

Ottimizzazione Combinatoria II

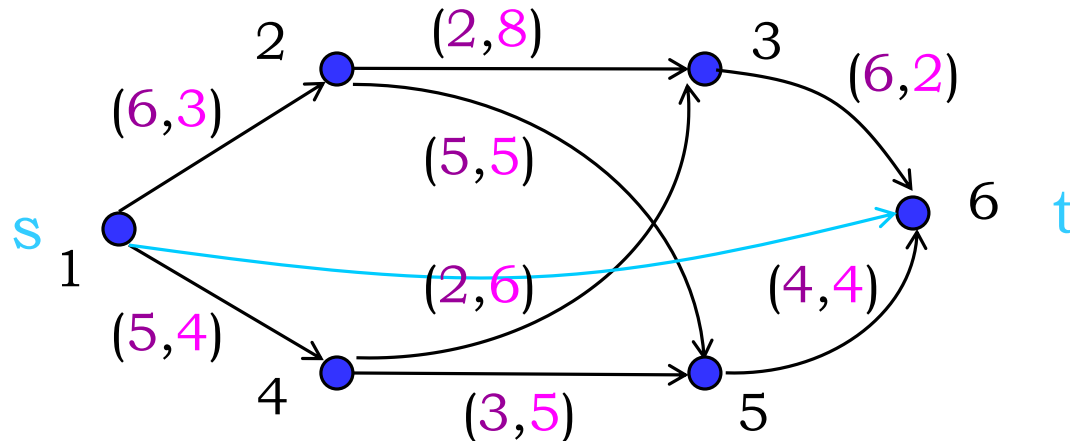
Metodo del Simplexso Dinamico

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Problema

Sia dato il grafo $G(V,E)$, dove V rappresenta un insieme di città, ed E rappresenta i collegamenti tra le città.



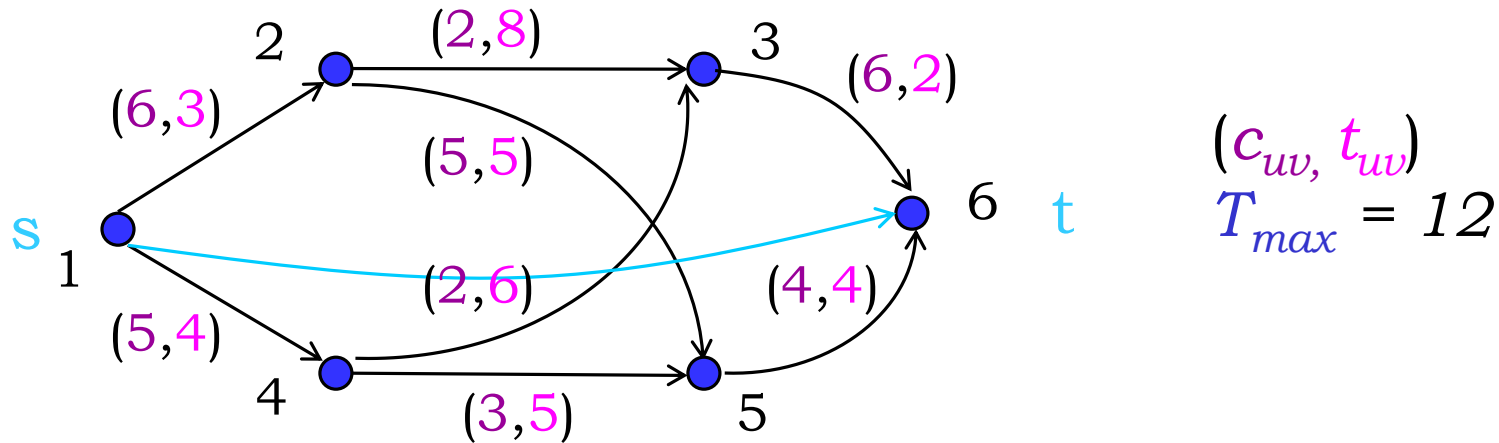
Ad ogni arco $(u,v) \in E$ sono associati:

- un costo di utilizzo c_{uv}
- un tempo di percorrenza t_{uv}

Un corriere preleva un pacco in 1 e deve consegnarlo in 6 , decidendo quale percorso scegliere per:

- minimizzare il costo di trasporto
- impiegare un tempo minore di $T_{max} = 12$

