Control Systems

Introduction

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course main topics

- analysis (time and frequency domain)
- general feedback control system
- controller design in the frequency domain (loop shaping)
- performance and limitations of a control system
- analysis and design using root locus
- state space design
- stability theory

goal

fundamental objective of the control systems course

- analysis
- control

of dynamical systems

dynamical system:
a system whose state evolves over time

e.g. for mechanical engineering
dynamics: study of forces/torques that produce motion

motion:
time evolution of the quantities which characterize a system

dynamics

- we need to describe how a quantity varies in time
- how do we represent such a variation?

variation in time of the quantity x

derivative
(continuous time C.T.)

$$\frac{dx(t)}{dt} = \dot{x}(t) = \dot{x}$$

$$t \in \mathbf{R}$$

difference
(discrete time D.T.)

$$x(t+1) - x(t)$$

$$t \in \mathbf{Z}$$

dynamics

examples of known relationships including time derivatives:

- mass-spring-damper system

  acceleration (a) - velocity (v) - position (p) - force (f)

$$m\,a(t) + b\,v(t) + k\,p(t) = f(t)$$

- capacitor: voltage (v) - current (i)

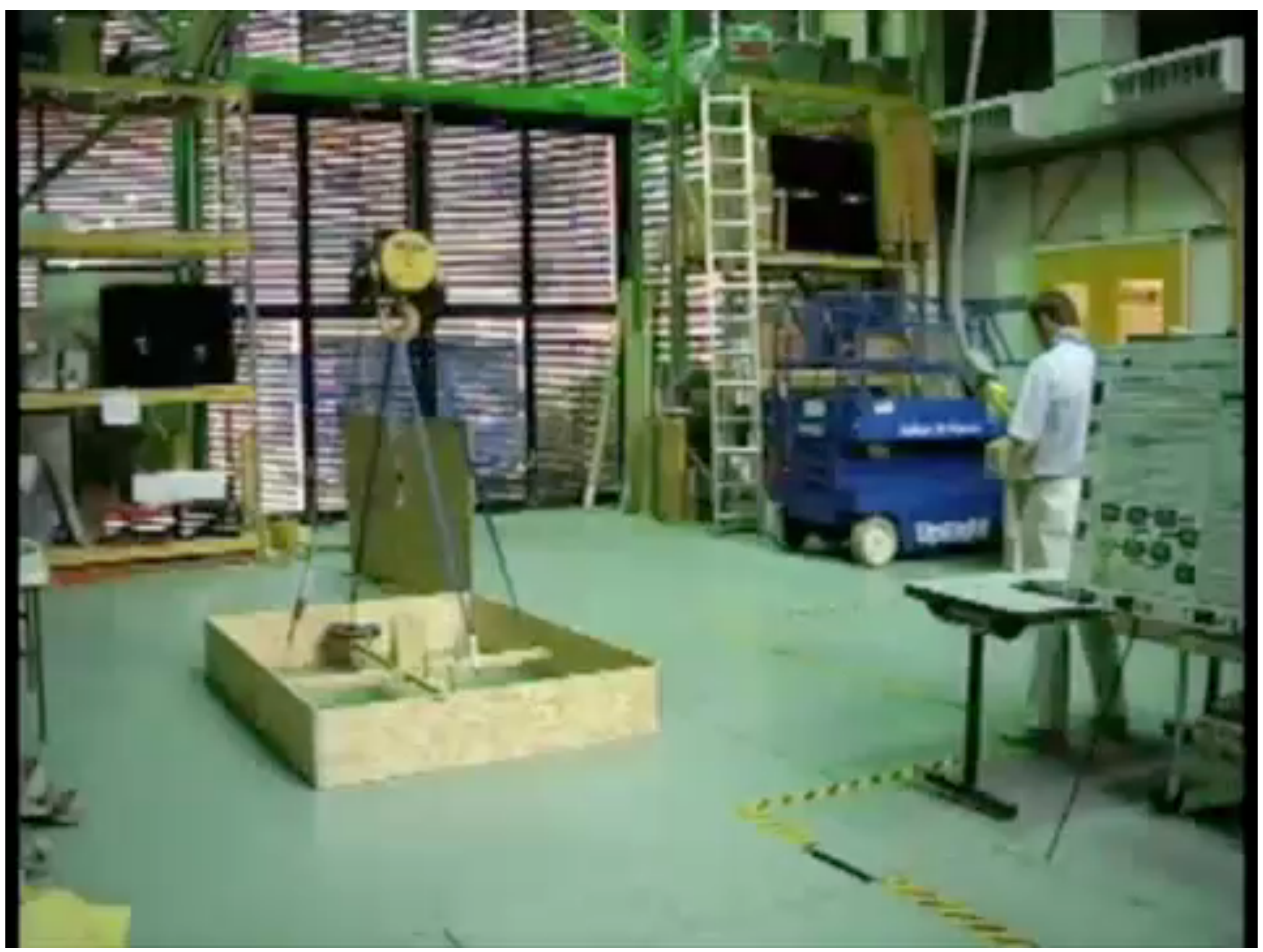
$$\frac{d\,v(t)}{dt} = \frac{1}{C} i(t)$$

