Ottimizzazione Combinatoria 2

Sistemi di Indipendenza: Tre Applicazioni

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Tre Applicazioni

- ***Aste Combinatorie**: **Massimo Insieme Stabile***
  - *Assegnazione di **carichi a trasportatori***
  - *Acquisto di beni (**e-procurement**)*
  - *Assegnazione di **frequenze***
  - *Assegnazione di **obiettivi a robot***
- ***AdWords**: **(Online) Matching con Budget***
  - *TV Ads (su YouTube)*
- ***TV Digitale**: **Massimo Sottosistema Ammissibile***
  - *Progetto di reti TV a **massima copertura***
  - *Determinazione ottima dei **ritardi***

CPS 296.1
Auctions &
Combinatorial Auctions

Vincent Conitzer

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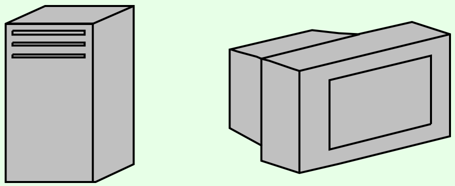
A few different 1-item auction mechanisms

- English auction:
  - Each bid must be higher than previous bid
  - Last bidder wins, pays last bid
- Japanese auction:
  - Price rises, bidders drop out when price is too high
  - Last bidder wins at price of last dropout
- Dutch auction:
  - Price drops until someone takes the item at that price
- Sealed-bid auctions (direct revelation mechanisms):
  - Each bidder submits a bid in an envelope
  - Auctioneer opens the envelopes, highest bid wins
    - First-price sealed-bid auction: winner pays own bid
    - Second-price sealed bid (or Vickrey) auction: winner pays second-highest bid

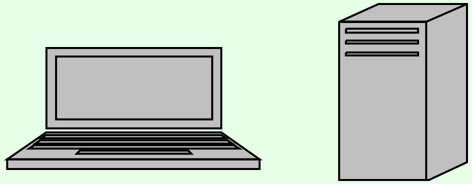
Complementarity and substitutability

- How valuable one item is to a bidder may depend on whether the bidder possesses another item

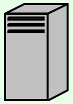
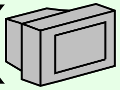
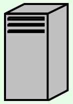
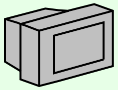
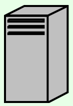
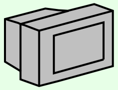
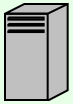
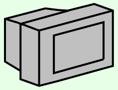
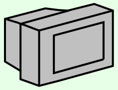
- Items a and b are complementary if v({a, b}) > v({a}) + v({b})
- E.g.



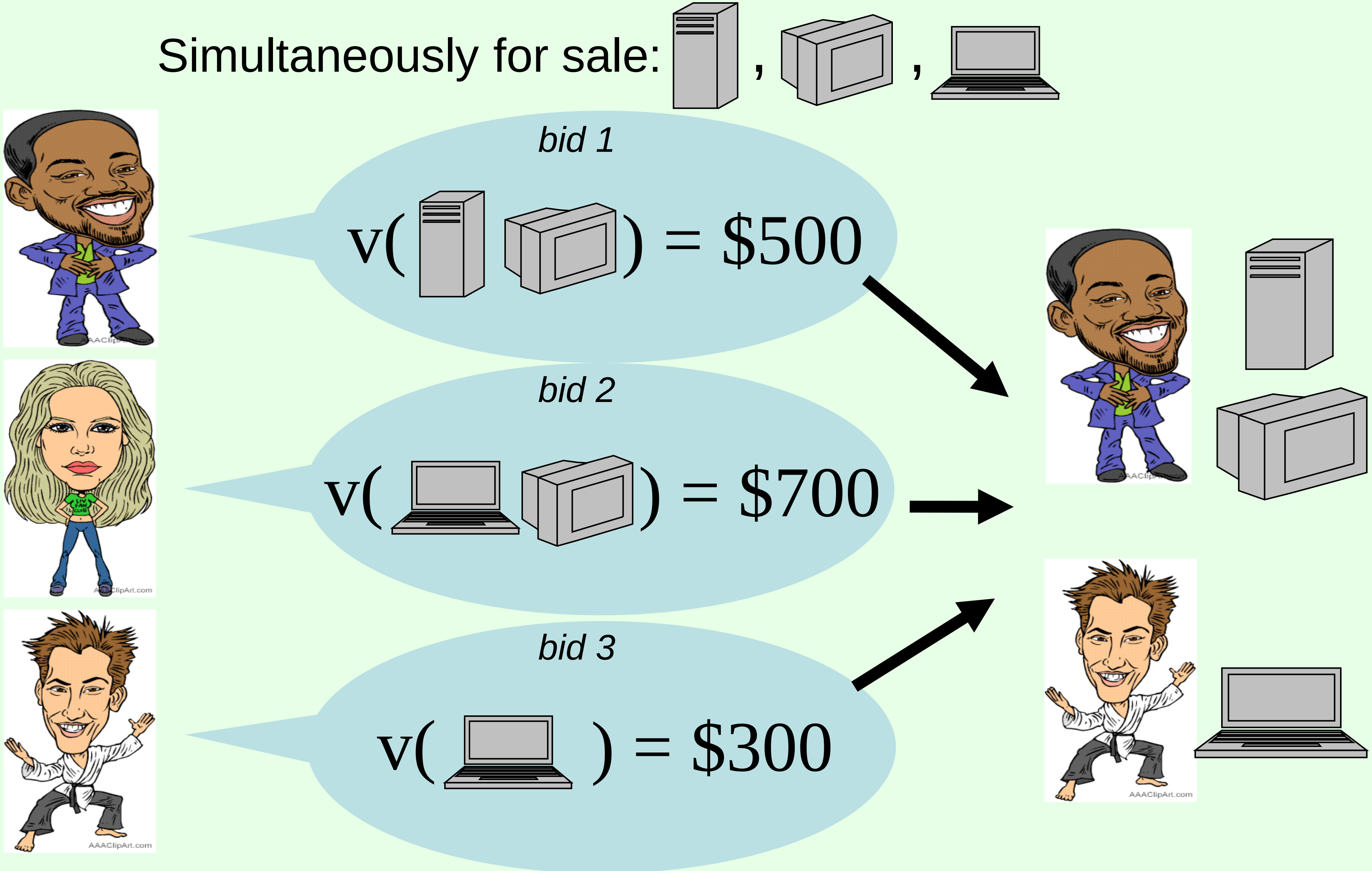
- Items a and b are substitutes if v({a, b}) < v({a}) + v({b})
- E.g.



Inefficiency of sequential auctions

- Suppose your valuation function is v() = $200, v() = $100, v( ) = $500
- Now suppose that there are two (say, Vickrey) auctions, the first one for  and the second one for 
- What should you bid in the first auction (for )?
- If you bid $200, you may lose to a bidder who bids $250, only to find out that you could have won  for $200
- If you bid anything higher, you may pay more than $200, only to find out that  sells for $1000
- Sequential (and parallel) auctions are inefficient

Combinatorial auctions



used in truckload transportation, industrial **procurement**, **radio spectrum allocation**, …

The winner determination problem (WDP)

- Choose a subset **X** (*the accepted bids*) of the bids **B**,
- to maximize $\mathbf{\Sigma_{b \text{ in } X} v_b}$,
- under the constraint that every item occurs at most once in **X**

- This is assuming free disposal, i.e., not everything needs to be allocated

WDP example

- Items A, B, C, D, E
- Bids:
- ({A, C, D}, 7)
- ({B, E}, 7)
- ({C}, 3)
- ({A, B, C, E}, 9)
- ({D}, 4)
- ({A, B, C}, 5)
- ({B, D}, 5)

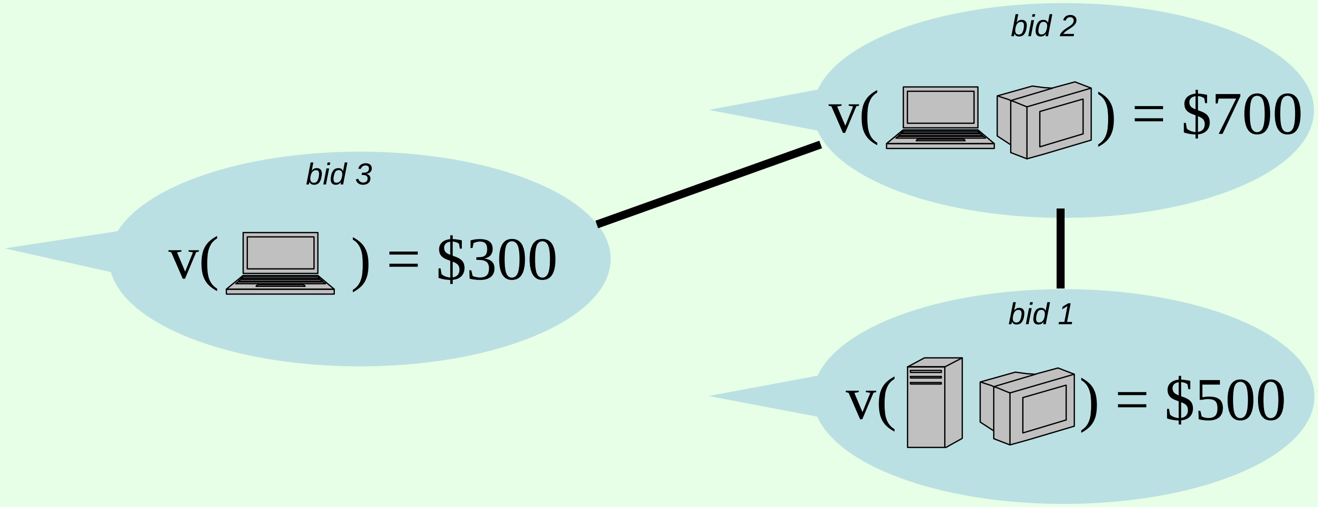
- *What's an optimal solution?*
- *How can we prove it is optimal?*

An integer program formulation

- x_b equals **1** if bid b is accepted, **0** if it is not
- maximize $\Sigma_b \, v_b x_b$
- subject to
  - for each item j, $\Sigma_{b:\, j \text{ in } b} \, x_b \leq 1$
- If each x_b can take any value in [0, 1], we say that bids can be partially accepted
- In this case, this is a linear program that can be solved in polynomial time
- This requires that
  - – each item can be divided into fractions
  - – if a bidder gets a fraction f of each of the items in his bundle, then this is worth the same fraction f of his value v_b for the bundle

