SAPIENZA
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A lot sizing problem

Production planning over a finite discrete horizon

(Wagner-Whitin model)

Operations Research MMER

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PRODUCTION PLANNING

A manufacturing company has the following demand for pairs of shoes for the next three months: 600 for the first month, 500 for the second month, 900 for the third month. The production of each pair of shoes takes 15 minutes and the cost of production changes with the month as reported in the table (euro/pair).

	Month 1	Month 2	Month 3
Demand	600	500	900
Unit Cost	5	7	6

Available hours per month 180.

The company has a warehouse. The cost of storage is 3 euro/month for each pair of shoes and at the beginning of the first month are 100 pairs of shoes.

Decision variables x_1, x_2, x_3

Pair of shoes manufactured during month 1,2,3

constraints

demand	time
$x_1 >= 600$	$15\ x_1 <= 180 \times 60$
$x_2 >= 500$	$15\ x_2 <= 180 \times 60$
$x_3 >= 900$	$15\ x_3 <= 180 \times 60$

$$\min \quad 5\,x_1 + 7\,x_2 + 6\,x_3$$

$$x_1 >= 600$$

$$x_2 >= 500$$

$$x_3 >= 900$$

$$15\,x_1 <= 180 \times 60$$

$$15\,x_2 <= 180 \times 60$$

$$15\,x_3 <= 180 \times 60$$

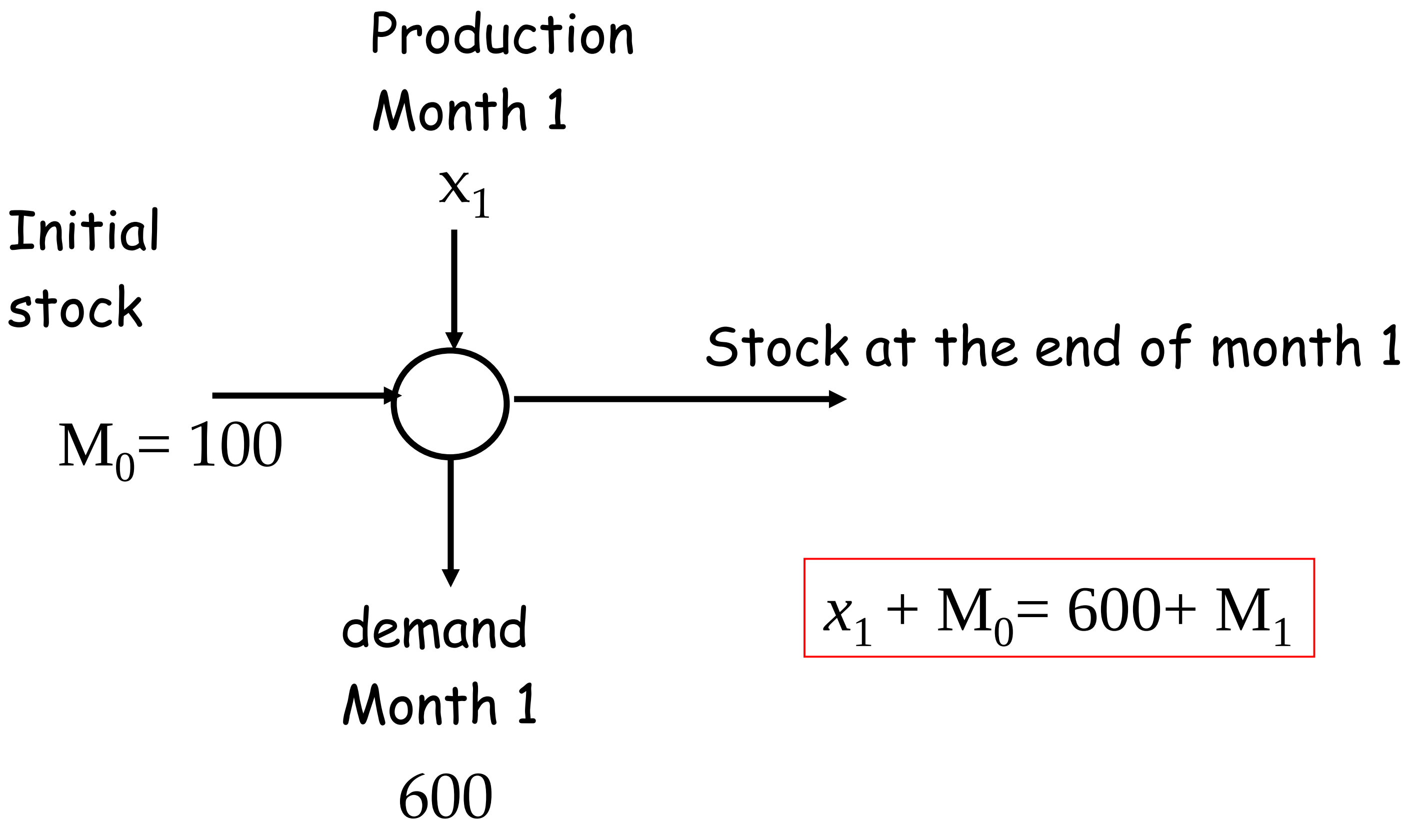
$$x_1,\ x_2,\ x_3 >= 0$$

There is non solution of this formulation beacuse of constraints

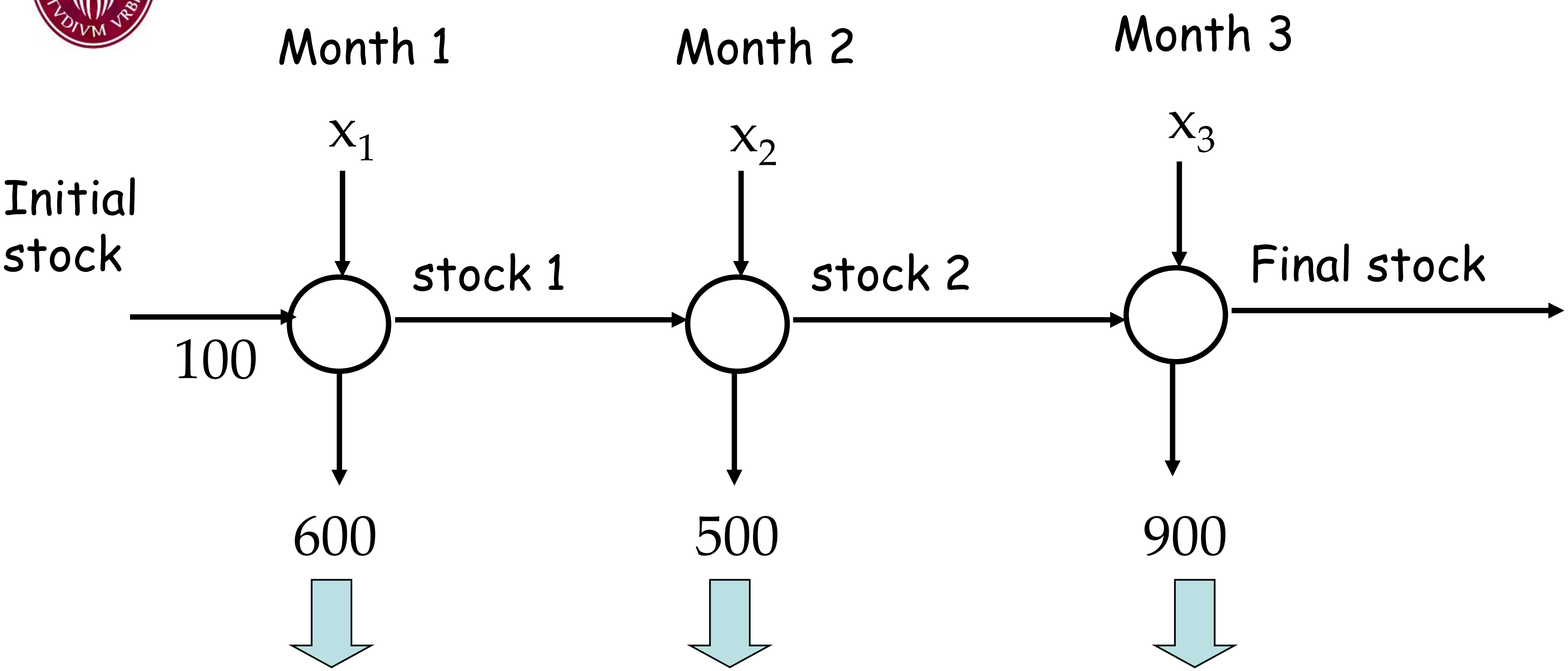
$$x_3 >= 900$$

$$x_3 <= 720$$

During months 1 and 2, the production capacity is not fully utilized



$$\boxed{x_1 + M_0 = 600 + M_1}$$



$\boxed{x_1 + M_0 = 600 + M_1}$ $\boxed{x_2 + M_1 = 500 + M_2}$ $\boxed{x_3 + M_2 = 900 + M_F}$