- inductor: current (i) - voltage (v)

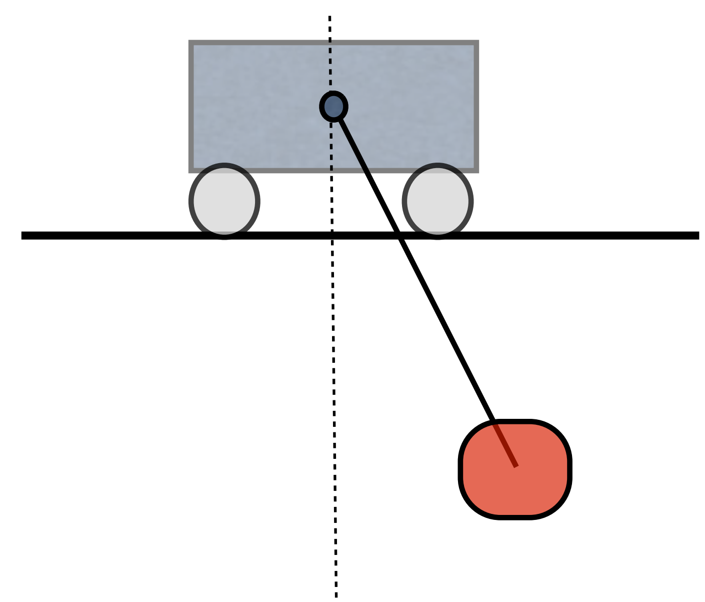
$$\frac{d\,i(t)}{dt} = \frac{1}{L} v(t)$$

importance of dynamics

- description of the motion (eg. satellite trajectory)
- simulation models (system behavior wrt to inputs)