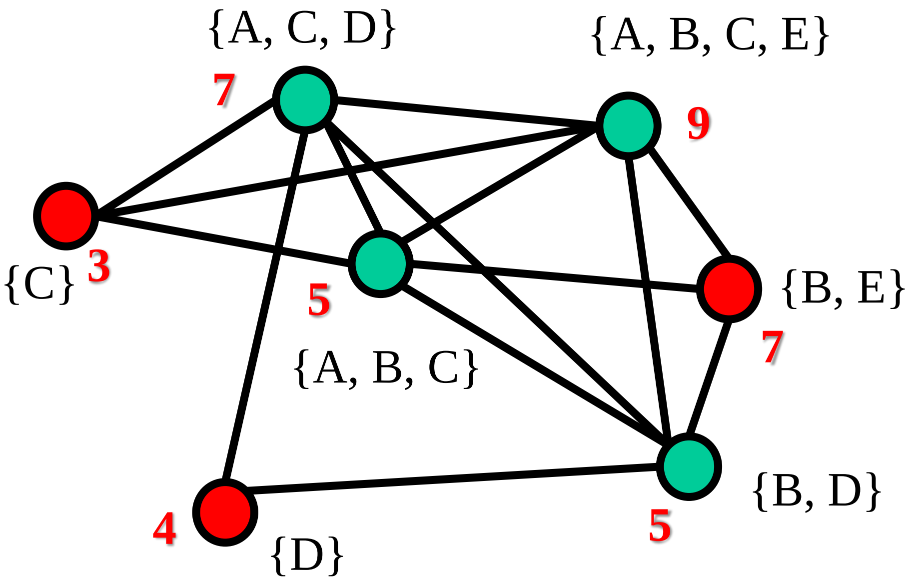
The winner determination problem as a **weighted independent set problem**

- **Each bid is a vertex**
- Draw an edge between two vertices if they share an item



- Optimal allocation = **maximum weight independent set**
- Can model any weighted independent set instance as a CA winner determination problem (1 item per edge (or clique))
- Weighted independent set is NP-hard, even to solve approximately [Håstad 96] - hence, so is WDP
  - [Sandholm 02] noted that this inapproximability applies to the WDP

- **Items A, B, C, D, E**
- ***Bids***:
- ({A, C, D}, 7)
- ({B, E}, 7)
- ({C}, 3)
- ({A, B, C, E}, 9)
- ({D}, 4)
- ({A, B, C}, 5)
- ({B, D}, 5)



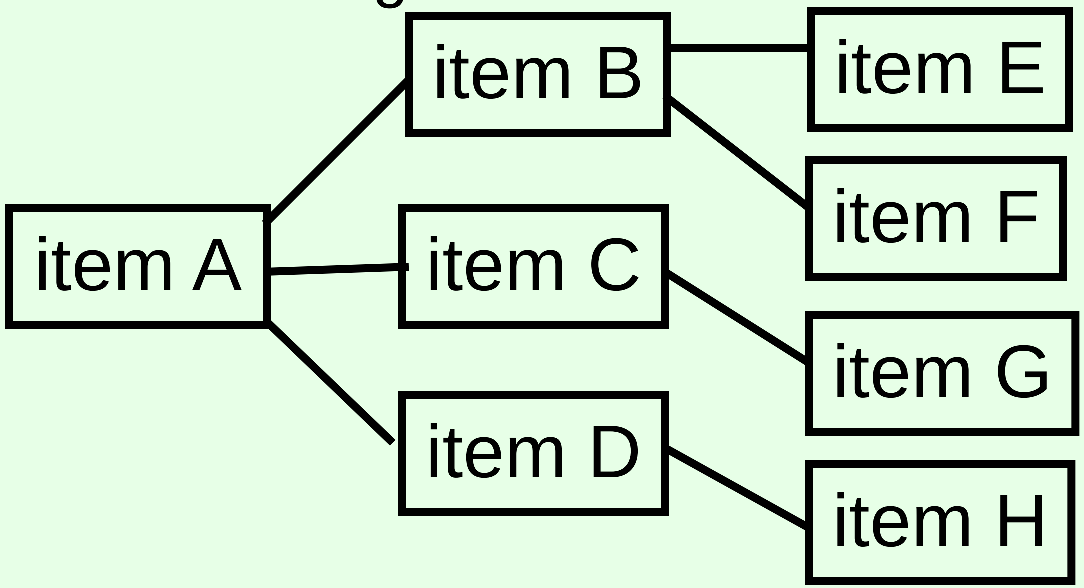
Maximal Stable Sets

- ({A, C, D}, {B, E}) → **14**
- ({B, E}, {C}, {D}) → **14**
- ({A, B, C, E}, {D}) → **13**
- ({A, B, C}, {D}) → **9**
- ({C}, {B, D}) → **8**
- …

- **Problema difficile** (NP-Hard)
- Esistono casi **risolvibili in tempo polinomiale**?
- Necessarie ***ipotesi sulle offerte***

Bids on connected sets of items in a tree

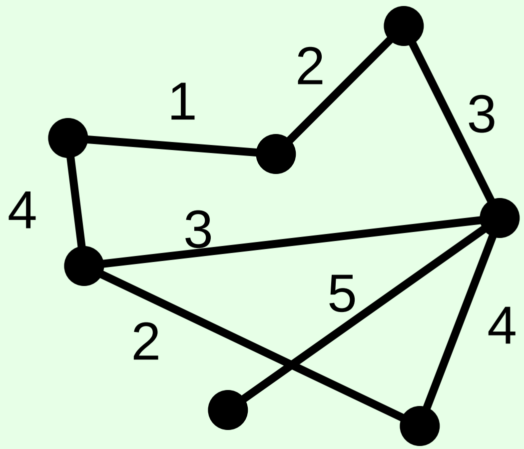
- Suppose items are organized in a tree



- Suppose each bid is on a **connected set of items**
  - E.g. *{A, B, C, G},* but ***not*** *{A, B, G}*
- Then the WDP can be solved in **polynomial time** (using dynamic programming) [Sandholm & Suri 03]
- Tree does not need to be given: can be constructed from the bids in polynomial time if it exists [Conitzer, Derryberry, Sandholm 04]
- More generally, WDP can also be solved in polynomial time for graphs of bounded treewidth [Conitzer, Derryberry, Sandholm 04]
  - Even further generalization given by [Gottlob, Greco 07]

Maximum weighted matching

(not necessarily on bipartite graphs)

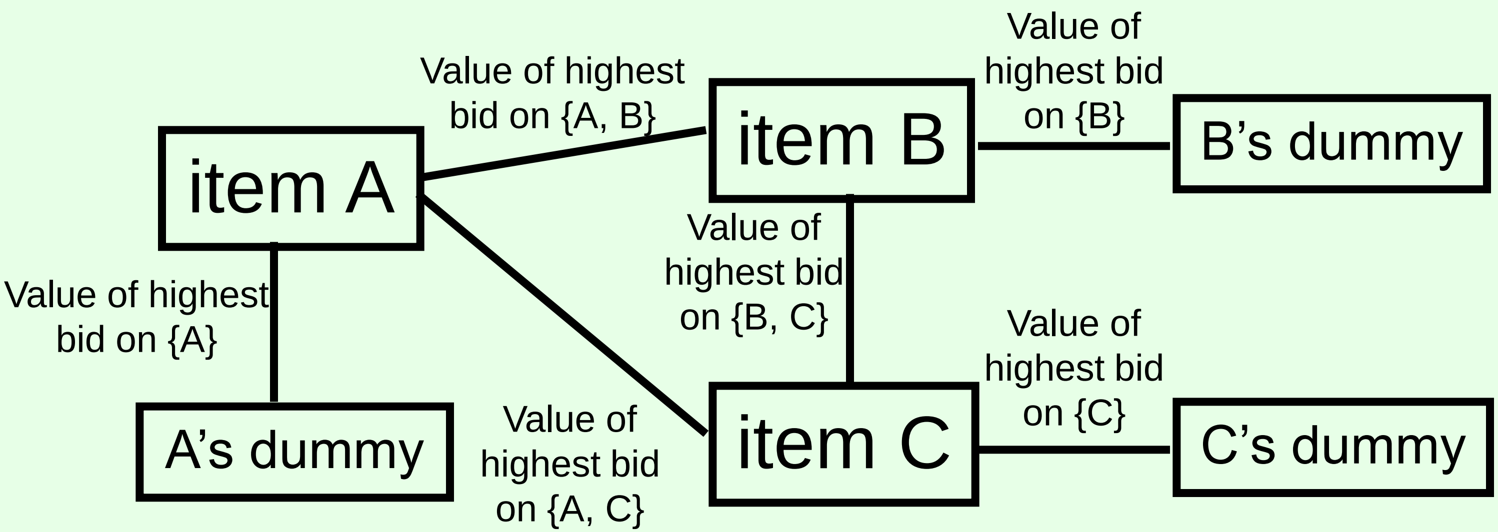


- Choose subset of the edges with maximum total weight,
- Constraint: no two edges can share a vertex

- Still solvable in polynomial time

Bids with few items [Rothkopf et al. 98]

- If each bid is on a bundle of at most two items,
- then the winner determination problem can be solved in polynomial time as a maximum weighted matching problem
  - 3-item example:



- If each bid is on a bundle of three items, then the winner determination problem is NP-hard again

Variants [Sandholm et al. 2002]: combinatorial reverse auction

- In a combinatorial reverse auction (CRA), the auctioneer seeks to buy a set of items, and bidders have values for the different bundles that they may sell the auctioneer
- ***minimize*** $\boldsymbol{\Sigma_b v_b x_b}$
- subject to
  - for each item j, $\boldsymbol{\Sigma_{b:\, j \text{ in } b}\, x_b \geq 1}$

Matching Online
&
Google Adwords

basato sulla survey:

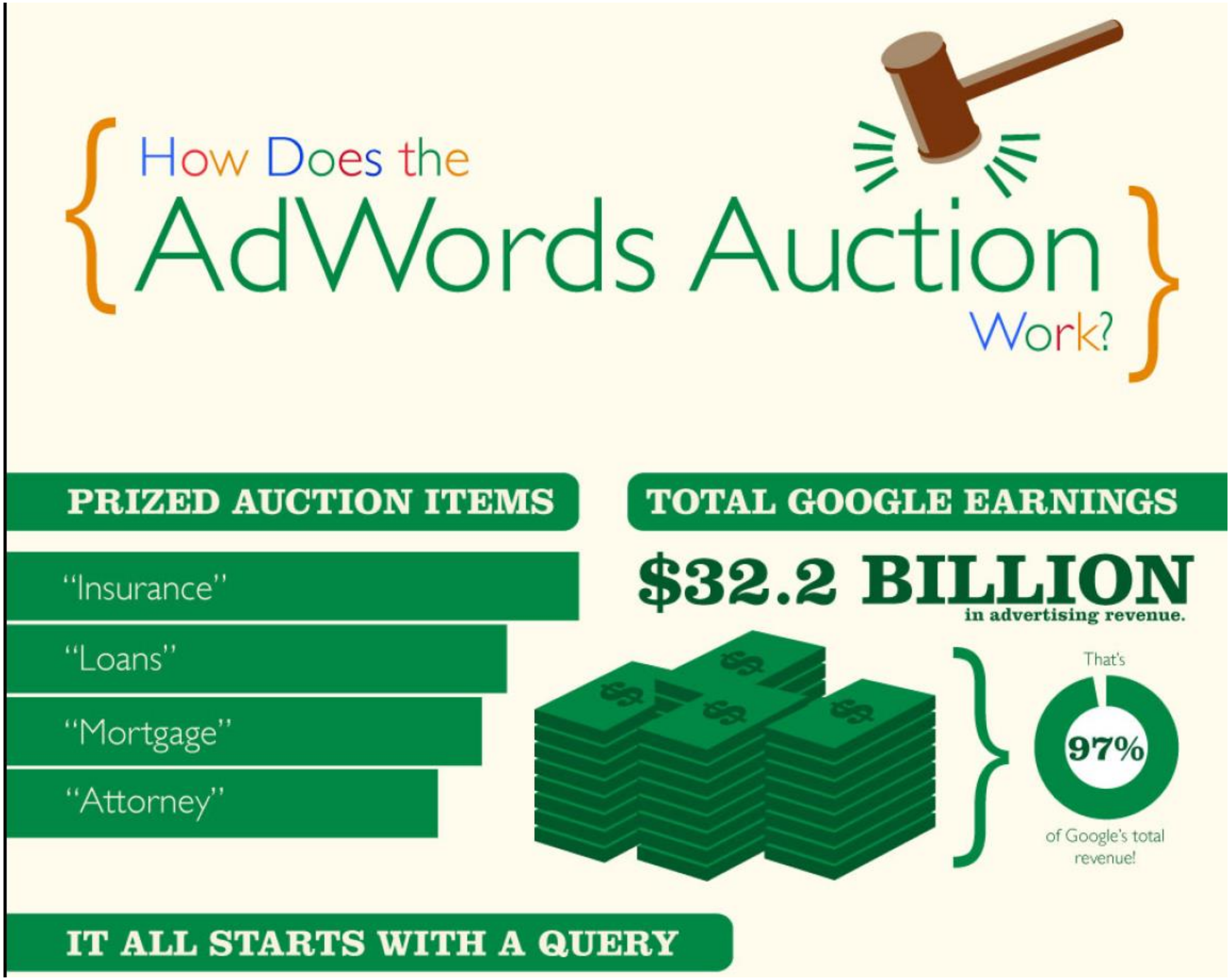
Foundations and Trends® in
Theoretical Computer Science
Vol. 8, No. 4 (2012) 265–368

DOI: 10.1561/0400000057

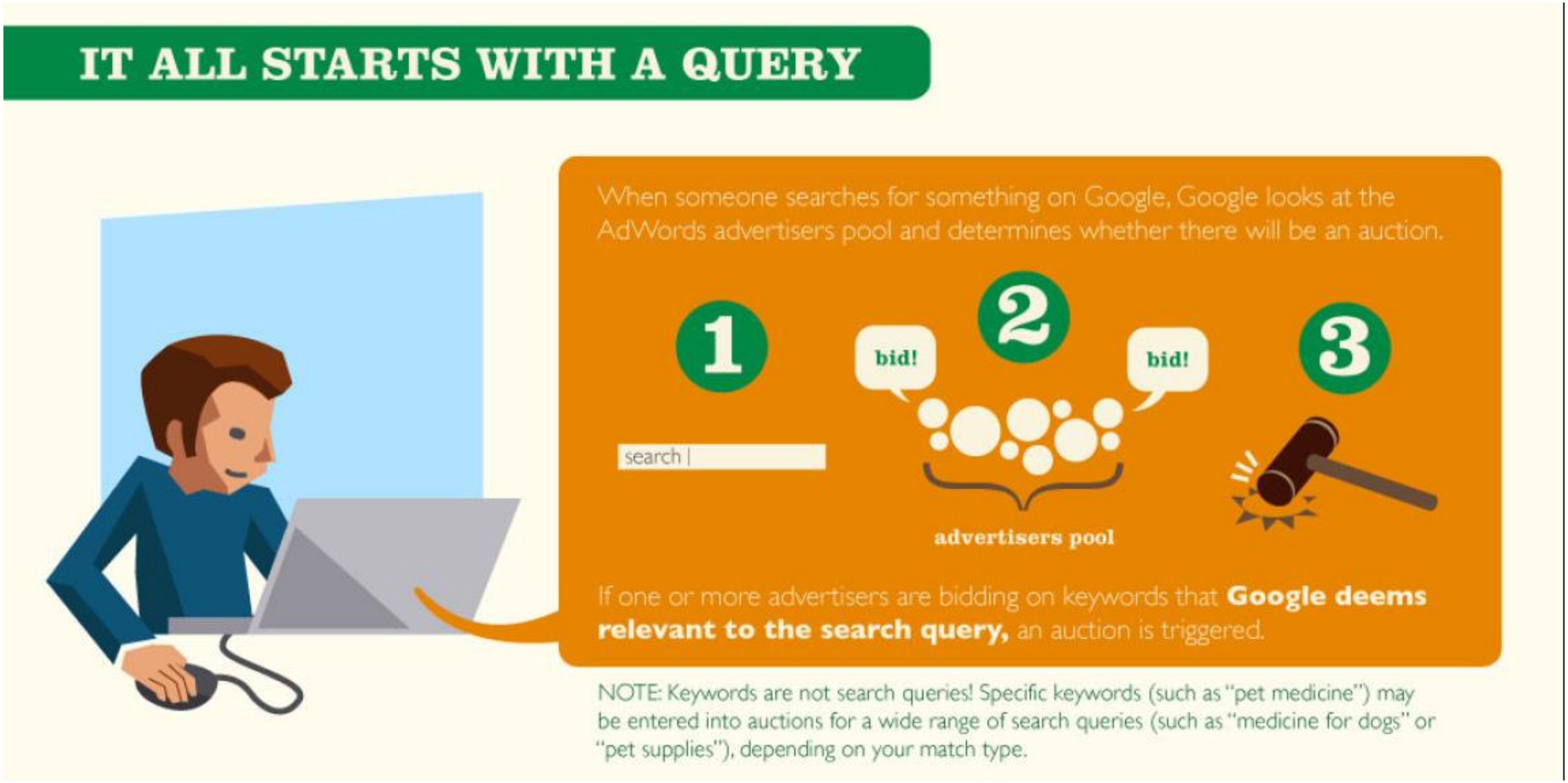
Online Matching and Ad Allocation

By Aranyak Mehta

AdWords I

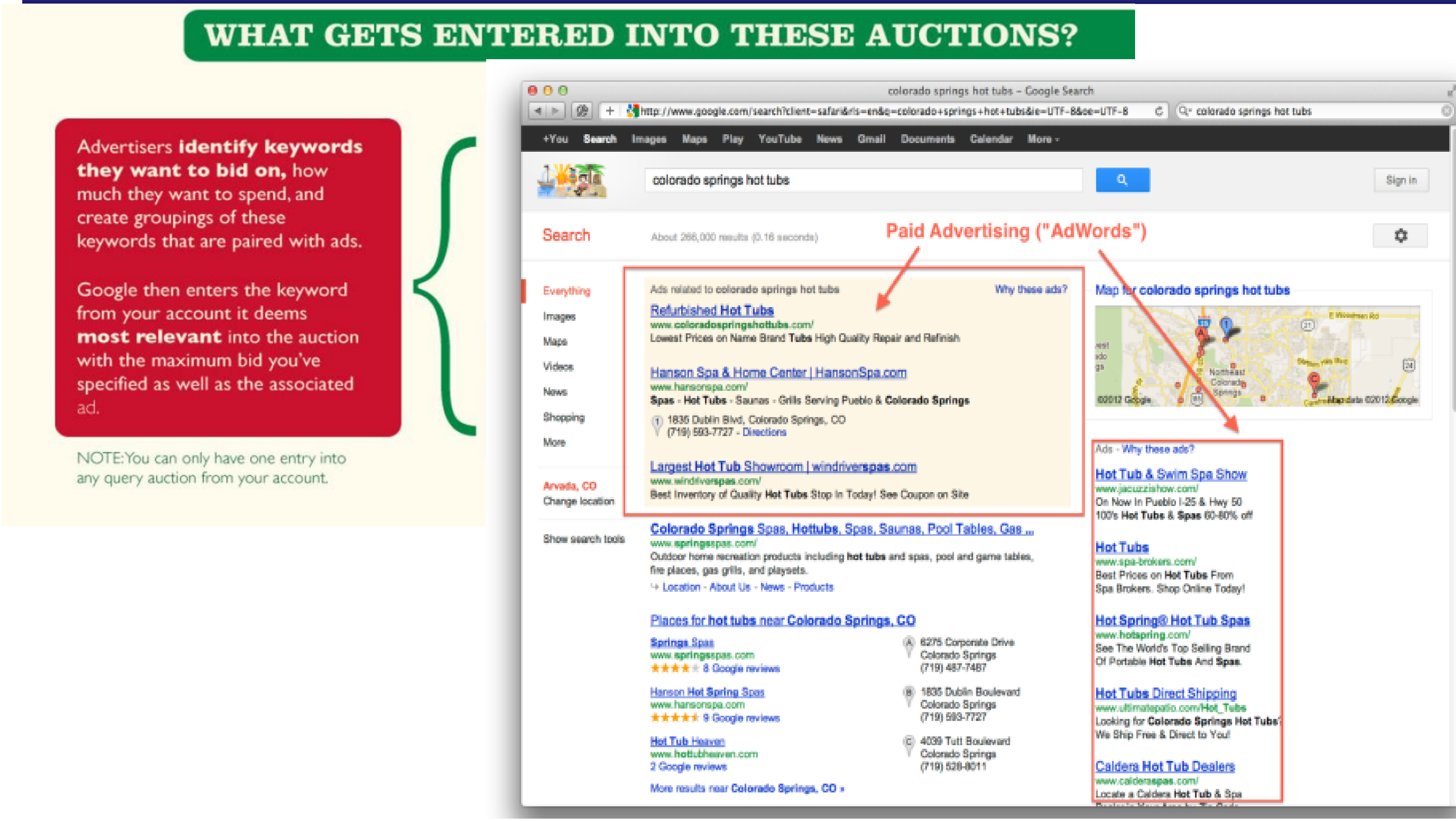

How Does the
AdWords Auction
Work?
PRIZED AUCTION ITEMS
"Insurance"
"Loans"
"Mortgage"
"Attorney"
TOTAL GOOGLE EARNINGS
$32.2 BILLION
in advertising revenue.
That's
97%
of Google's total revenue!
IT ALL STARTS WITH A QUERY

