Self assessment - 00A

November 7, 2016

1 Exercise

Given the matrices

$$A_1 = \begin{pmatrix} 1 & 1 & 1 \\ -2 & -1 & -1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

1. Find the nullspace of A_1 and A_2.
2. Prove that vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ generate the same subspace than $\mathbf{w}_3$ and $\mathbf{w}_4$ with

$$\mathbf{w}_1 = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \quad \mathbf{w}_2 = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{w}_3 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{w}_4 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

3. Prove that both $(\mathbf{w}_1, \mathbf{w}_2)$ and $(\mathbf{w}_3, \mathbf{w}_4)$ generate the nullspace of A_2.

2 Exercise

Given the matrices

$$A_1 = \begin{pmatrix} 3 & 1 & 1 \\ -3 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

1. Find the eigenvalues of A_1 and their geometric multiplicities.
2. Find the eigenvalues of A_2 and their geometric multiplicities.

3 Exercise

Consider the following plant with $\alpha \in \mathbb{R}$ a real parameter.

$$\begin{aligned} \dot{x}_1 &= x_1 + x_3 + u \\ \dot{x}_2 &= u \\ \dot{x}_3 &= -2x_3 \\ y &= \alpha x_1 + x_2 + x_3 \end{aligned}$$

1. Find (A, B, C, D) of the state space representation.

2. Compute the eigenvalues of A and their corresponding eigenvectors. What are the natural modes of the system?
3. Which initial conditions guarantee that the state ZIR will converge to zero asymptotically?
4. Which initial conditions guarantee that the state ZIR will not diverge?
5. Can we avoid, with a proper choice of the output through α, the divergence of the output ZIR for every initial condition?
6. Can we avoid divergence of the impulse response with a proper choice of α?

4 Exercise

Consider the horizontal motion of a point mass under the action of a force $f(t)$ and a friction force proportional, with coefficient $\mu > 0$, to the mass velocity. The following questions need to be solved symbolically, without assigning particular numeric values for the system parameters m and μ.

1. Find the state space representation by considering that we are also interested in the mass position.
2. If possible, find the change of coordinates (similarity transformation) that will diagonalize the dynamic matrix.
3. Write the matrix exponential in the original state (position displacement and velocity).
4. Assuming the mass is pushed from its rest position with a unit impulse force $f(t) = \delta(t)$, where will the mass stop?
5. Find explicitly the position $p(t)$ and velocity $\dot{p}(t)$ time evolution when no input is applied but the system starts from a generic initial condition $(p_0, \dot{p}_0)$, in other words find the state Zero Input Response (ZIR). How is the found ZIR related to the natural modes of the system?
6. For the state ZIR, find the relationship between $\dot{p}(t)$ and $p(t)$, i.e. write the solution $\dot{p}(t)$ in terms of the solution $p(t)$ so that we can plot the ZIR in the $(p, \dot{p})$ phase plane. The obtained relationship will also depend upon the initial condition $(p(0), \dot{p}(0)) = (p_0, \dot{p}_0)$. Comment the typical system trajectories in the phase plane.
7. Find the set of initial conditions $(p_0, \dot{p}_0)$ such that the ZIR tends asymptotically to the origin $(0, 0)$. Plot this set in the phase plane $(p, \dot{p})$.
8. Find explicitly the position and velocity time evolution when the system starts from the rest configuration $(p_0, \dot{p}_0) = (0, 0)$ and a unit constant force $f(t) = 1$ is applied from $t = 0$.
9. Assume that the constant unit force is applied only for a finite time interval of length T, i.e. $f(t) = 1$ for $t \in [0, T]$ and $f(t) = 0$ for $t > T$. Write the state forced response.
10. Write the state evolution when the constant applied force during the interval of duration T has amplitude α, i.e. $f(t) = \alpha$ for $t \in [0, T]$?
11. Assume we start for the initial condition $(p_0, 0)$, we want to find α (if it exists) such that the input $f(t) = \alpha$ for $t \in [0, T]$ will lead to a state evolution that will asymptotically tend to the origin. To do so, note that the given input will transfer the state from its original value to a new value reached at time $t = T$. From that state the system evolves with no input applied. Use the previous results in order to solve the problem.