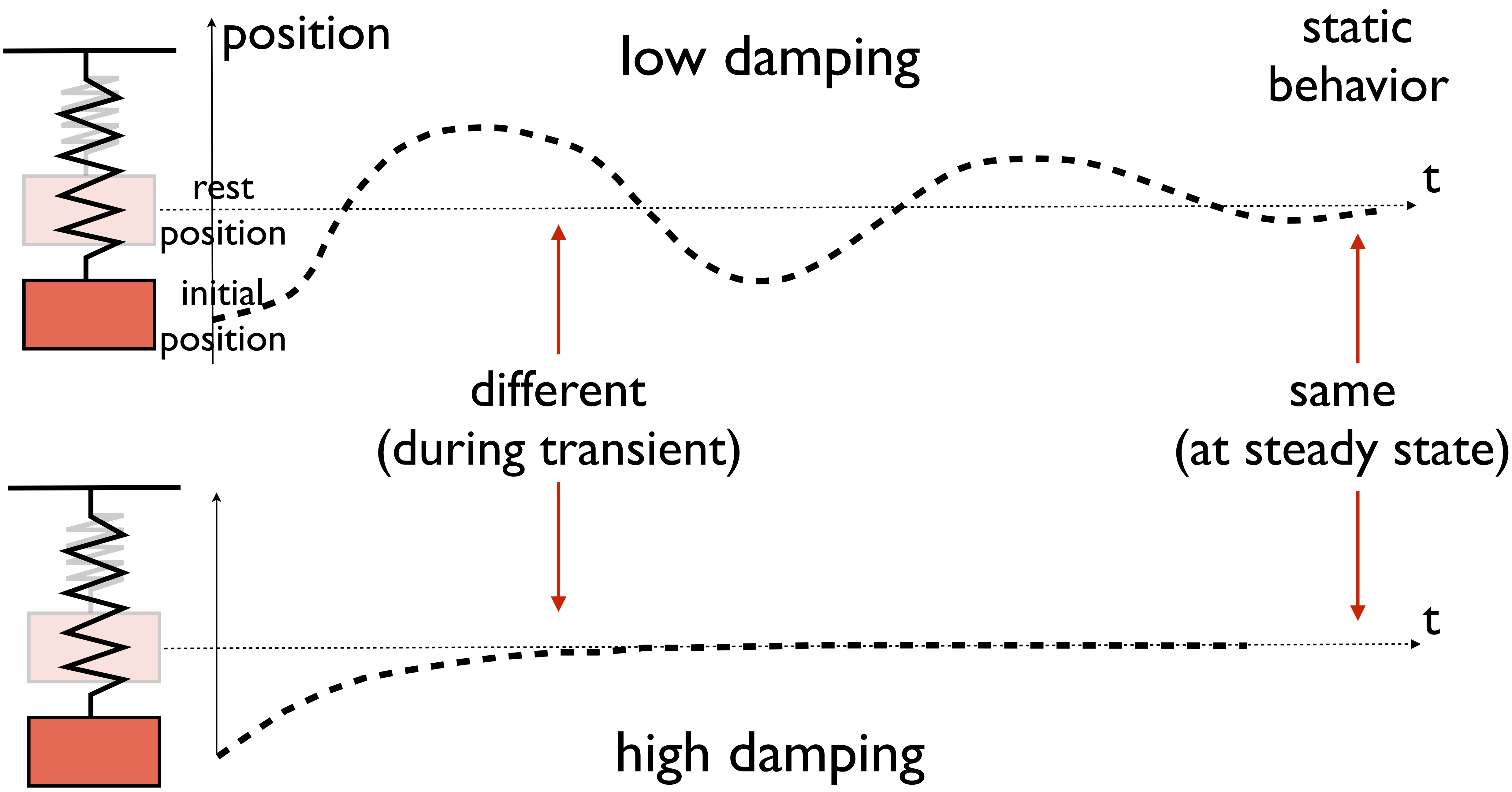
simplified model



crane control:
input shaping technique
(Georgia Tech)

importance of dynamics

- same static behavior, different dynamic one
  (two similar systems but with different parameters, starting from same initial position)



analysis of dynamic properties

- infer important properties from few basic quantities
  - e.g. stability from dynamic matrix eigenvalues
  - or possibility to influence the dynamics through the input from controllability analysis
  - or understand the internal dynamics from the observability analysis
- these will allow a clear formulation of specifications for the control system design
- other uses: forecast, prediction

qualitative analysis of systems of differential equations

analysis

- model based

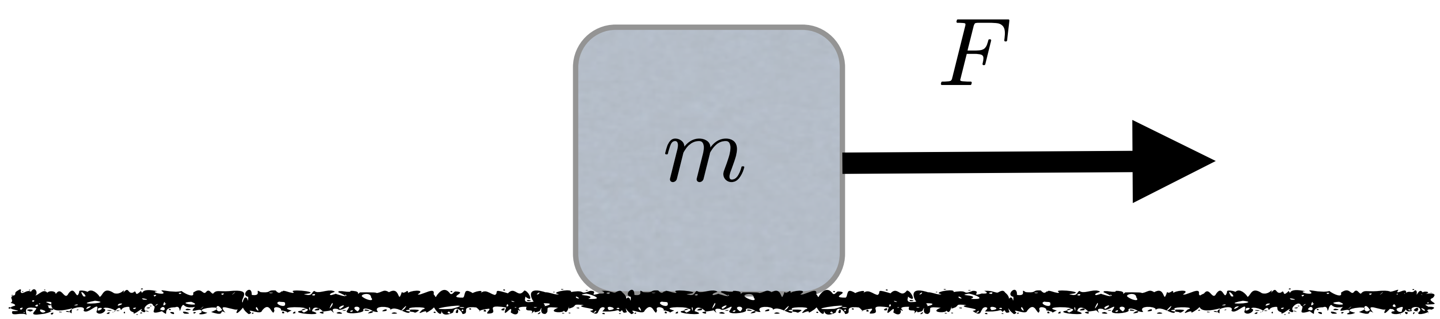
  representation of the real system through a **model**, usually with approximations