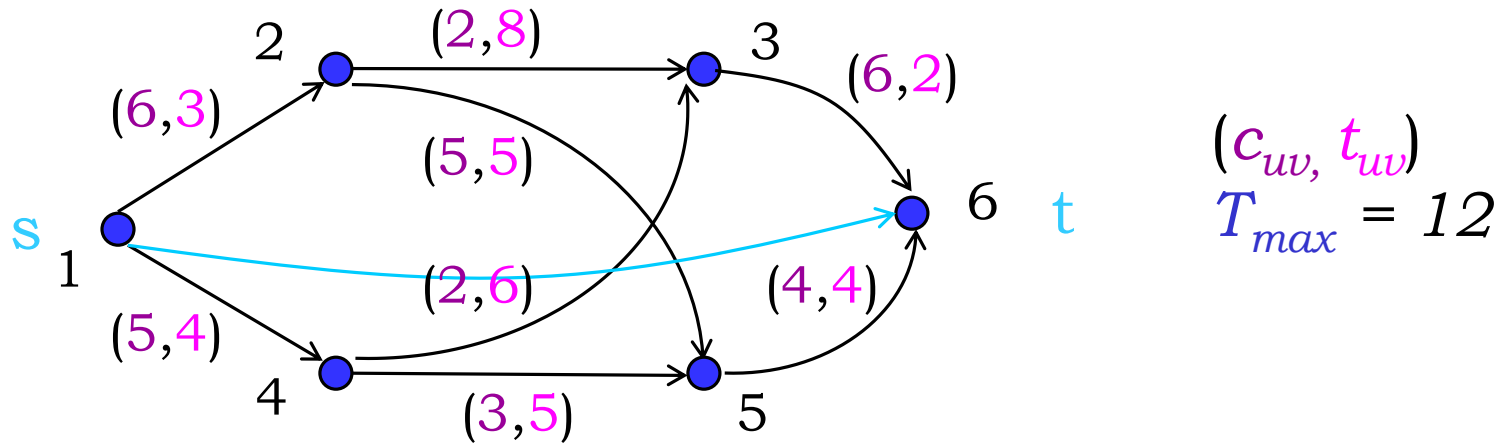
Formulazione



- insieme ammissibile **S** :
vettori di incidenza di cammini tra s e t con ritardo minore di T

$$x_{uv} = \begin{cases} 1 & (u,v) \in \pi_{st} \\ 0 & (u,v) \notin \pi_{st} \end{cases}$$

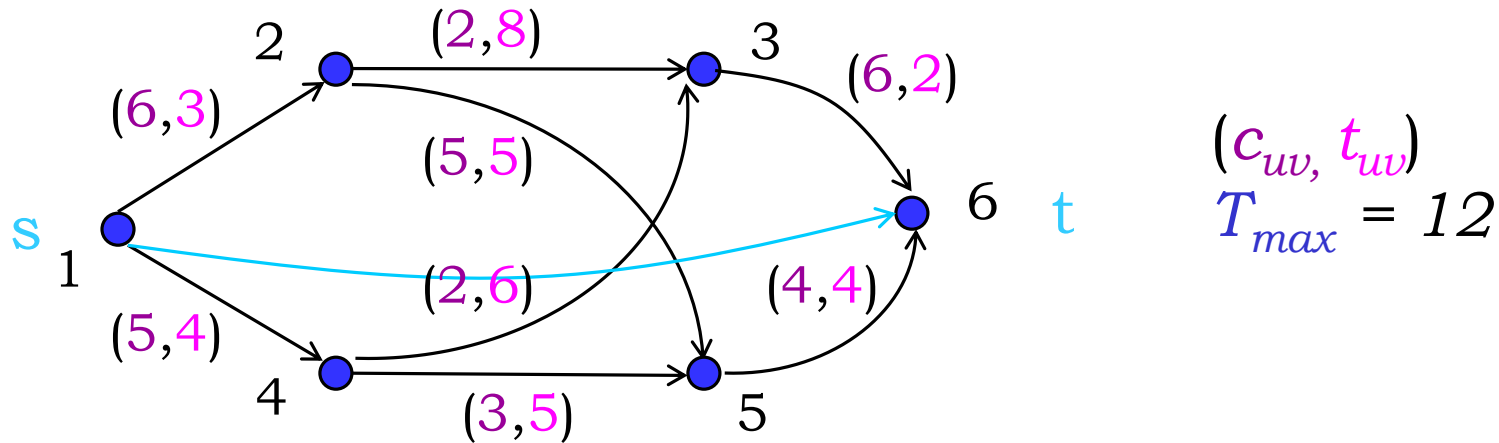
Formulazione



- formulazione **P** :

$$\left\{ \begin{array}{ll} \sum_{(u,v) \in E} t_{uv} x_{uv} \leq T & \Rightarrow \text{vincolo sul ritardo} \\ \sum_{(u,v) \in K} x_{uv} \geq 1 & \text{K st - taglio} \Rightarrow \text{cammino st} \\ 0 \leq x_{uv} \leq 1 & (u,v) \in E \Rightarrow \text{vincoli di box} \end{array} \right.$$