AdWords II



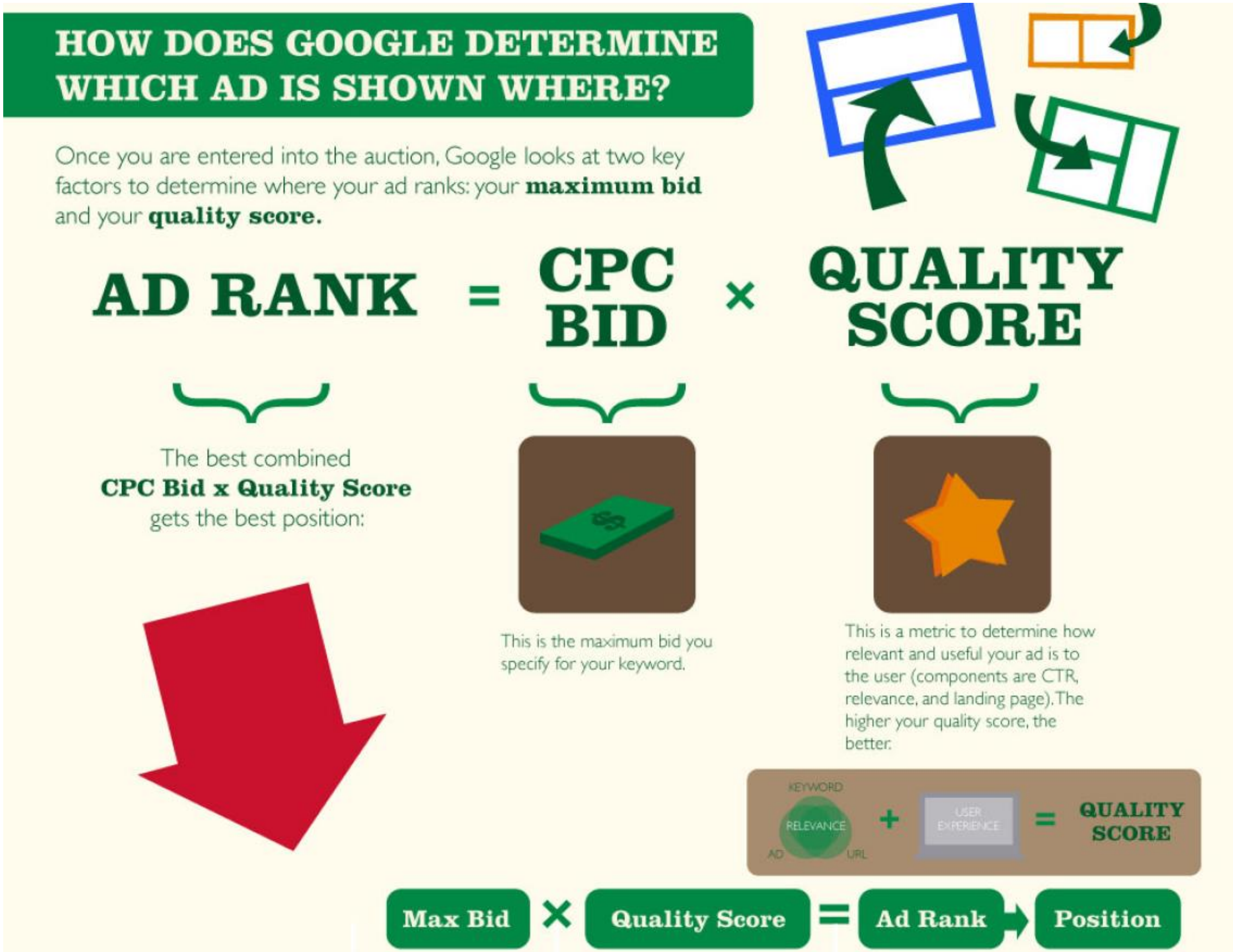
- L'asta si svolge ogni volta che qualcuno effettua una ricerca (!)
- L'offerta (***bid***) indica quanto si offre per un click (***CPC***)
- ***GOOGLE*** decide cosa è rilevante (sulla base di regole note)

AdWords III

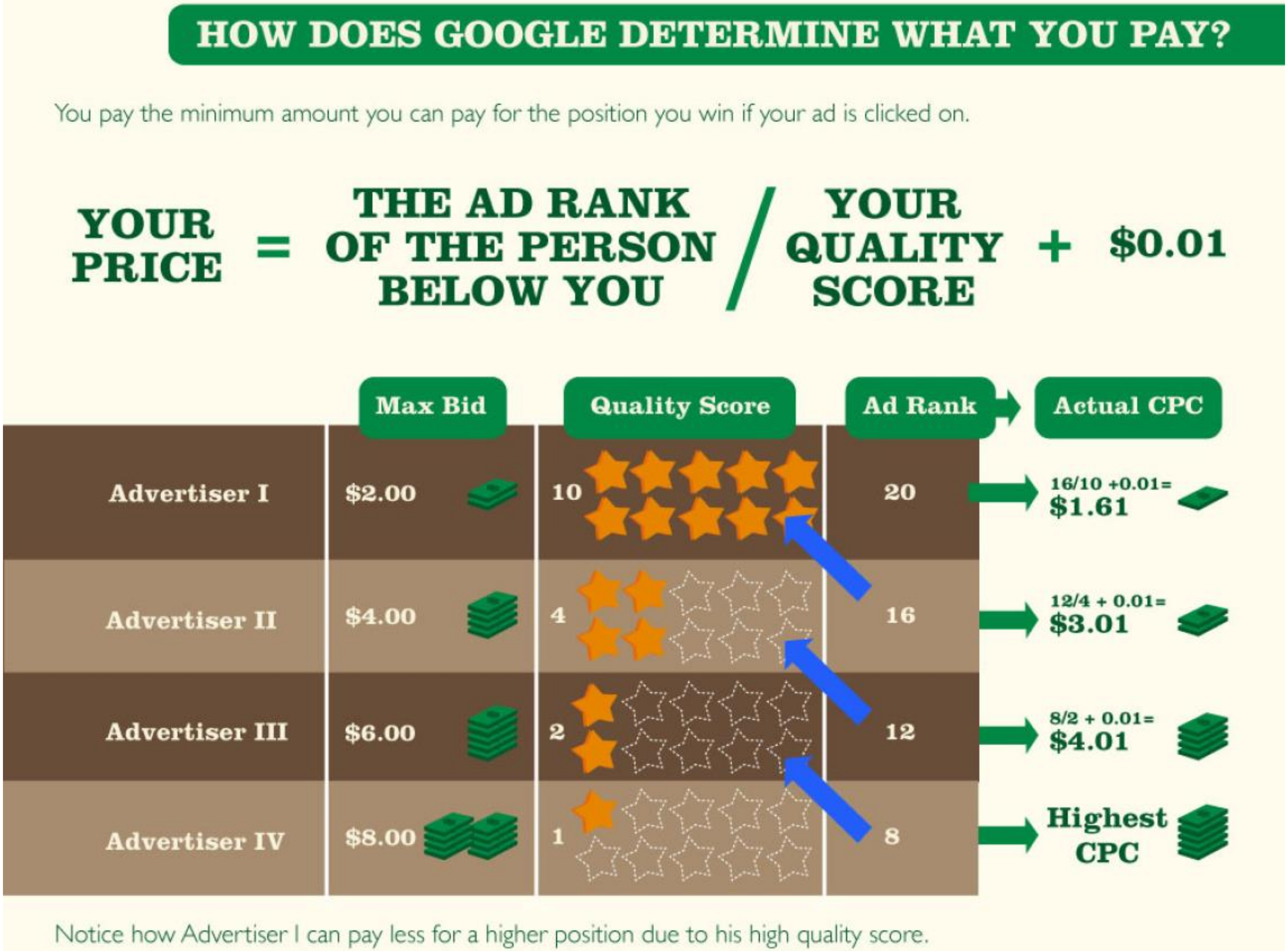


- I *vincitori* ottengono una posizione nella ***pagina dei risultati***
- *Pagheranno* *sulla base* del bid fino ad un ***budget massimo***
- ***GOOGLE*** decide cosa è rilevante (sulla base di regole note)

AdWords IV


HOW DOES GOOGLE DETERMINE WHICH AD IS SHOWN WHERE?
Once you are entered into the auction, Google looks at two key factors to determine where your ad ranks: your maximum bid and your quality score.
AD RANK
=
CPC BID
×
QUALITY SCORE
The best combined CPC Bid x Quality Score gets the best position:
This is the maximum bid you specify for your keyword.
This is a metric to determine how relevant and useful your ad is to the user (components are CTR, relevance, and landing page). The higher your quality score, the better.
KEYWORD
RELEVANCE
AD
URL
+
USER EXPERIENCE
=
QUALITY SCORE
Max Bid
×
Quality Score
=
Ad Rank
Position