- in particular **mathematical model**

  we will consider systems of differential equations

- study of the system mathematical description looking for quantities that characterize the system motion

example



- mass m moving on a line (one-dimensional motion) under the action of a force F
- hyp: no friction

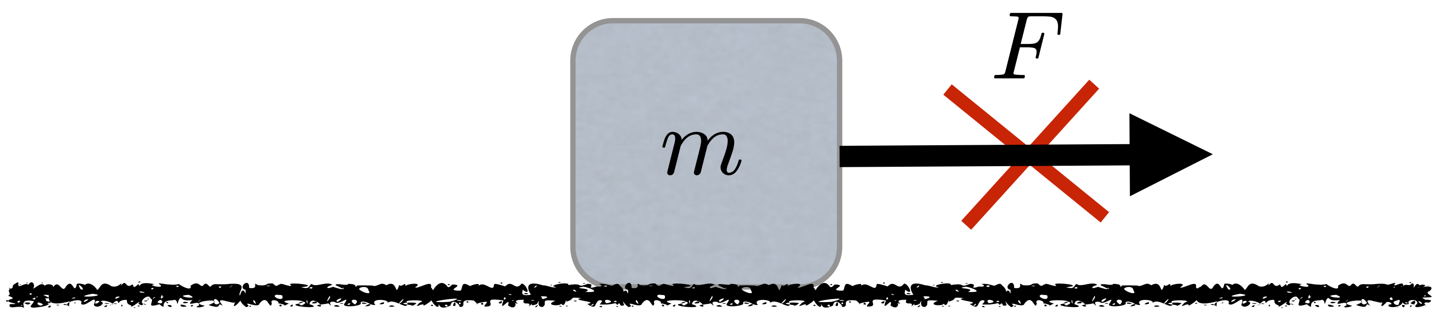
$$m\,\dot{v} = F$$

mathematical model

this mathematical relationship tells us how the variation of the mass velocity is related to the applied force under the assumed hypothesis: it is our **model**

+ other tacit hypothesis
(ex. m constant otherwise linear momentum)

example (cont.)



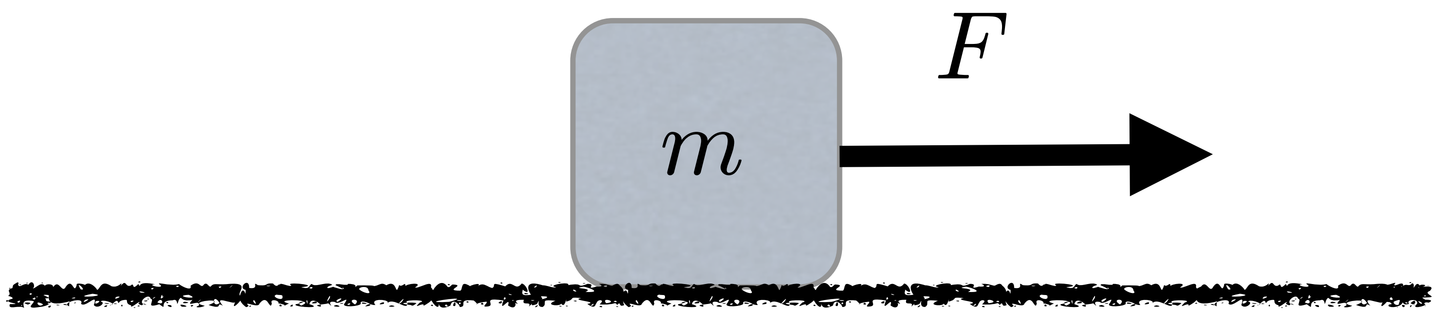
- if $F = 0$ do we still have motion?

model becomes $\dot{v} = 0$ solution is $v(t) = v(0)$

- if we have a non-zero initial velocity, the mass moves (at constant speed)

- we need to learn how to read the information hidden in the mathematical model

example (cont.)