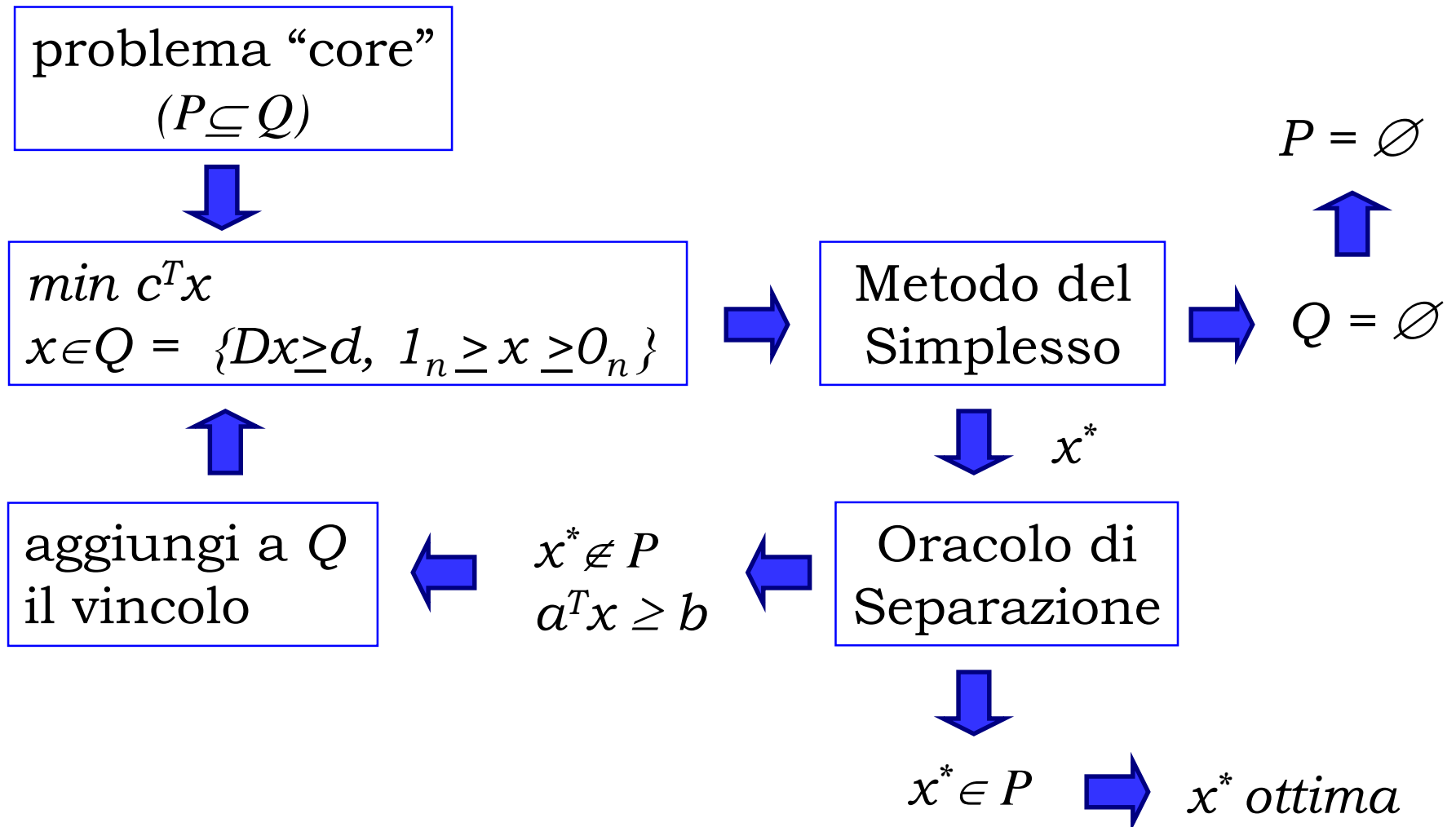
Formulazione



$$\min (6x_{12} + 5x_{14} + 2x_{23} + 5x_{25} + 2x_{43} + 3x_{45} + 6x_{36} + 4x_{56})$$

$$\left\{ \begin{array}{l} 3x_{12} + 4x_{14} + 8x_{23} + 5x_{25} + 6x_{43} + 5x_{45} + 2x_{36} + 4x_{56} \leq 12 \\ \sum_{(u,v) \in K} x_{uv} \geq 1 \quad K \text{ st - taglio} \\ 0 \leq x_{uv} \leq 1 \quad (u,v) \in E \end{array} \right.$$