$$\min \quad 5\,x_1 + 7\,x_2 + 6\,x_3 + 3\,(M_1 + M_2 + M_F)$$

$$x_1 + M_0 = 600 + M_1$$

$$x_2 + M_1 = 500 + M_2$$

$$x_3 + M_2 = 900 + M_F$$

$$15\,x_1 <= 180 \times 60$$

$$15\,x_2 <= 180 \times 60$$

$$15\,x_3 <= 180 \times 60$$

$$M_0, M_1, M_2, M_F >= 0$$

$$M_0 = 100$$

$$x_1, x_2, x_3 >= 0$$

$$\min \quad 5x_1 + 7x_2 + 6x_3 + 3(M_1 + M_2 + M_F)$$

$$x_1 + 100 - 600 = M_1 >= 0$$

$$x_2 + (x_1 + 100 - 600) - 500 = M_2 >= 0$$

$$x_3 + (x_2 + (x_1 + 100 - 600) - 500) - 900 = M_F >= 0$$

$$15\, x_1 <= 180 \times 60$$

$$15\, x_2 <= 180 \times 60$$

$$15\, x_3 <= 180 \times 60$$

$$x_1,\ x_2,\ x_3 >= 0$$

Adding constraints

The warehouse has a capacity of up to 130.

$$M_1, M_2, M_F <= 130$$

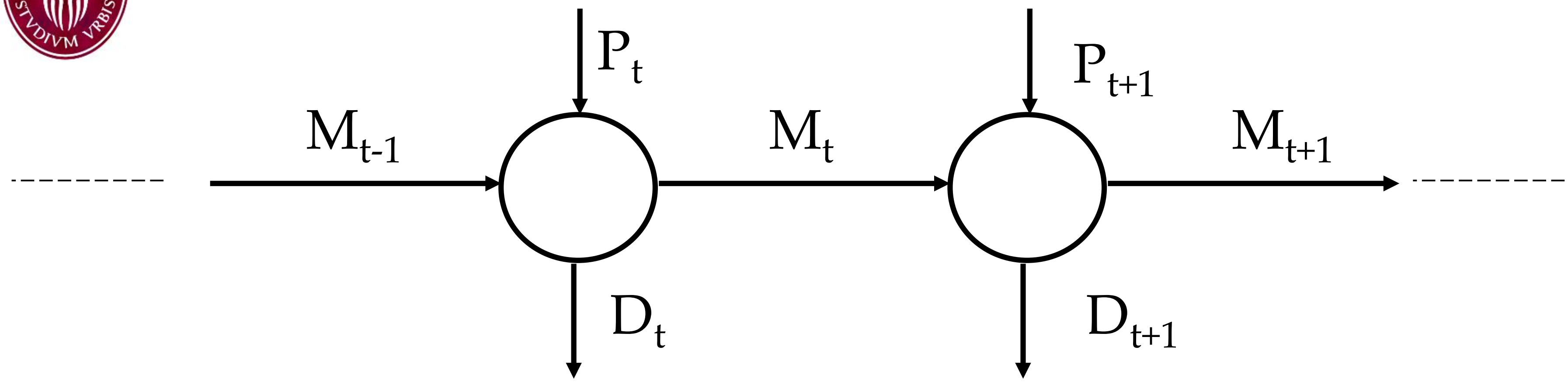
Lot sizing problem
Wagner-Whitin model

1. A finite and discrete horizon {1,….T} (planning horizon)
2. A variable demand over the discrete intervals $D_t > 0$ fort=1, …T
3. No backlogging (it not possible to satisfy the demand of the current period in future periods, i.e. with delay)

In each interval t the decision variables are

1. M_t the stock inventory
2. P_t the production level

Modello Wagner-Whitin



No backlogging so that

1. Demand in each period must be satisfied with the production P_t and stock inventory M_{t-1}
2. Production "surplus" define the stock inventory

$$M_t = M_{t-1} + P_t - D_t \quad t = 1, \ldots, T$$

$$M_t \geq 0,$$

$$P_t \geq 0$$

Additional constraints

- Initial and final stock are usually fixed (often zero)
- The stock is limited by the capacity of the warehouse

$$M_0 = \overline{M}_0$$

$$M_T = \overline{M}_T$$

$$M_t \leq M_{\max}$$

- Limits on the production

$$0 \leq P_t \leq P_{\max} \quad t = 1, \ldots, T$$

Objective function

Costs

1. Fixed charge production cost
2. Production costs
3. Fixed charge production cost
4. Stock costs