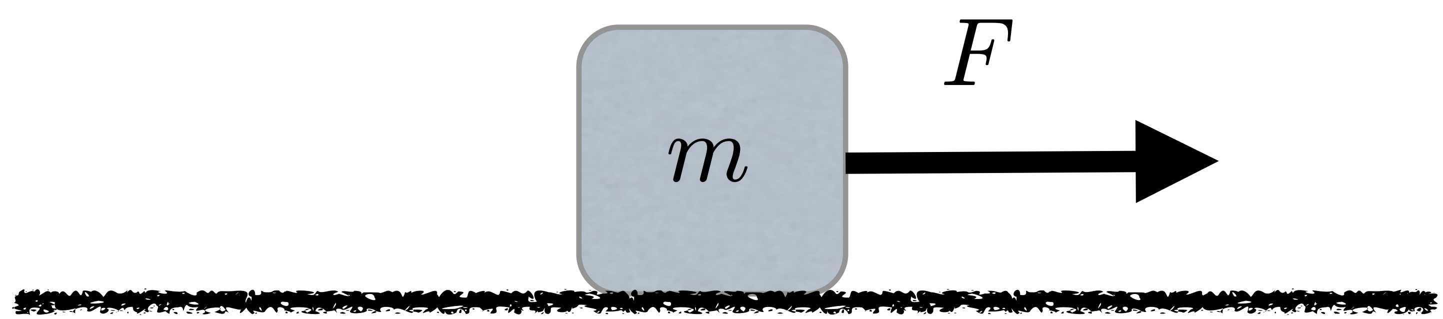
model

$$\begin{aligned} \dot{v} &= \frac{F}{m} \\ v(0) &= v_0 \end{aligned}$$

we have noticed that the motion is generated by two causes

- forcing term $F(t)$ (will be called input to the system)
- initial condition v_0

example (cont.)



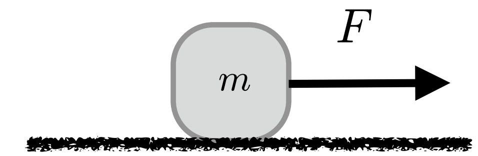
solution of the differential equation (model)

$$v(t) = v_0 + \frac{1}{m}\int_0^t F(\tau)d\tau$$

tells us how the velocity depends upon the initial condition **and** the applied force. Knowing the applied force and the initial velocity we know how the velocity of the point mass behaves in the future

new capacity: **analysis** & **prediction**

example (cont.) - linearity



- with initial condition $v_0 = 0$ and $F \neq 0$ we have velocity v

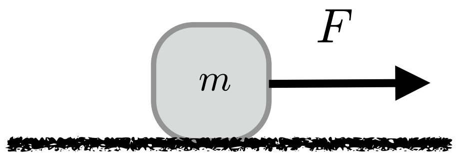
  if we apply $2F$ instead of F what happens to velocity?

$$\tilde{v}(t) = \cancel{v_0} + \frac{1}{m}\int_0^t 2F(\tau)d\tau$$

  the velocity will also double to $2\ v$

linear behavior wrt to F

example (cont.) - linearity



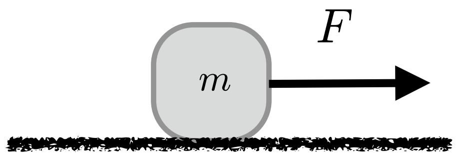
- if we apply no force $F = 0$ and start with non-zero $v_0 \neq 0$, the velocity will be $v = v_0$

  clearly, if the initial velocity changes to $3v_0$ the velocity will also triple

$$\tilde{v}(t) = 3v_0 + \cancel{\frac{1}{m}\int_0^t F(\tau)d\tau}$$

linear behavior wrt to the initial condition v_0

example (cont.) - linearity



- Att. $F \neq 0$ and $v_0 \neq 0$ simultaneously

  if $F \longrightarrow 2F$

  and $v_0 \longrightarrow 3v_0$

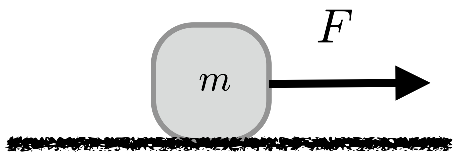
  what happens to velocity?

