OPERATIONS RESEARCH

EXAM February 9, 2017

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Report your answers to the exercises in the appropriate boxes below.

You must include the sheets with your calculus and explanations that will be used to verify the correct procedure.

Calculation errors that lead to a wrong answer may not be taken into account.

Please note that $\boxed{\textbf{YES}}$ $\boxed{\textbf{NO}}$ answers are not valid if you do not explain them.

Exercise 1. (Score 7) Consider the following nonlinear programming problem

$$\begin{array}{ll} \min\limits_{x\in I\!R^4} & x_1^2 - 2x_2^2 + x_3^2 + x_4^2 + x_1x_4 + 2x_1 - 4x_3 \\ & x_1 - 3x_3 + 2x_4 \geq 5 \\ & 2x_1 + x_2 - x_3 \leq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

(i) (score 1) Consider the unconstrained problem (remove the constraints). Is there any unconstrained stationary point ? Is this a global minimizer, local minimizer, saddle point or maximizer ?

(ii) (score 1) Are the KKT conditions necessary and sufficient for optimality ?

(iii) (score 2) Consider the point $\widehat{x} = (2,\ 0,\ 1,\ 3)^T$ and write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

(iv) (score 1) Write the KKT conditions in $\widehat{x}$. Are the KKT conditions satisfied ? Evaluate the multipliers

(v) (score 2) Find a a feasible direction along which it is possible to move from $\tilde{x} = (2,\ 0,\ 2,\ \frac{9}{2})^T$ finding an additional active constraint. Find the stepsize $t^{\max}$ and the corresponding new point y. Is the direction also a descent one?

Exercise 2. (Score 3) Consider the following Linear programming problem

$$\begin{array}{ll} \min & 3x_1 - 2x_2 + x_3 + 5x_4 \\ & x_1 - 3x_3 + 2x_4 \geq 5 \\ & 2x_1 + x_2 - x_3 \leq 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

(i) (score 1) Is the point $(2,\ 0,\ 1,\ 3)^T$ a vertex of the feasible polyedron ?

(ii) (score 1) Write the problem in the standard form for the simplex method. ($Ax = b,\ x \geq 0$ with $b \geq 0$).

(iii) (score 1) Write a BFS for the problems at point (ii) and specify the corresponding matrices B and N.

Exercise 3. (Score 7) Consider the following Linear programming problem

$$\begin{array}{ll} \min & 3x_1 - 2x_2 + x_3 + 5x_4 \\ & x_1 - 3x_3 + 2x_4 \geq 5 \\ & 2x_1 + x_2 - x_3 \leq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

(i) (score 2) Write the dual problem

(ii) (score 2) Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) (score 2) Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.

(iv) (score 1) State how the value of the optimal solution change if the rhs of the first constraint change from 5 to $5 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

Exercise 4. (score 5) Consider the following Multiobjective Linear Programming problem

$$\begin{array}{ll} \max & \{5x_1 - 3x_2, -x_1 - x_2\} \\ & x_1 - 2x_2 \leq 3 \\ & 2x_1 \leq 5 \\ & -3x_1 + x_2 \leq 1 \\ & x_1 \geq 0 \end{array}$$

(i) (score 1) Find the two optimal point (x^{*1}, x^{*2}) of each of the two objectives and the corresponding ideal vector (z^{*1}, z^{*2})

(ii) (score 2) Plot the imagine of the feasible region into the Objective function space. Find the Pareto front and plot them in the graph.

(iii) (score 2) Write the KKT conditions. Are the KKT satisfied in the point $\hat{x} = (2,\ 0)^T$? Justify your answer by evaluating the multipliers.

Exercise 5. (Score 4) Consider the following problem

$$\begin{array}{ll} \max & 5x_1 - 3x_2 \\ & x_1 - 2x_2 \leq 3 \\ & 2x_1 \leq 5 \\ & -3x_1 + x_2 \leq 1 \\ & x_1 \geq 0 \\ & x \text{integer} \end{array}$$

Let $(1,\ 0)^T$ be a feasible point.

(i) (score 1) Solve the linear relaxation and find the upper bound to the optimal integer solution.

(ii) (score 1) Write the two subproblem $\mathcal{P}_1, \mathcal{P}_2$ obtained by branching with respect to fractional variable

(iii) (score 1) Solve problem $\mathcal{P}_1$. Explain if it is possible to close it.

(iv) (score 1) Solve problem $\mathcal{P}_2$. Explain if it is possible to close it.

Exercise 6. (Score 5)

The Pigskin Company produces footballs. Pigskin must decide how many footballs to produce each month. The company has decided to use a six-month planning horizon. The forecasted monthly demands for the next six months are 10 000, 15 000, 30 000, 35 000, 25 000, and 10 000. Pigskin wants to meet these demands on time, knowing that it currently has 5000 footballs in inventory and that it can use a given months production to help meet the demand for that month. (For simplicity, we assume that production occurs during the month, and demand occurs at the end of the month.) During each month there is enough production capacity to produce up to 30 000 footballs, and there is enough storage capacity to store up to 10,000 footballs at the end of the month, after demand has occurred. The forecasted production costs per football for the next six months are 12.50, 12.55, 12.70, 12.80, 12.85, and 12.95, respectively. The holding cost per football held in inventory at the end of any month is figured at 5% of the production cost for that month. (This cost includes the cost of storage and also the cost of money tied up in inventory.) The selling price for footballs is not considered relevant to the production decision because Pigskin will satisfy all customer demand exactly when it occursat whatever the selling price is. Therefore, Pigskin wants to determine the production schedule that minimizes the total production and holding costs.

Write an LP problem to find the cheapest way for planning Pigskin's production.