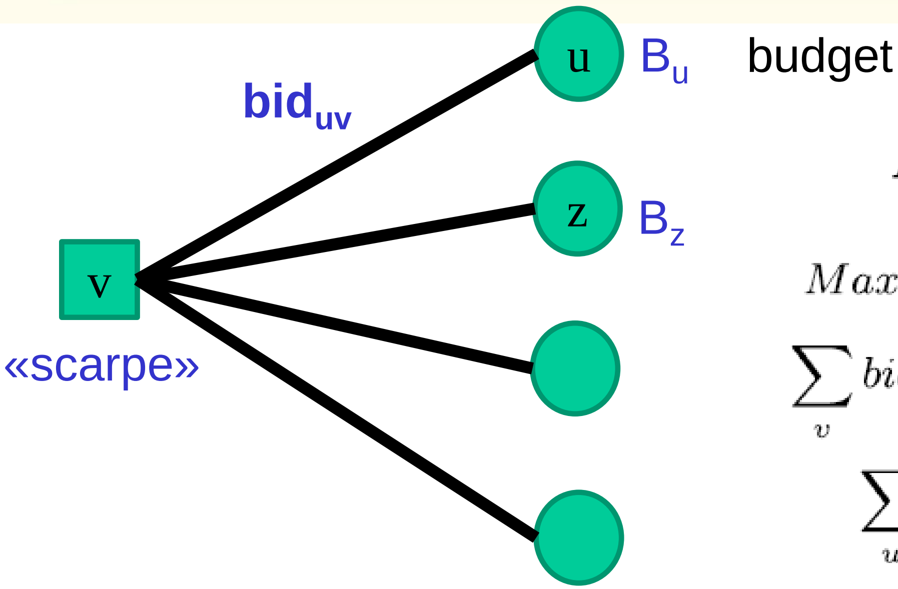
AdWords V



HOW DOES GOOGLE DETERMINE WHAT YOU PAY?
You pay the minimum amount you can pay for the position you win if your ad is clicked on.
YOUR PRICE = THE AD RANK OF THE PERSON BELOW YOU / YOUR QUALITY SCORE + $0.01
Max Bid
Quality Score
Ad Rank
Actual CPC
Advertiser I
$2.00
10
20
16/10 +0.01=
$1.61
Advertiser II
$4.00
4
16
12/4 + 0.01=
$3.01
Advertiser III
$6.00
2
12
8/2 + 0.01=
$4.01
Advertiser IV
$8.00
1
8
Highest CPC
Notice how Advertiser I can pay less for a higher position due to his high quality score.

AdWords VI

> The Auction gets run billions of times each month. The results are such that
> - Users find ads that are relevant to what they're looking for
> - Advertisers connect with potential customers at lowest possible prices
> - Google rakes in billions of dollars in revenue



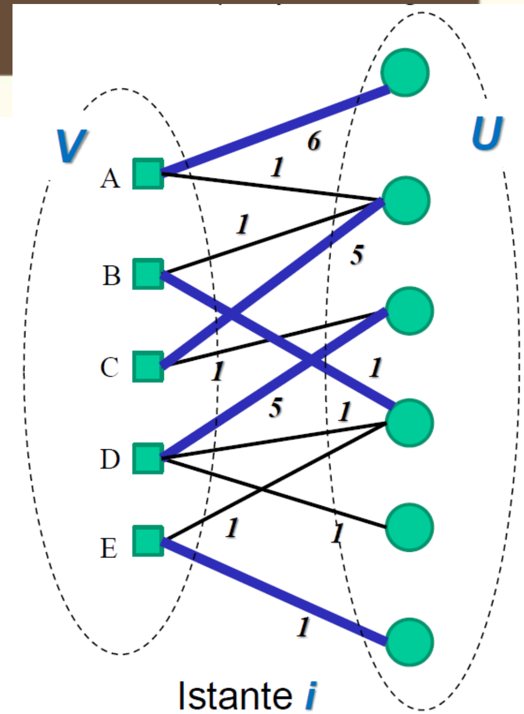
Adwords

$$Max \sum bid_{uv} x_{uv}$$

$$\sum_v bid_{uv} x_{uv} \le B_u \quad \forall u$$

$$\sum_u x_{uv} \le 1 \quad \forall v$$

$$x_{uv} \in \{0,1\}$$

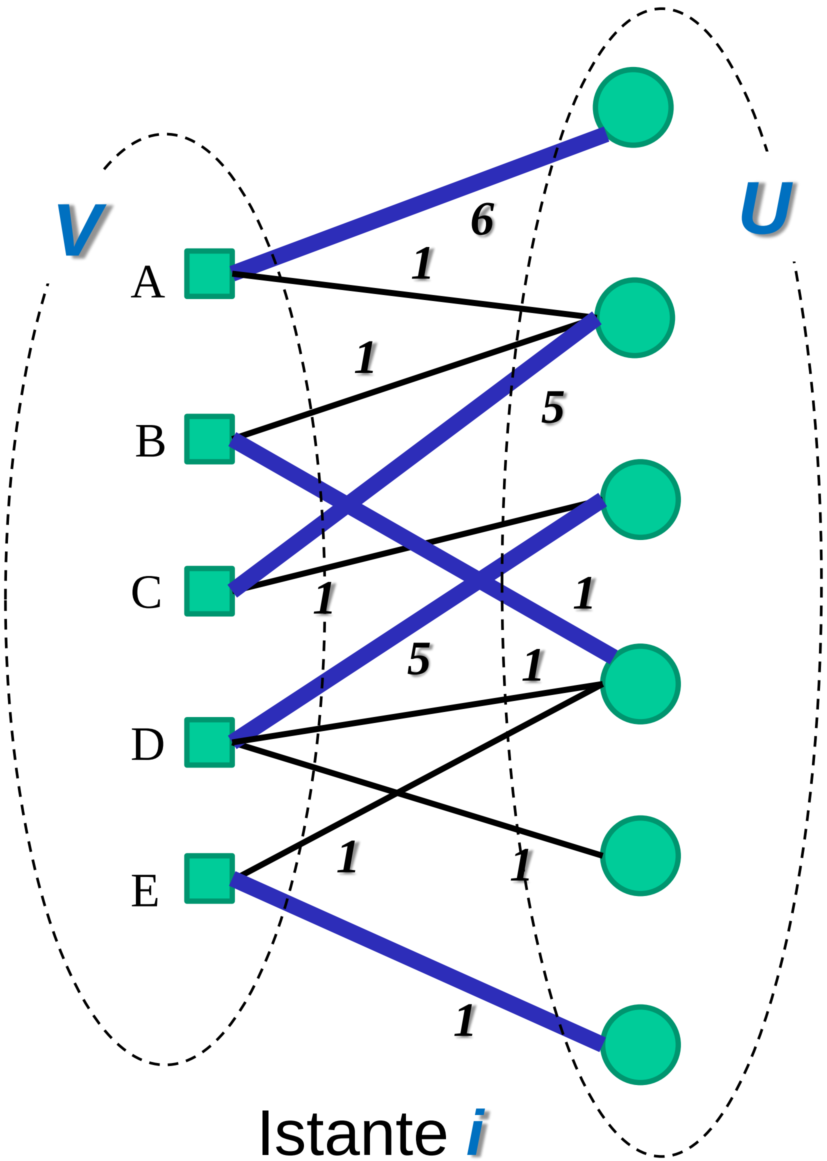


- **Intersezione** di due Sistemi di indipendenza
- ***Matching ∩ Knapsack (a coefficienti positivi)***
- ***Intersezione delle formulazioni***

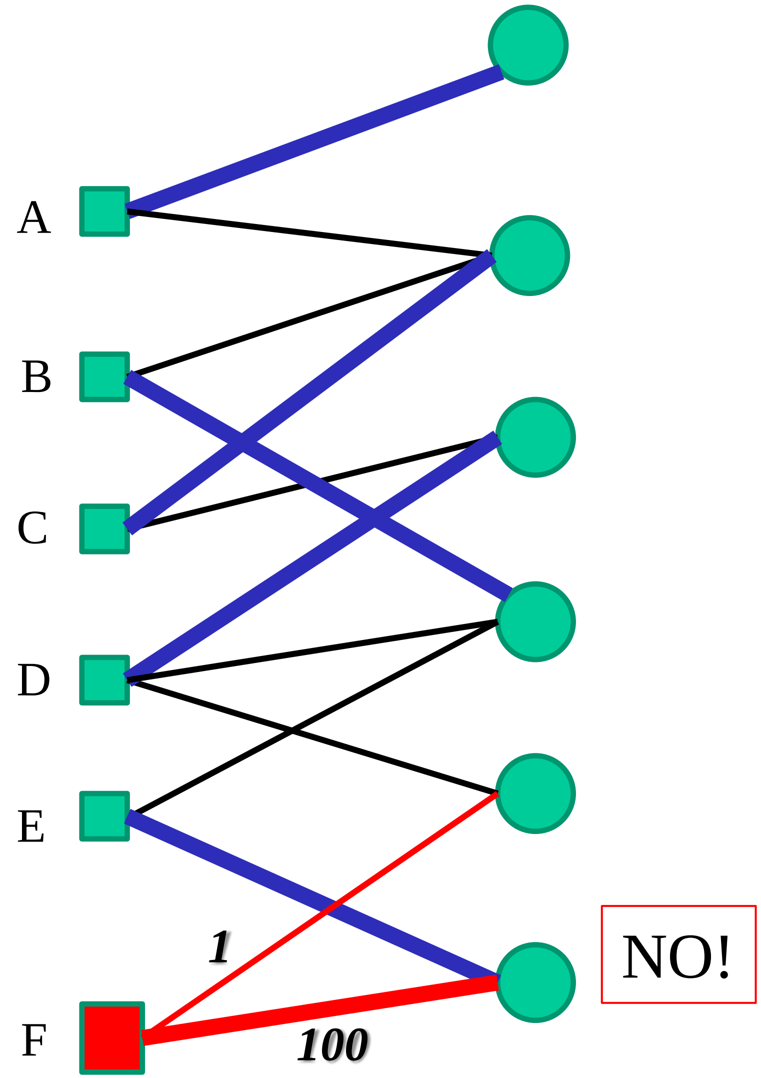
- ***Formulazione Naturale*** del *Knapsack*
- ***Formulazione Rango*** del *Matching*

Matching Online

I nodi di ***V*** (e i pesi degli archi incidenti) **noti in istanti successivi**



Istante ***i***



Istante ***i+1***

Progetto dei ritardi di una Rete OFDM

(rete digitale terrestre SFN)