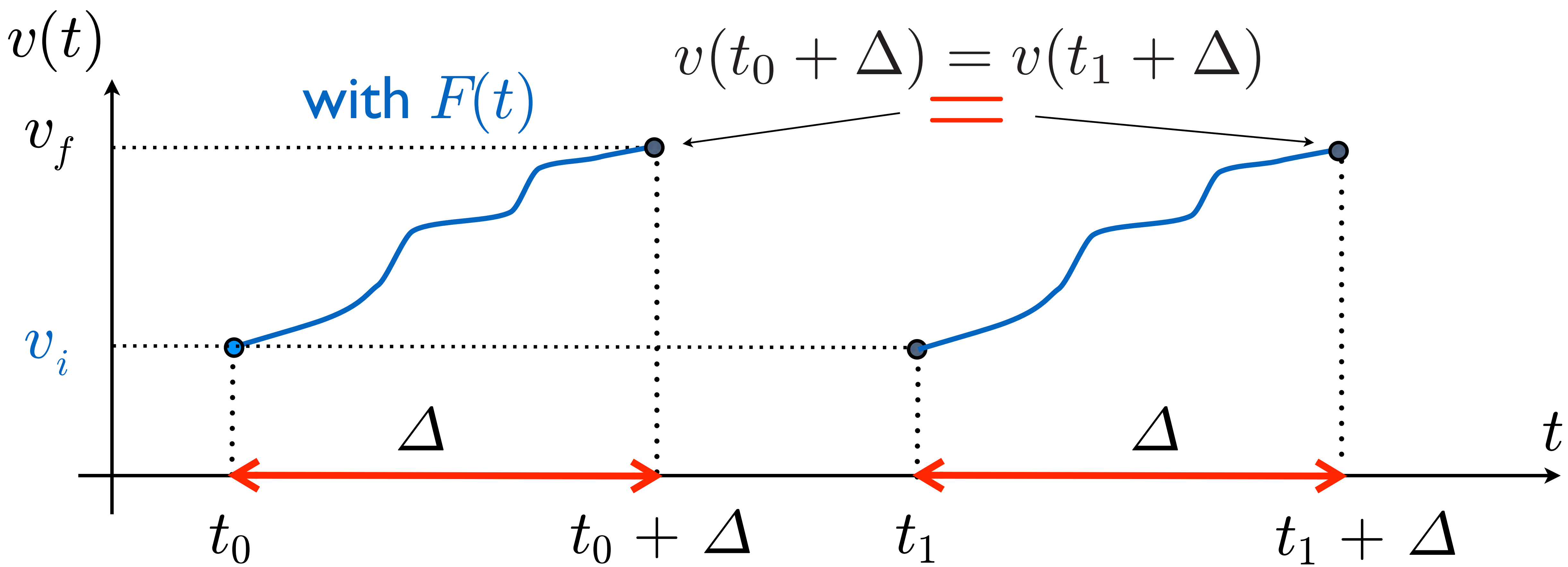
linear behavior wrt the motion causes

cause $= (v_0, F)$

this linearity comes from the **differential equation** being **linear**

example (cont.) - time invariance





same initial condition v_i and same input (force) $F(t)$ after same time interval Δ leads to the same state

- state evolution does not depend on the initial time t_0 but only on the elapsed time Δ
- this time invariance comes from the **differential equation** having **constant coefficients**

general mathematical model

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$
$$x(0) = x_0$$

Linear Time Invariant (LTI) dynamical system (Continuous Time)

$x(t)$ **state** $x \in \mathbf{R}^n$

$u(t)$ **input** $u \in \mathbf{R}^m$ multi input (we consider $m = 1$, single input)

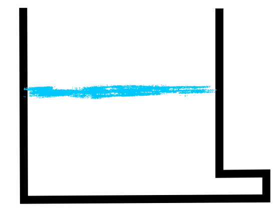
$y(t)$ **output** $y \in \mathbf{R}^p$ multi output (we consider $p = 1$, single output)

SISO (single input/single output) linear time-invariant system

control example: water level in a tank

unknown output flow
(disturbance)

input flow
(control input)



real system

water level in the tank
(output & controlled variable)

problem: we want to maintain the water level at a desired height regardless of the unknown output flow and any other disturbance

understand how to choose the (control) input in order to guarantee a **desired behavior** of the output

