957). On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangents. *American Journal of Mathematics* **79**, pp. 497–516.

Moutarlier, P., B. Mirtich and J. Canny (1996). Shortest paths for a car-like robot to manifolds in configuration space. *Int. Journal of Robotics Research* **15**(1), 36–60.

Pontryagin, L.S., V.G. Boltyansky, R.V. Gamkrelidze and E.F. Mischenko (1962). *The Mathematical Theory of Optimal Processes*. Wiley, New York.

Souères, P. and J.-D. Boissonnat (1998). Optimal trajectories for nonholonomic mobile robots. In: *Robot Motion Planning and Control* (Jean-Paul Laumond, Ed.). pp. 93–169. Springer.

Souères, P., J-Y. Fourquet and J-P. Laumond (1994). Set of reachable positions for a car. *IEEE Trans. Automat. Contr.* **39**, 1626–1630.

Sussmann, H. J. and W. Tang (1991). Shortest paths for the reeds-sheep car: a worked out example of the use of geometric techniques in nonlinear optimal control. Report SYCON-91-10. Rutgers University.

Vendittelli, M., J.P. Laumond and C. Nissoux (1999). Obstacle distance for car-like robots. *IEEE Trans. on Robotics and Automation* **15**(4), 678–691.