La tecnologia OFDM

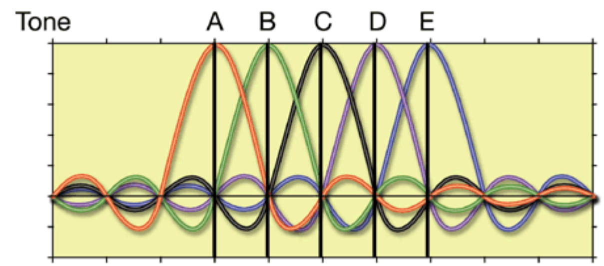
***O**rthogonal **F**requency **D**ivision **M**ultiplexing*

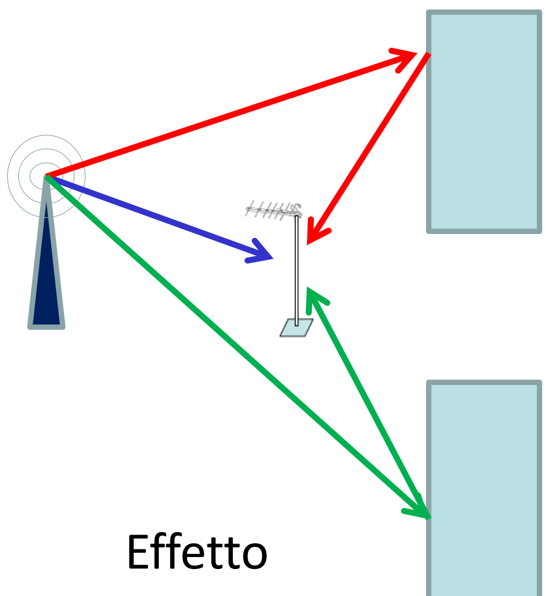
Decompone un bitstream in *bitstream* ***paralleli*** *ma* ***più "lenti"***

1 Megabit/s → **T** stream di **1/T** Megabit/s

Stream **A**	S1	S2	S8	S8	S7	S2	S9
Stream **B**							
Stream **C**							
Stream **D**							
Stream **E**							

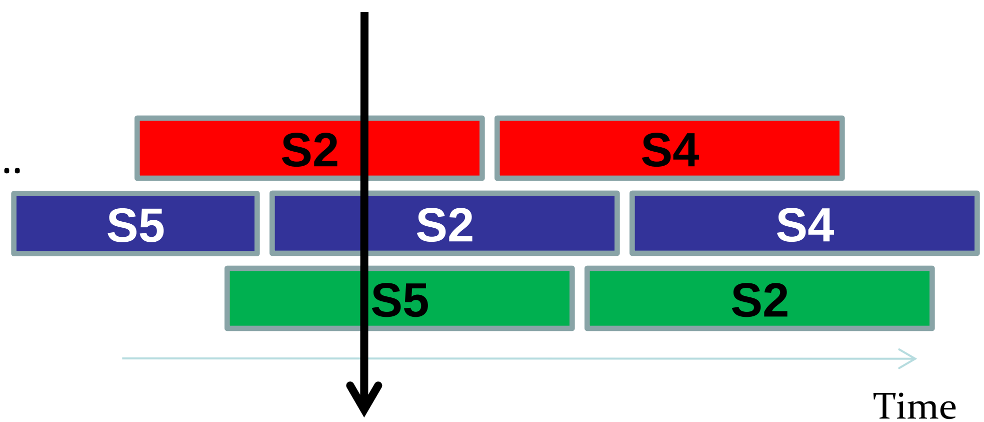
Simboli





***Stesso stream** …*

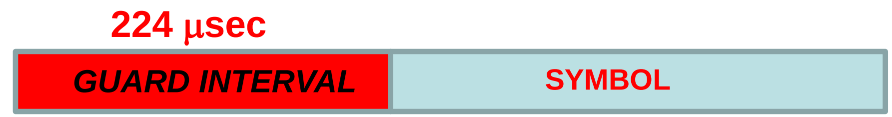
*… ma **ritardato***



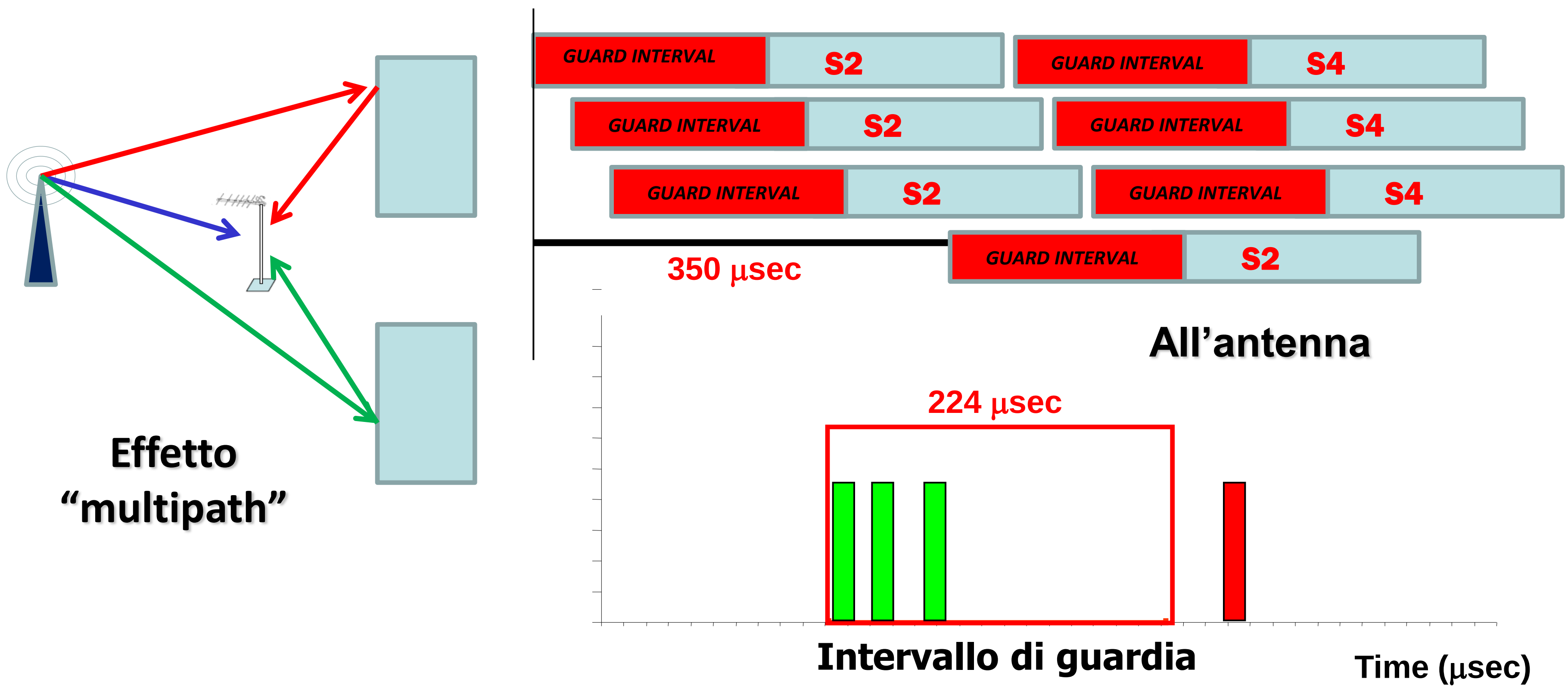
Effetto "multipath"

Interferenza Inter-Simbolo (S2 o S5??)

Intervallo di guardia



- *L'OFDM aggiunge un **prefisso** ad ogni simbolo*
- *Se il **prefisso** è **più lungo del ritardo***
- *.. si può evitare **l'interferenza intersimbolo***



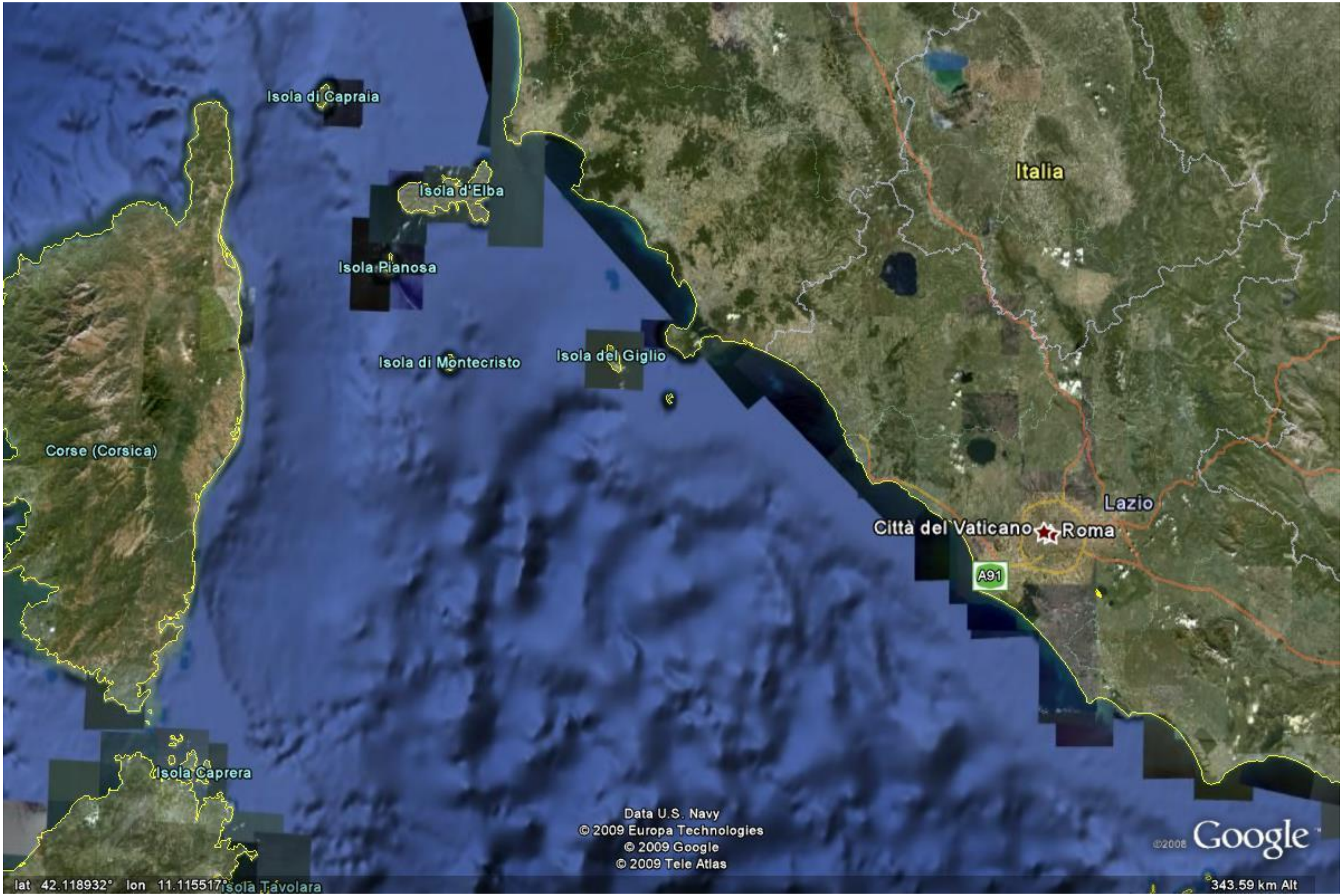
OFDM: Conclusions

- *Better spectral efficiency (more bits per Hz)*
- *Better use of spectrum (SFN)*
- *Resilience to "selective frequency fading"*
- *Turns Multipath from "bad" to "good"*