disturbance $d(t)$

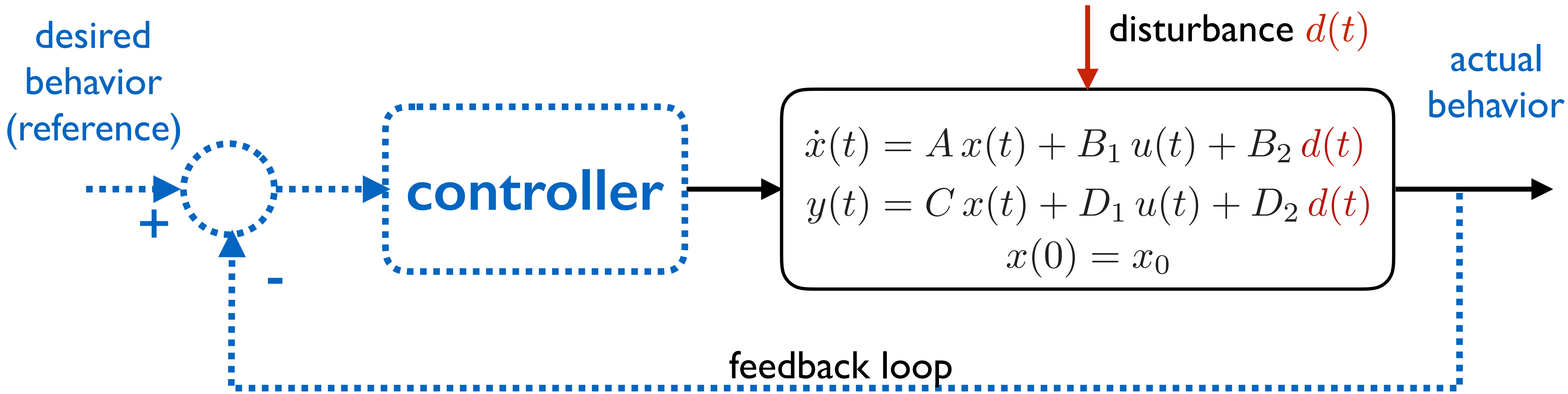
input $u(t)$
(control input)

$$\dot{x}(t) = A\,x(t) + B_1\,u(t) + B_2\,d(t)$$
$$y(t) = C\,x(t) + D_1\,u(t) + D_2\,d(t)$$
$$x(0) = x_0$$

output $y(t)$
(controlled variable)

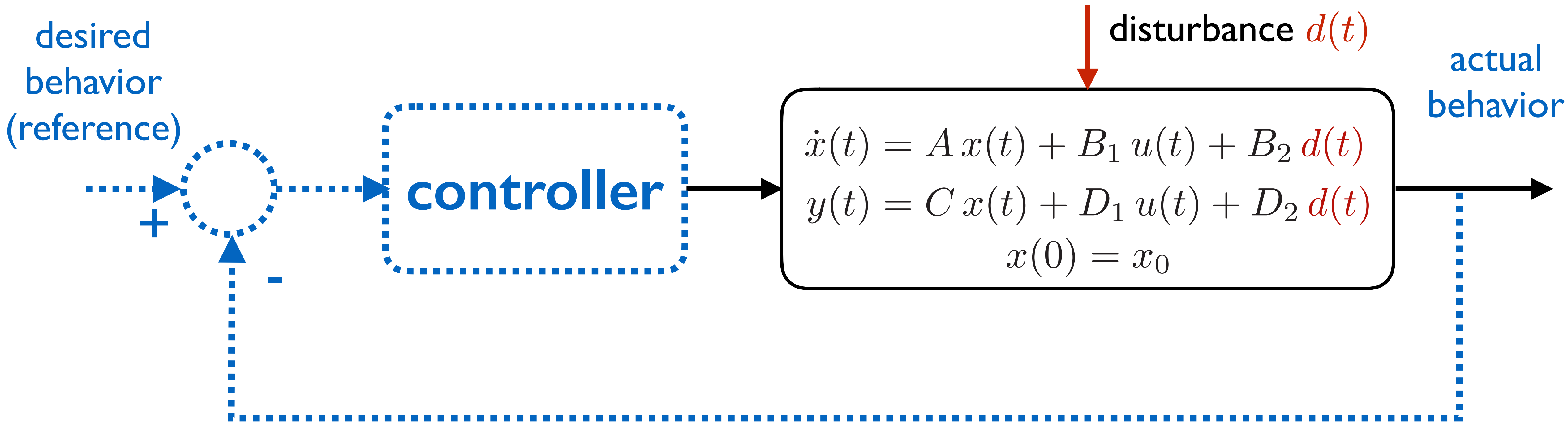
model

control example: water level in a tank



- schematic diagram of an **automatic control system** based on **feedback**
- the design of such a control system requires the determination (design) of the **controller**
- need a systematic procedure in order to design the controller
- design (and controller) will be based on the **plant model**

control example: water level in a tank



control scheme is **implemented** on the real system