OPERATIONS RESEARCH

EXAM January 19, 2017

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Report your answers to the exercises in the appropriate boxes below.

You must include the sheets with your calculus and explanations that will be used to verify the correct procedure.

Calculation errors that lead to a wrong answer may not be taken into account.

Please note that $\boxed{\text{YES}}$ $\boxed{\text{NO}}$ **answers are not valid if you do not explain them.**

Exercise 1. (Score 7) Consider the following nonlinear programming problem

$$\begin{aligned} \min_{x \in \mathbb{R}^3} \quad & x_1^2 + \frac{1}{2}x_2^2 + 2x_3^2 + x_1x_2 + x_2x_3 - 2x_1 - x_2 - 4x_3 \\ & x_1 + 2x_2 + 6x_3 \leq 4 \\ & -x_1 + 2x_3 = 1 \\ & x_1,\ x_2,\ x_3 \geq 0. \end{aligned}$$

(i) (score 1) Consider the unconstrained problem (remove the constraints). Is there any unconstrained stationary point ? Is this a global minimizer, local minimizer, saddle point or maximizer ?

(ii) (score 1) Are the KKT conditions necessary and sufficient for optimality ?

(iii) (score 2) Consider the point $\widehat{x} = (\frac{1}{4},\ 0,\ \frac{5}{8})^T$ and write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

(iv) (score 1) Write the KKT conditions in $\widehat{x}$. Are the KKT conditions satisfied ? Evaluate the multipliers

(v) (score 2) Find a a feasible direction along which it is possible to move from $\tilde{x} = (\frac{1}{5},\ \frac{1}{10},\ \frac{3}{5})^T$ finding an additional active constraint. Find the stepsize $t^{\max}$ and the corresponding new point y. Is the direction also a descent one?

Exercise 2. (Score 3) Consider the following Linear programming problem

$$\begin{aligned} \min \quad & -2x_1 + x_2 - 6x_3 \\ & x_1 + 2x_2 + 6x_3 \leq 4 \\ & -x_1 + 2x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(i) (score 1) Is the point $(0,\ 0,\ \frac{1}{2})^T$ a vertex of the feasible polyedron ?

(ii) **(score 1)** Write the problem in the standard form for the simplex method. ($Ax = b$, $x \geq 0$ with $b \geq 0$).

(iii) **(score 1)** Write a BFS for the problems at point (ii) and specify the corresponding matrices B and N.

Exercise 3. **(Score 7)** Consider the following Linear programming problem

$$\begin{array}{ll} \min & -2x_1 + x_2 - 6x_3 \\ & x_1 + 2x_2 + 6x_3 \leq 4 \\ & -x_1 + 2x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

(i) **(score 2)** Write the dual problem

(ii) **(score 2)** Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) **(score 2)** Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.

(iv) **(score 1)** State how the value of the optimal solution change if the rhs of the first constraint change from 4 to $4 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

Exercise 4. **(score 5)** Consider the following Multiobjective Linear Programming problem

$$\begin{array}{ll} \max & \{4x_1 + x_2, -x_1 - x_2\} \\ & x_1 - x_2 \geq 2 \\ & 2x_1 \leq 15 \\ & 6x_1 + 2x_2 \geq 6 \\ & x_1 \geq 0 \end{array}$$

(i) **(score 1)** Find the two optimal point (x^{*1}, x^{*2}) of each of the two objectives and the corresponding ideal vector (z^{*1}, z^{*2})

(ii) **(score 2)** Plot the imagine of the feasible region into the Objective function space. Find the Pareto front and plot them in the graph.

(iii) **(score 2)** Write the KKT conditions. Are the KKT satisfied in the point $\hat{x} = (3,\ 1)^T$? Justify your answer by evaluating the multipliers.

Exercise 5. **(Score 4)** Consider the following Knapsack problem

$$\begin{array}{ll} \max & 3x_1 + 2x_2 + 4x_3 + x_4 \\ & 3x_1 + x_2 + 5x_3 + 3x_4 \leq 7 \\ & x \in \{0, 1\}^4. \end{array}$$

(i) **(score 1)** Solve the linear relaxation and find the upper bound to the optimal integer solution.

(ii) **(score 1)** Write the two subproblem $\mathcal{P}_1, \mathcal{P}_2$ obtained by branching with respect to fractional variable

(iii) **(score 1)** Solve problem $\mathcal{P}_1$. Explain if it is possible to close it.

(iv) (score 1) Solve problem $\mathcal{P}_2$. Explain if it is possible to close it.

Exercise 6. (Score 5)

DeMont Chemical Company manufactures fertilizer in three plants, referred to as P1, P2, and P3. The company ships its products from plants to two central DCs, designated D_1 and D_2, and then from the DCs to five regional warehouses, $W_1 \dots W_5$. At the DCs, no demand occurs but a capacity limits exist. Demand is associated with the warehouses, and production limit exist at the plants. The system is described in the following two tables, one for each stage. The first table reports the transportation costs from plant to DCs, the capacity at DCs and the production limit at the plants. The second table reports the transportation costs from DCs to warehouses and the demand at the warehouses. The units for capacity and demand are pounds of fertilizer, and the unit costs are given per pound.

	D1	D2	Production
P1	1.36	1.28	2400
P2	1.28	1.35	2750
P3	1.68	1.55	2500
Capacity	4500	3200	

	W1	W2	W3	W4	W5
D1	0.60	0.36	0.32	0.44	0.72
D2	0.80	0.56	0.42	0.40	0.55
Requirement	1250	1000	1600	1750	1500

Write an LP problem to find the cheapest way for planing the transportation .