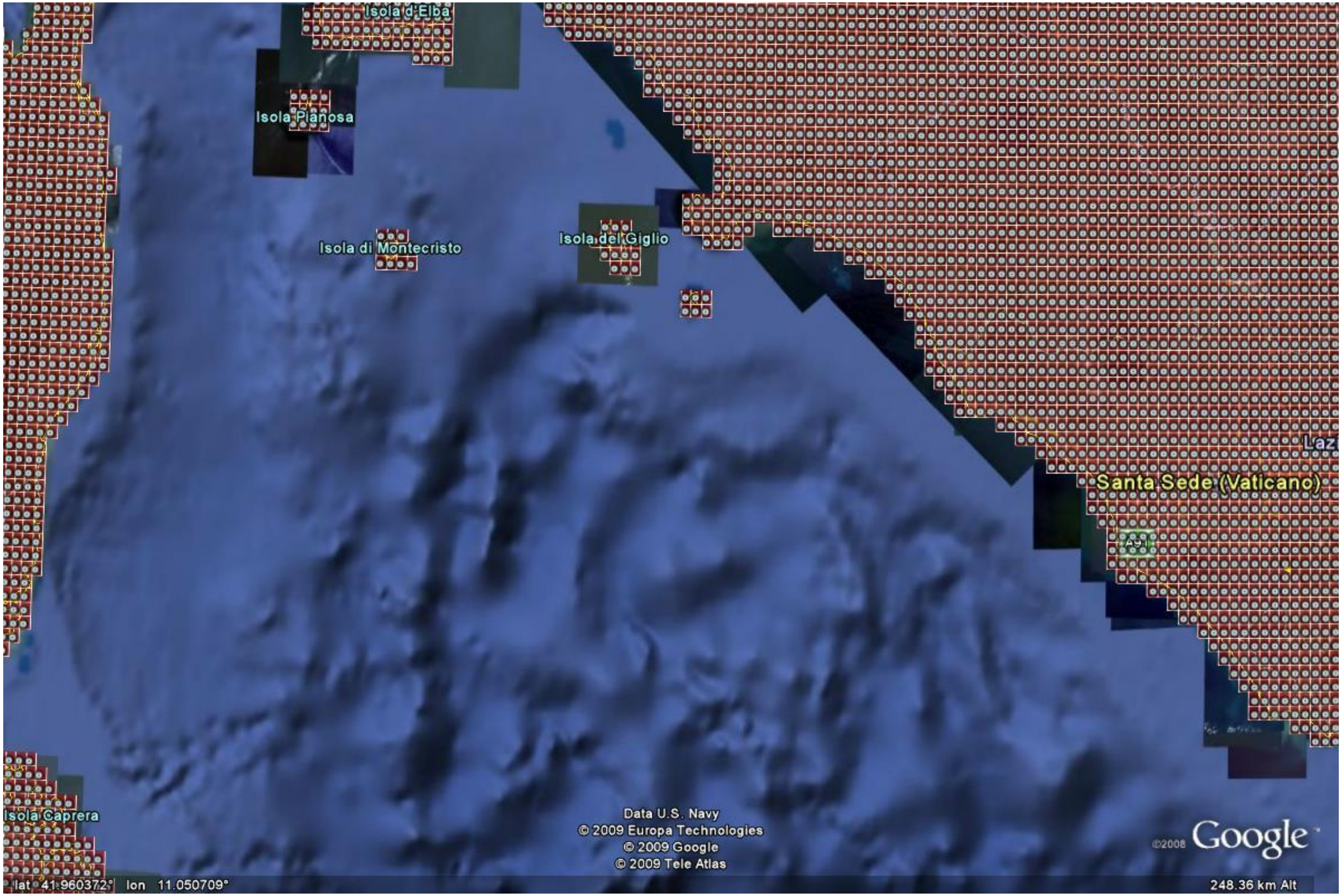
And in fact …. It will be the basic technology for:

- ***Digital Broadcasting***
- ***Wimax** (WiBro)*
- ***LTE** (Long Term Evolution of 3G)*
- *… 4^{th} **generation** (Anytime, Anywhere …)*
- *Short Range (UltraWideBand)*

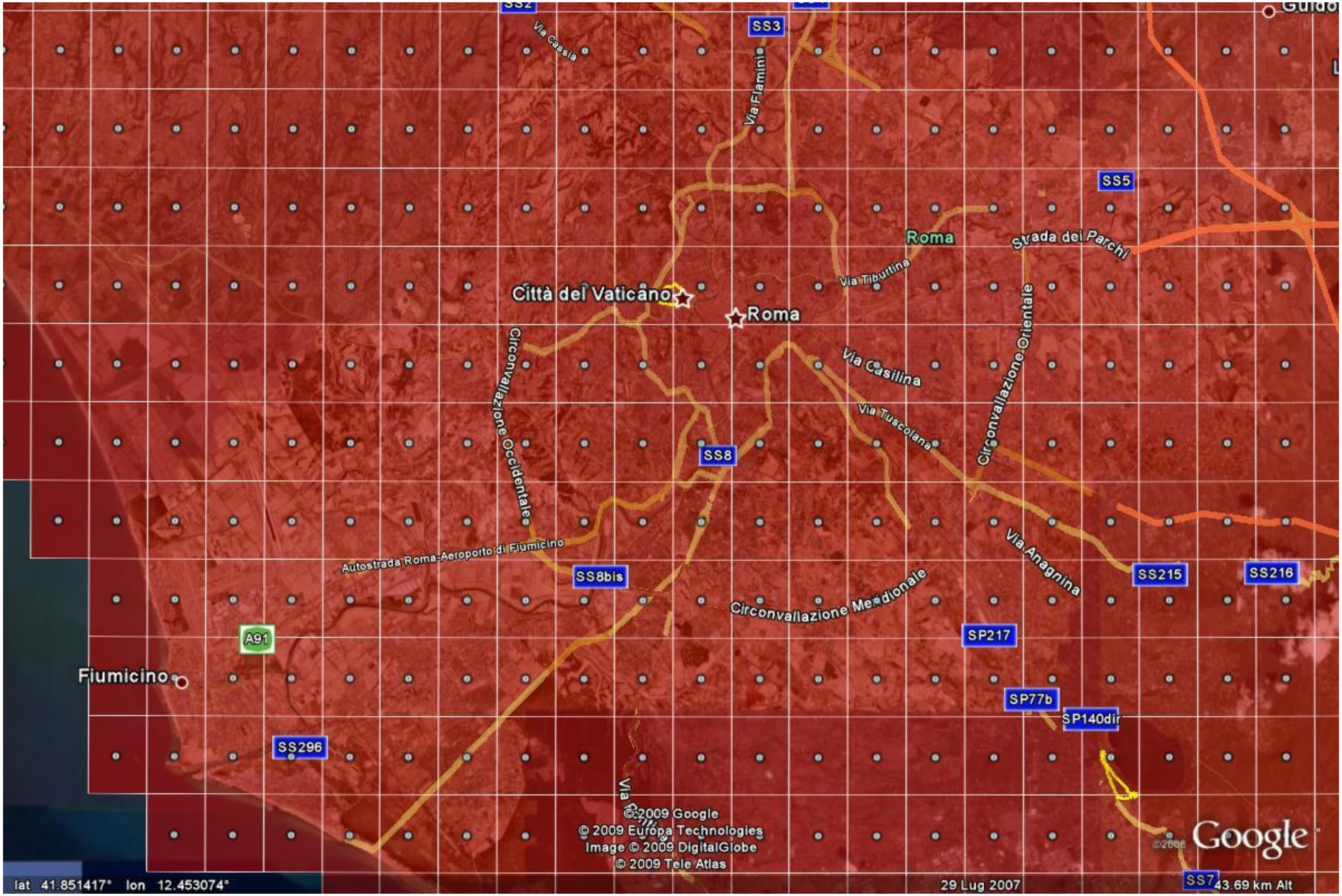
Si vuole *servire* in tecnologia digitale terrestre una specifica *area geografica*



