OPERATIONS RESEARCH - TEXT A

EXAM February 17, 2018

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Please note that YES NO answers are NOT valid if you do not give an explanation.

SURNAME **NAME**

GRADE

I **authorize** the publication (paper and electronic) of the results obtained in the examination in accordance with the Italian Law 675/96 and subsequent amendments

SIGNATURE

I solved the following exercise

1. **(Score 6)**
2. **(Score 5)**
3. **(Score 8)**
4. **(Score 6)**
5. **(Score 6)**

ORAL PART: questions

GRADE

Exercise 1. **(Score 6)** Consider the following nonlinear programming problem

$$\begin{array}{ll} \min_{x\in I\!R^2} & (-x_1^2+1)^2+(x_2-6)^2 \\ & -2x_1-3x_2\leq 6 \\ & 2x_1-x_2\leq 2 \\ & -x_1+2x_2\leq 9 \\ & x_1+x_2\leq 7 \\ & x_2\geq 0 \end{array}$$

Unconstrained problem (score 2)

(i) **(score 1)** Consider the unconstrained problem (remove the constraints). Find the unconstrained stationary point and determine the nature of the points.

(ii) **(score 1)** Consider the point $x^0=(1,\ 1)^T$ and write the first iteration of the gradient method with exact line search to obtain the new point x^1

Constrained problem (score 4)

(iii) **(score 1)** State if the problems is convex or strictly convex or none of the two.

(iv) **(score 2)** Consider the point $\widehat{x}=(3,\ 4)^T$ and write the KKT conditions in $\widehat{x}$. Are the KKT conditions satisfied ? Evaluate the multipliers

(v) **(score 1)** Write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

Exercise 2. **(Score 5)** Consider the following Linear programming problem

$$\begin{array}{ll} \min & 6x_1+2x_2+9x_3+7x_4 \\ & -2x_1+2x_2-x_3+x_4=1 \\ & 3x_1+x_2-2x_3-x_4\leq -4 \\ & x_1,x_2,x_3,x_4\geq 0 \end{array}$$

(i) **(score 2)** Find a a feasible direction along which it is possible to move from $\widehat{x}=(0,\ \frac{1}{2},\ 2,\ 2)^T$ finding an additional active constraint. Find the stepsize $t^{\max}$ and the corresponding new point y. Is the direction also a descent one?

(ii) **(score 0,5)** Write the problem in the standard form for the simplex method.

(iii) **(score 0,5)** Write the auxiliary problem to be solved in Phase I of the simplex method.

(iv) **(score 1)** Check if there exists a vertex corresponding to the active constraints

a) $I=\{1,2,3,4\}$
b) $I=\{2,3,4,5\}$
c) $I=\{1,2,3,6\}$

(v) **(score 1)** Write a BFS for the problems at point (ii) and specify the corresponding matrices B and N.

Exercise 3. **(Score 8)** Consider the following Linear programming problem

$$\begin{array}{ll} \min & 6x_1+2x_2+9x_3+7x_4 \\ & -2x_1+2x_2-x_3+x_4=1 \\ & 3x_1+x_2-2x_3-x_4\leq -4 \\ & x_1,x_2,x_3,x_4\geq 0 \end{array}$$

(i) **(score 1.5)** Write the dual problem

(ii) **(score 2)** Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) **(score 2)** Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.

(iv) **(score 1)** State how the value of the optimal solution change if the rhs of the first constraint change from 1 to $1+\varepsilon$ with $\varepsilon > 0$ and sufficiently small.

(v) **(score 1.5)** State which is the maximum value of $\varepsilon > 0$ for which the analysis in point (iv) holds.

Exercise 4. (Score 6) Consider the following integer linear programming problem

$$\begin{array}{ll} \max & x_1 + 4x_2 \\ & -2x_1 - 3x_2 \leq 6 \\ & 2x_1 - x_2 \leq 2 \\ & -x_1 + 2x_2 \leq 9 \\ & x_1 + x_2 \leq 7 \\ & x_2 \geq 0 \\ & x \text{ integer} \end{array}$$

Let $(1,\ 0)^T$ be a feasible point.

(i) **(score 1)** Report a lower and an upper bound to the optimal value of the objective function.

(ii) **(score 1)** Write the two subproblem $\mathcal{P}_1, \mathcal{P}_2$ obtained by branching with respect to the fractional variable x_1

(iii) **(score 2)** Solve problem $\mathcal{P}_1$. Explain if it is possible to update the incumbent and/or to close it (you can use the graphical solution).

(iv) **(score 2)** Solve problem $\mathcal{P}_2$. Explain if it is possible to update the incumbent and/or to close it (you can use the graphical solution).

Exercise 5. (Score 6) Europa Auto Company is an automaker with six manufacturing plants P_i, $i = 1, \ldots, 6$ and six vehicles V_j $j = 1, \ldots, 6$ to produce this year. The firm has learned that it makes sense to produce each vehicle at a unique plant, even though some of the plants are older and less efficient than others. For each possible assignment of a vehicle V_j to a plant P_i, the firm has estimated the annual cost c_{ij} (in millions of dollars) of implementing the assignment. The cost data take the form shown in the following table.

	Compact	Coupe	Sedan	SUV	Truck	Van
Akron	80	56	43	62	46	58
Buffalo	94	50	88	64	63	52
Columbus	94	46	50	40	55	73
Detroit	98	79	71	65	91	59
Evansville	61	59	89	98	45	52
Flint	77	49	65	95	72	91

Write an LP problem to minimize the total cost of the assignment.