Metodo del Simpleso Dinamico

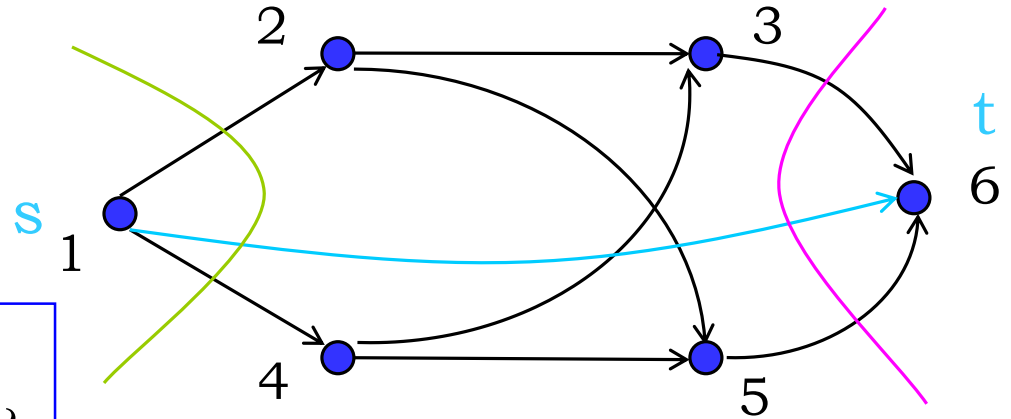


Metodo del Simpleso Dinamico

problema "core"
 $(P \subseteq Q)$



$\min c^T x$
 $x \in Q = \{Dx \geq d, 1_n \geq x \geq 0_n\}$



$$\min (6x_{12} + 5x_{14} + 2x_{23} + 5x_{25} + 2x_{43} + 3x_{45} + 6x_{36} + 4x_{56})$$

$$\left\{ \begin{array}{l} 3x_{12} + 4x_{14} + 8x_{23} + 5x_{25} + 6x_{43} + 5x_{45} + 2x_{36} + 4x_{56} \leq 12 \\ x_{12} + x_{14} \geq 1 \\ x_{36} + x_{56} \geq 1 \\ 0 \leq x_{uv} \leq 1 \quad (u,v) \in E \end{array} \right.$$

Metodo del Simpleso Dinamico

$$\min (6x_{12} + 5x_{14} + 2x_{23} + 5x_{25} + 2x_{43} + 3x_{45} + 6x_{36} + 4x_{56})$$

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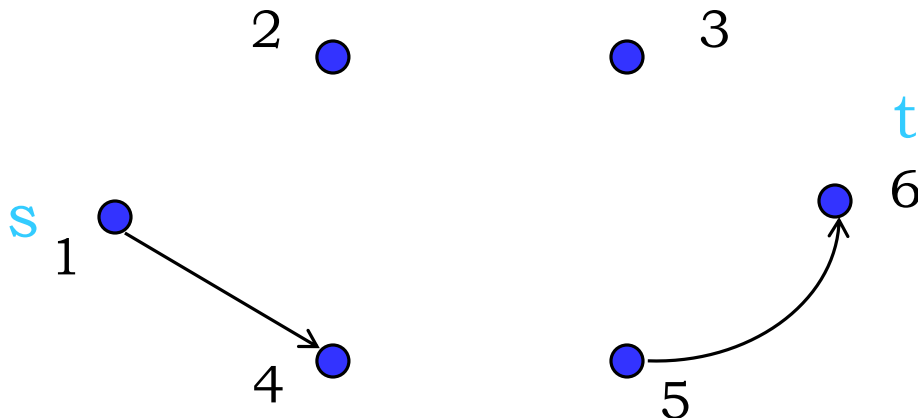
$$\min c^T x$$

