OPERATIONS RESEARCH - TEXT A

EXAM October 19, 2018

IMPORTANT: READ CAREFULLY

The grade on the written exam is valid at most for three months.

Check the score of each exercise.

Please note that YES NO answers are NOT valid if you do not give an explanation.

SURNAME **NAME**

I **authorize** the publication (paper and electronic) of the results obtained in the examination in accordance with the Italian Law 675/96 and subsequent amendments

SIGNATURE

GRADE

I solved the following exercise

1. ☐ **(Score 6)**
2. ☐ **(Score 5)**
3. ☐ **(Score 8)**
4. ☐ **(Score 6)**
5. ☐ **(Score 6)**

ORAL PART: questions

GRADE

Exercise 1. **(Score 6)** Consider the following nonlinear programming problem

$$\begin{array}{ll} \min & 4x_1^2 + 4x_1x_2 + x_2^2 + 10x_1 + 5x_2 \\ & 3x_1 - x_2 \le 15 \\ & 2x_1 + 3x_2 \le 6 \\ & -3x_1 - 2x_2 \le 6 \\ & x_1 \ge 0 \end{array}$$

Unconstrained problem (score 2)

(i) **(score 1)** Consider the point $x^0 = (0,\ 1)^T$ and write the first iteration of the gradient method with exact line search to obtain the new point x^1

(ii) **(score 1)** Consider the unconstrained problem (remove the constraints). Find the unconstrained stationary point and determine the nature of the points.

Constrained problem (score 4)

(iii) **(score 1)** State if the problems is convex or strictly convex or none of the two.

(iv) **(score 2)** Consider the point $\widehat{x} = (8/3,\ -7)^T$ and write the KKT conditions in $\widehat{x}$. Are the KKT conditions satisfied ? Evaluate the multipliers

(v) **(score 1)** Write the system to get a feasible and descent direction in $\widehat{x}$. Does a solution of the system (namely a feasible and descent direction) exist?

Exercise 2. **(Score 5)** Consider the following Linear programming problem

$$\begin{array}{ll} \min & 15x_1 + 6x_2 + 6x_3 \\ & 3x_1 + 2x_2 - 3x_3 \ge 1 \\ & -x_1 + 3x_2 - 2x_3 = 2 \\ & x_1, x_2, x_3 \ge 0 \end{array}$$

(i) **(score 2)** Find a a feasible direction along which it is possible to move from $\widehat{x} = (1,\ 1,\ 0)^T$ finding an additional active constraint. Find the stepsize $t^{\max}$ and the corresponding new point y. Is the direction also a descent one?

(ii) **(score 0,5)** Write the problem in the standard form for the simplex method.

(iii) **(score 0,5)** Write the auxiliary problem to be solved in Phase I of the simplex method.

(iv) **(score 1)** Check if the following points are vertex

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 4/3 \\ 1/2 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \frac{4}{3} \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ \frac{2}{3} \\ 0 \end{pmatrix}$$

(v) **(score 1,25)** Write a BFS for the problems at point (ii) and specify the corresponding matrices B and N.

Exercise 3. **(Score 8)** Consider the following Linear programming problem

$$\begin{array}{ll} \min & 15x_1 + 6x_2 + 6x_3 \\ & 3x_1 + 2x_2 - 3x_3 \ge 1 \\ & -x_1 + 3x_2 - 2x_3 = 2 \\ & x_1, x_2, x_3 \ge 0 \end{array}$$

(i) **(score 1.5)** Write the dual problem

(ii) **(score 2)** Solve graphically the dual: plot the feasible region, the level lines of the objective function, identify graphically the solution and find its value.

(iii) **(score 2)** Using duality theory, state if the primal problem has an optimal solution and in the affirmative case find it.

(iv) **(score 1)** State how the value of the optimal solution change if the rhs of the second constraint change from 2 to $2 + \varepsilon$ with $\varepsilon > 0$ and sufficiently small.

(v) **(score 1.5)** State which is the maximum value of $\varepsilon > 0$ for which the analysis in point (iv) holds.

Exercise 4. (Score 6) Consider the following integer Knapsack problem:

$$\begin{aligned} \max \quad & 6x_1 + x_2 + 4x_3 + 0.9x_4 \\ & 2x_1 + 4x_2 + 4x_3 + 3x_4 \leq 7 \\ & x_i \in \{0, 1\}, i = 1, \ldots, 4. \end{aligned}$$

(i) **(score 2)** Report a lower and an upper bound to the optimal value of the objective function.

(ii) **(score 4)** Solve the problem

Exercise 5. (Score 6) The four sales departments (V1, V2, V3, V4) of a company require daily to be supplied by the three production departments (P1, P2, P3) with pre-defined quantities of a new product to be placed on the market. The table below shows the quantities (in quintals) required by each of the sales departments on a daily basis, together with unit costs (in euros per quintal) of the transport of one quintal of product from each of the productive departments to each one of sales departments.

	V1	V2	V3	V4
quantities required	250	380	420	195
Transportation cost	V1	V2	V3	V4
P1	9	7.5	8	8.5
P2	6.5	7	7.8	8
P3	7	6.7	8.2	7.9

The following table shows, for each production department, its maximum daily production capacity (expressed in quintals), the unit manufacturing cost (in euros per quintal).

	P1	P2	P3
maximum daily production	850	500	530
unit manufacturing cost	3	3.5	4.5

Write an LP problem that satisfy the demand exactly and minimize the total cost of the firm.