

Esercitazione n° 4 per il corso di Ricerca Operativa

Modello di miscelazione

Sunco Blending Problem: Problem Statement and Summary Source of the problem: page 86, W. L. Winston, "Operations Research, Application and Algorithms", 4th Edition, Thomson Learning, 2004, ISBN: 0-534-38058-1

Sunco Oil manufactures three types of gasoline (gas 1, gas 2, gas 3). Each type is produced by blending three types of crude oil (crude 1, crude 2, crude 3). The sales prices per barrel of gasoline are as follows:

	Sales price per Barrel		Purchase Price per Barrel
Gas 1	\$70	Crude 1	\$45
Gas 2	\$60	Crude 2	\$35
Gas 3	\$50	Crude 3	\$25

Sunco can purchase up to 5000 barrels of each type of crude oil.

The three types of gasoline differ in their octane rating and sulfur content. The crude oil blends to form gas 1 must have an average octane rating of at least 10 and contain at most 1

	Octane Rating	Sulfur Content
Crude 1	12	0.5%
Crude 2	6	2.0%
Crude 3	8	3.0%

It costs \$4 to transform one barrel of gasoline, and Sunco's refinery can produce up to 14,000 barrels of gasoline daily. Sunco's customers require the following amounts of each gasoline: gas 1- 3000 barrels/day, gas 2- 2000 barrels/day, and gas 3- 1000 barrels/day. The company considers it an obligation to meet these demands. Sunco also has the option of advertising to stimulate demand for its products. Each dollar spent daily in advertising a particular type of gas increases the daily demand for that type of gas by 10 barrels. For example, if Sunco decides to spend \$20 daily in advertising gas 2, the daily demand for gas 2 will increase by $20(10) = 200$ barrels. Formulate an LP that will enable Sunco to maximize daily profits (profits= revenue - costs).

Summary of problem (an efficient tool to assist formulation):

Sulfur	Octane	Sales price \$/barrel	Meet demand Barrel/day	GAS	Decision variables			
At most 1%	At least 10	\$70	3000	1	X11	X21	X31	A_1
At most 2%	At least 8	60	2000	2	X12	X22	X32	A_2
At most 1%	At least 6	50	1000	3	X13	X23	X33	A_3
OIL					1	2	3	
Purchase price \$/barrel					\$45	35	25	
Available oil, max barrel/day					5000	5000	5000	
Octane rating					12	6	8	
Sulfur content					0.5%	2.0%	3.0%	

* for each type of gas: 1 \$ ad/day will increase demand by 10 barrel/day

* \$4 to transform 1 barrel oil into one barrel gas

* Max gas to produce 14000 barrel /day

Decision variables: A_i = dollars spent daily on advertising gas i ($i = 1, 2, 3$)

X_{ij} = barrel of crude oil i used daily to produce gas j ($i = 1, 2, 3; j = 1, 2, 3$)

OF: max profit

Z = profit = (revenue - purchase cost- production cost) * (amount of each oil used for each gas) - advertising cost For example: profit from Oil 1 used for Gas 1

$$z = (70 - 45 - 4) * X_{11} = 21 * X_{11}$$

$$\max Z = 21X_{11} + 11X_{12} + 1X_{13} + 31X_{21} + 21X_{22} + 11X_{23} + 41X_{31} + 31X_{32} + 21X_{33} - 1A_1 - 1A_2 - 1A_3$$

Constraints for Gas (1, 2, and 3) demand:

$$C1 : 1X_{11} + 1X_{21} + 1X_{31} - 10A_1 = 3000$$

$$C2 : 1X_{12} + 1X_{22} + 1X_{32} - 10A_2 = 2000$$

$$C3 : 1X_{13} + 1X_{23} + 1X_{33} - 10A_3 = 1000$$

Constraints for Oil (1, 2, and 3) availability to be purchased:

$$C4 : 1X_{11} + 1X_{12} + 1X_{13} \leq 5000$$

$$C5 : 1X_{21} + 1X_{22} + 1X_{23} \leq 5000$$

$$C6 : 1X_{31} + 1X_{32} + 1X_{33} \leq 5000$$

Constraint for refinery capacity limit:

$$C7 : 1X_{11} + 1X_{12} + 1X_{13} + 1X_{21} + 1X_{22} + 1X_{23} + 1X_{31} + 1X_{32} + 1X_{33} \leq 14000$$

Constraints for minimum average required Octane level in Gas (1, 2, and 3):

$$(12X_{11} + 6X_{21} + 8X_{31}) / (X_{11} + X_{21} + X_{31}) \geq 10$$

will result in C8 constraint

$$C8 : 2X_{11} - 4X_{21} - 2X_{31} \geq 0$$

$$C9 : 4X_{12} - 2X_{22} \geq 0 \quad C10 : 6X_{13} + 2X_{33} \geq 0$$

Constraints for maximum sulfur% content in Gas (1, 2, and 3):

$$(0.005X_{11} + 0.02X_{21} + 0.03X_{31}) / (X_{11} + X_{21} + X_{31}) \leq 0.01$$

will result in C11 constraint

$$C11 : -0.005X_{11} + 0.01X_{21} + 0.02X_{31} \leq 0$$

$$C12 : -0.015X_{12} + 0.01X_{32} \leq 0$$

$$C13 : -0.005X_{13} + 0.01X_{23} + 0.02X_{33} \leq 0$$