$$x \in Q = \{Dx \geq d, 1_n \geq x \geq 0_n\}$$



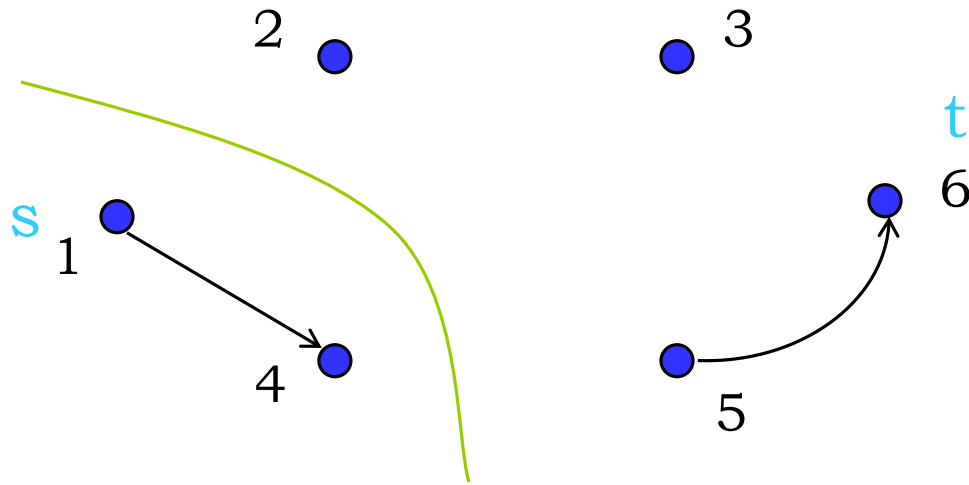
Metodo del Simpleso

$$z^* = 9$$



$$\begin{cases} x_{14}^* = x_{56}^* = 1 \\ x_{36}^* = x_{56}^* = x_{43}^* = x_{12}^* = \\ = x_{23}^* = x_{25}^* = x_{45}^* = 0 \end{cases}$$

Metodo del Simpleso Dinamico



$$C_{\min} = 0$$



$$x_{12} + x_{43} + x_{45} \geq 1$$

$$\begin{cases} x_{14}^* = x_{56}^* = 1 \\ x_{36}^* = x_{56}^* = x_{43}^* = x_{12}^* = z^* = 9 \\ x_{23}^* = x_{25}^* = x_{45}^* = 0 \end{cases}$$

Metodo del Simpleso



Oracolo di Separazione

aggiungi a Q il vincolo



$$x^* \notin P \\ a^T x \geq b$$



Metodo del Simpleso Dinamico

$$\min (6x_{12} + 5x_{14} + 2x_{23} + 5x_{25} + 2x_{43} + 3x_{45} + 6x_{36} + 4x_{56})$$

$$\left\{ \begin{array}{l} 3x_{12} + 4x_{14} + 8x_{23} + 5x_{25} + 6x_{43} + 5x_{45} + 2x_{36} + 4x_{56} \leq 12 \\ x_{12} + x_{14} \geq 1 \\ x_{36} + x_{56} \geq 1 \\ x_{12} + x_{43} + x_{45} \geq 1 \\ 0 \leq x_{uv} \leq 1 \quad (u,v) \in E \end{array} \right.$$



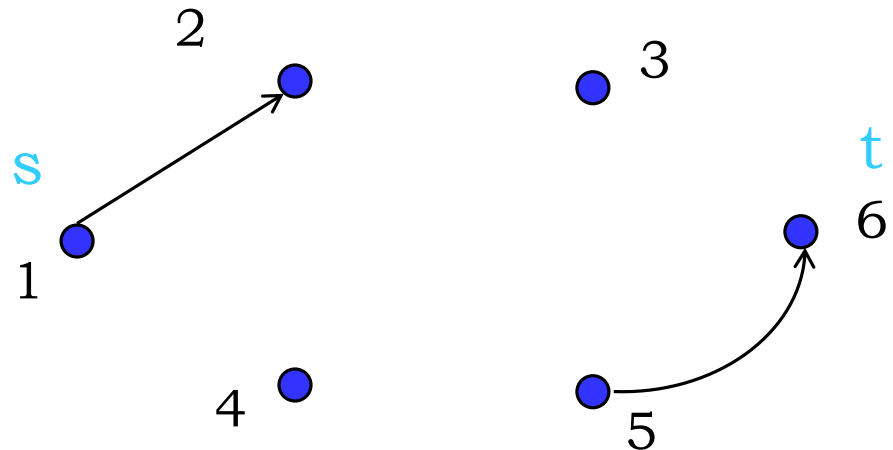
Metodo del Simpleso



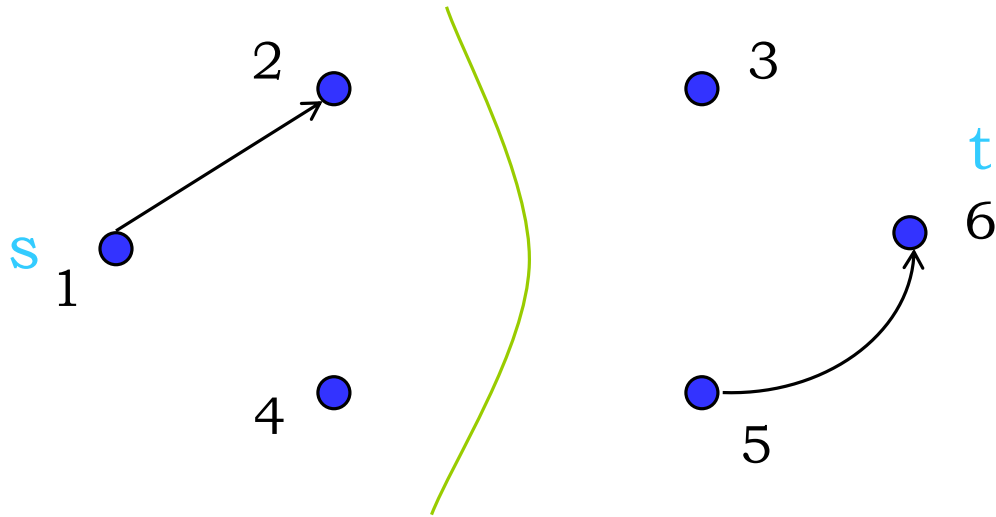
x^*

$z^* = 10$

$$\left\{ \begin{array}{l} x_{12}^* = x_{56}^* = 1 \\ x_{36}^* = x_{56}^* = x_{43}^* = x_{14}^* = \\ = x_{23}^* = x_{25}^* = x_{45}^* = 0 \end{array} \right.$$



Metodo del Simpleso Dinamico



$$C_{\min} = 0$$



$$x_{23} + x_{25} + x_{43} + x_{45} \geq 1$$

$$\begin{cases} x_{12}^* = x_{56}^* = 1 \\ x_{36}^* = x_{56}^* = x_{43}^* = \\ = x_{14}^* = x_{23}^* = x_{25}^* = x_{45}^* = 0 \end{cases} \quad z^* = 10$$

Metodo del Simpleso



Oracolo di Separazione

aggiungi a Q
il vincolo



$$x^* \notin P \\ a^T x \geq b$$



Metodo del Simpleso Dinamico

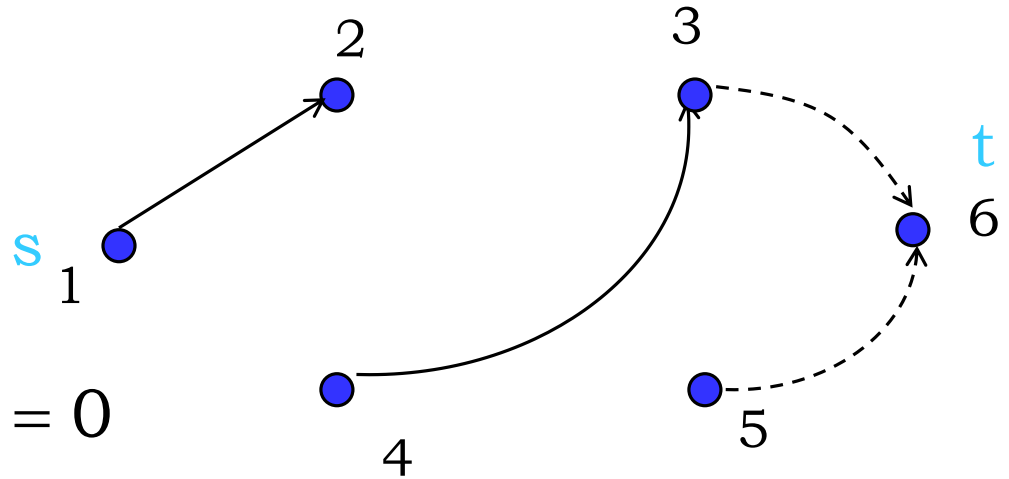
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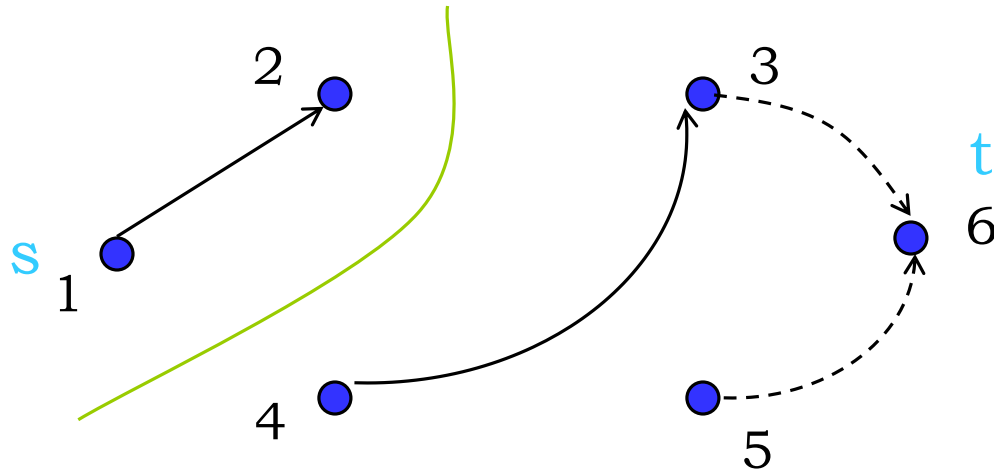
Metodo del Simpleso

$x^* \quad z^* = 13$

$$\begin{cases} x_{12}^* = x_{43}^* = 1 \\ x_{36}^* = x_{56}^* = 1/2 \\ x_{14}^* = x_{23}^* = x_{25}^* = x_{45}^* = 0 \end{cases}$$



Metodo del Simpleso Dinamico



$$C_{\min} = 0$$



$$x_{14} + x_{25} + x_{23} \geq 1$$

$$\begin{cases} x_{12}^* = x_{43}^* = 1 \\ x_{36}^* = x_{56}^* = 1/2 \\ x_{14}^* = x_{23}^* = x_{25}^* = x_{45}^* = 0 \end{cases} \quad z^* = 13$$

Metodo del Simpleso



x^*

Oracolo di Separazione

$$x^* \notin P \quad a^T x \geq b$$

aggiungi a Q
il vincolo

Metodo del Simplexso Dinamico

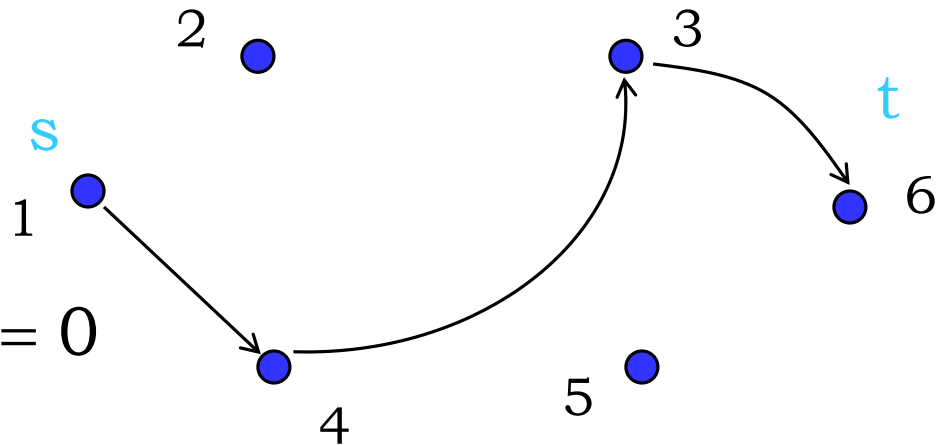
$$\min (6x_{12} + 5x_{14} + 2x_{23} + 5x_{25} + 2x_{43} + 3x_{45} + 6x_{36} + 4x_{56})$$

$$\left\{ \begin{array}{l} 3x_{12} + 4x_{14} + 8x_{23} + 5x_{25} + 6x_{43} + 5x_{45} + 2x_{36} + 4x_{56} \leq 12 \\ x_{12} + x_{14} \geq 1 \\ x_{36} + x_{56} \geq 1 \\ x_{12} + x_{43} + x_{45} \geq 1 \\ x_{23} + x_{25} + x_{43} + x_{45} \geq 1 \\ x_{14} + x_{25} + x_{23} \geq 1 \\ 0 \leq x_{uv} \leq 1 \quad (u,v) \in E \end{array} \right.$$

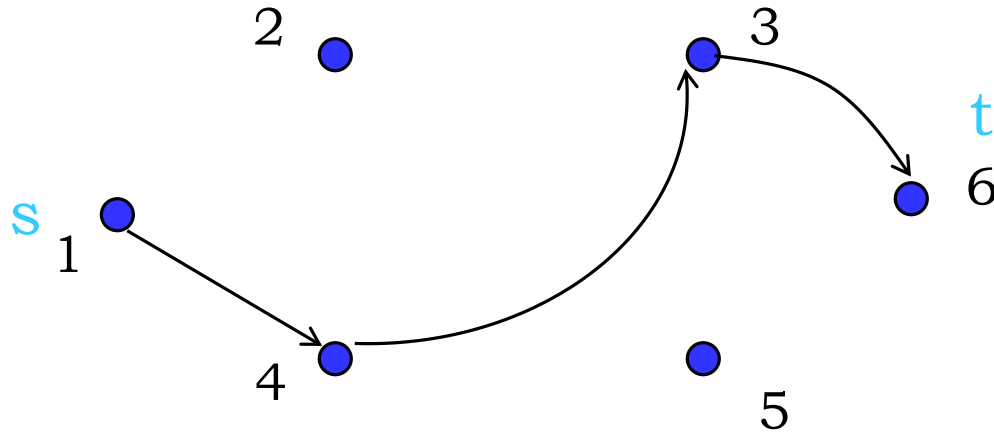
Metodo del Simplexso

x^* $z^* = 13$

$$\left\{ \begin{array}{l} x_{14}^* = x_{43}^* = x_{36}^* = 1 \\ x_{12}^* = x_{23}^* = x_{25}^* = x_{45}^* = x_{56}^* = 0 \end{array} \right.$$



Metodo del Simpleso Dinamico



$$z^* = 13$$

$$\begin{cases} x_{14}^* = x_{43}^* = x_{36}^* = 1 \\ x_{12}^* = x_{23}^* = x_{25}^* = x_{45}^* = x_{56}^* = 0 \end{cases}$$

Metodo del Simpleso

↓ x^*

Oracolo di Separazione

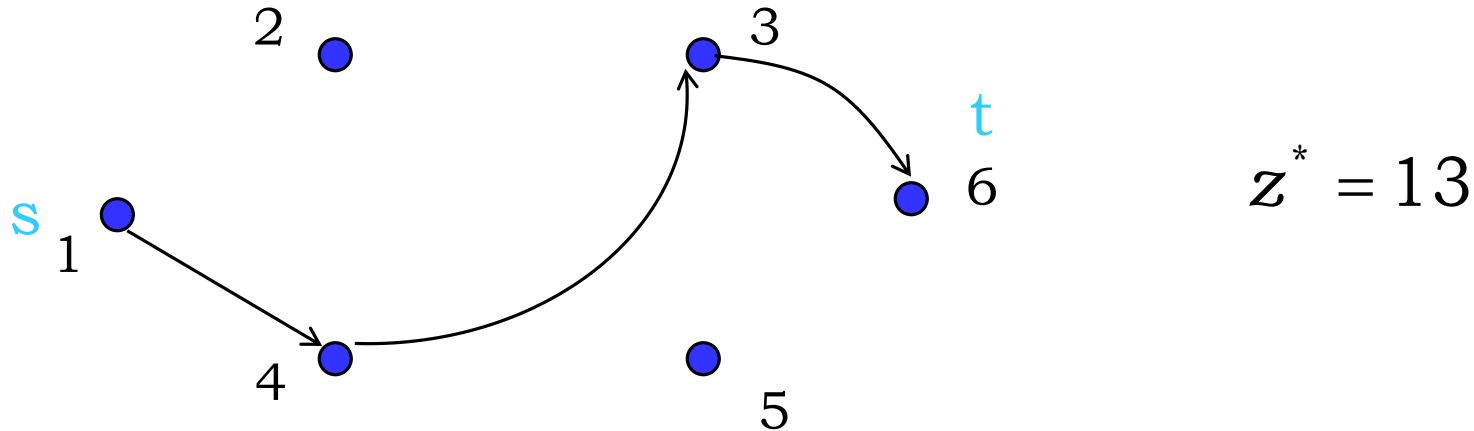
↓

$x^* \in P$ → x^* ottima

➔ la capacità del taglio minimo è 1

➔ non c'è nessun vincolo violato

Metodo del Simpleso Dinamico



$$\begin{cases} x_{14}^* = x_{43}^* = x_{36}^* = 1 \\ x_{12}^* = x_{23}^* = x_{25}^* = x_{45}^* = x_{56}^* = 0 \end{cases}$$

- x^* soluzione ottima per P
- x^* intera



x^* ottima per il problema intero