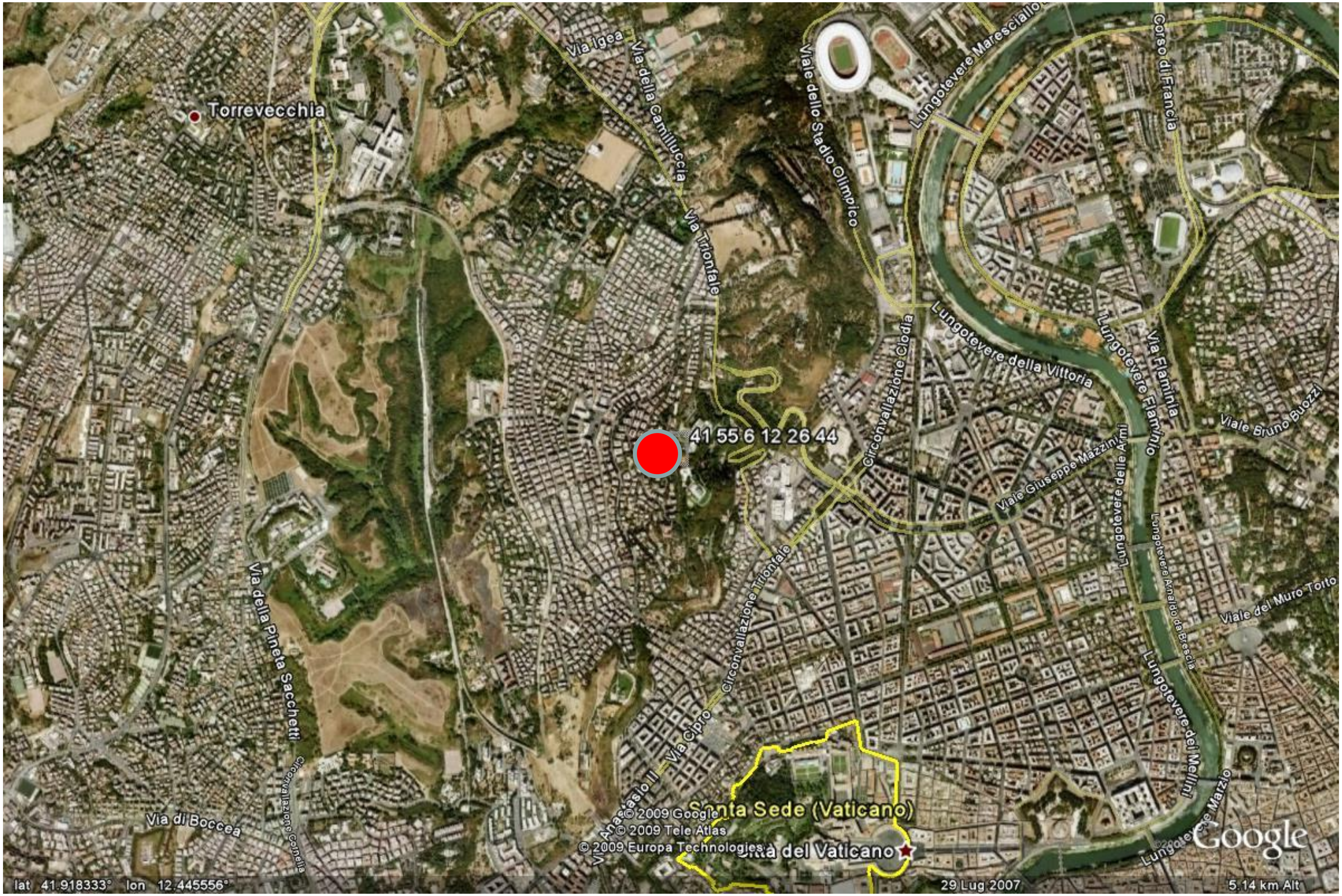
L'area geografica viene *rappresentata* come una *griglia di rettangoli* di lato *96 secondi* *(di grado)*



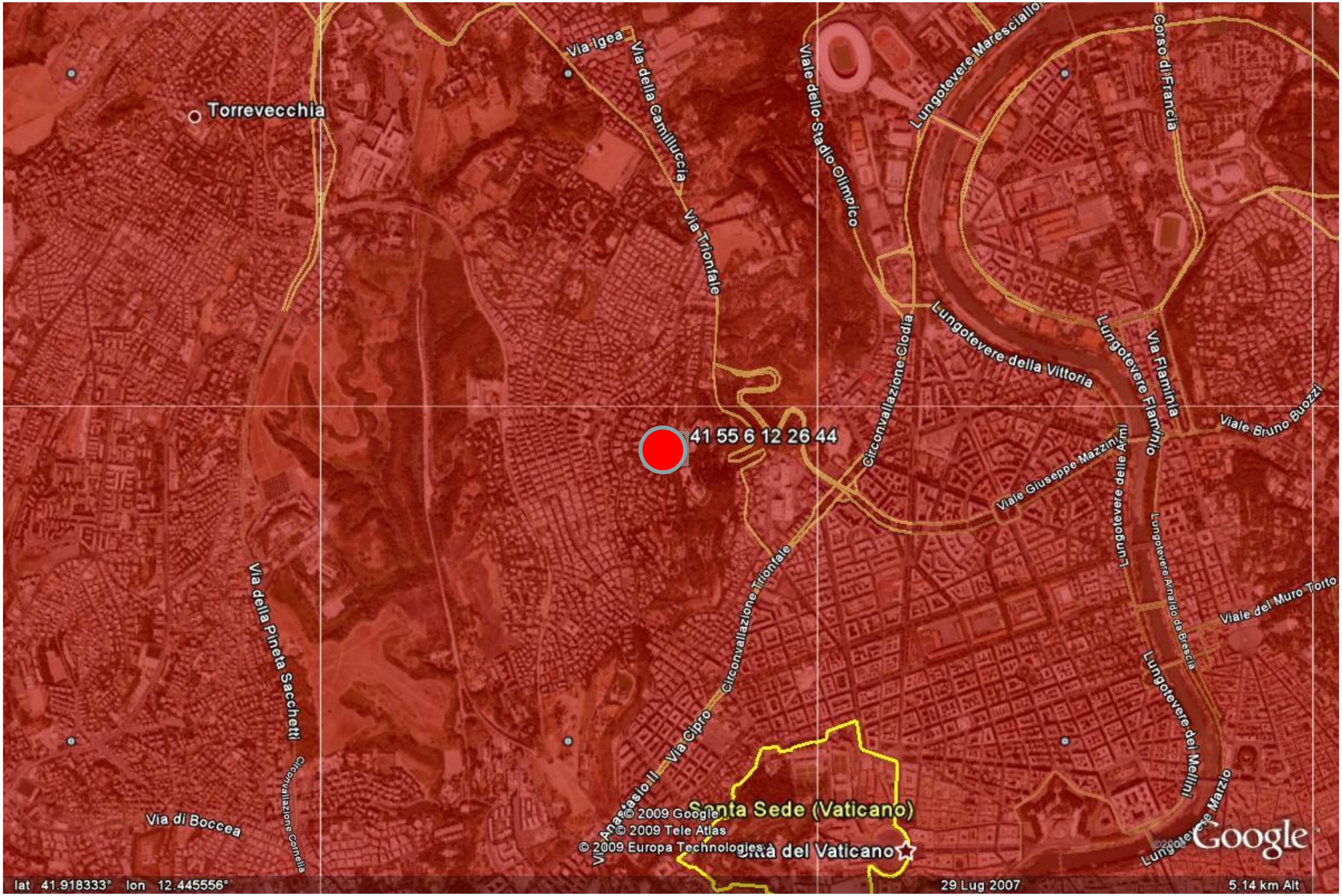
Ogni rettangolo della griglia è rappresentato dal suo baricentro: il *testpoint (o Punto di Verifica – PDV)*



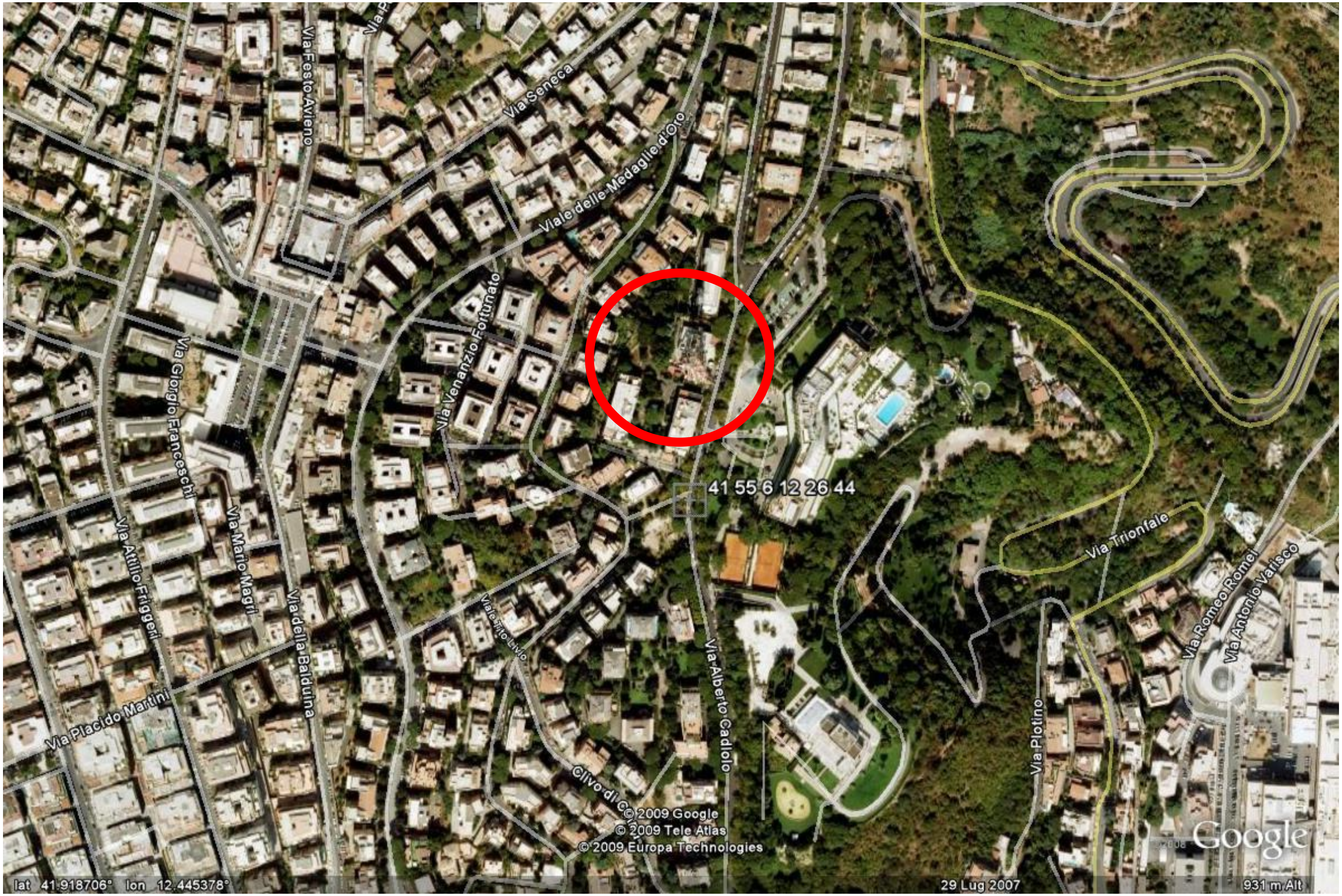
Ogni rettangolo della griglia è rappresentato dal suo baricentro: il *testpoint*



I siti di trasmissione sono anch'essi posizionati sulla "griglia": ad esempio Roma Monte Mario



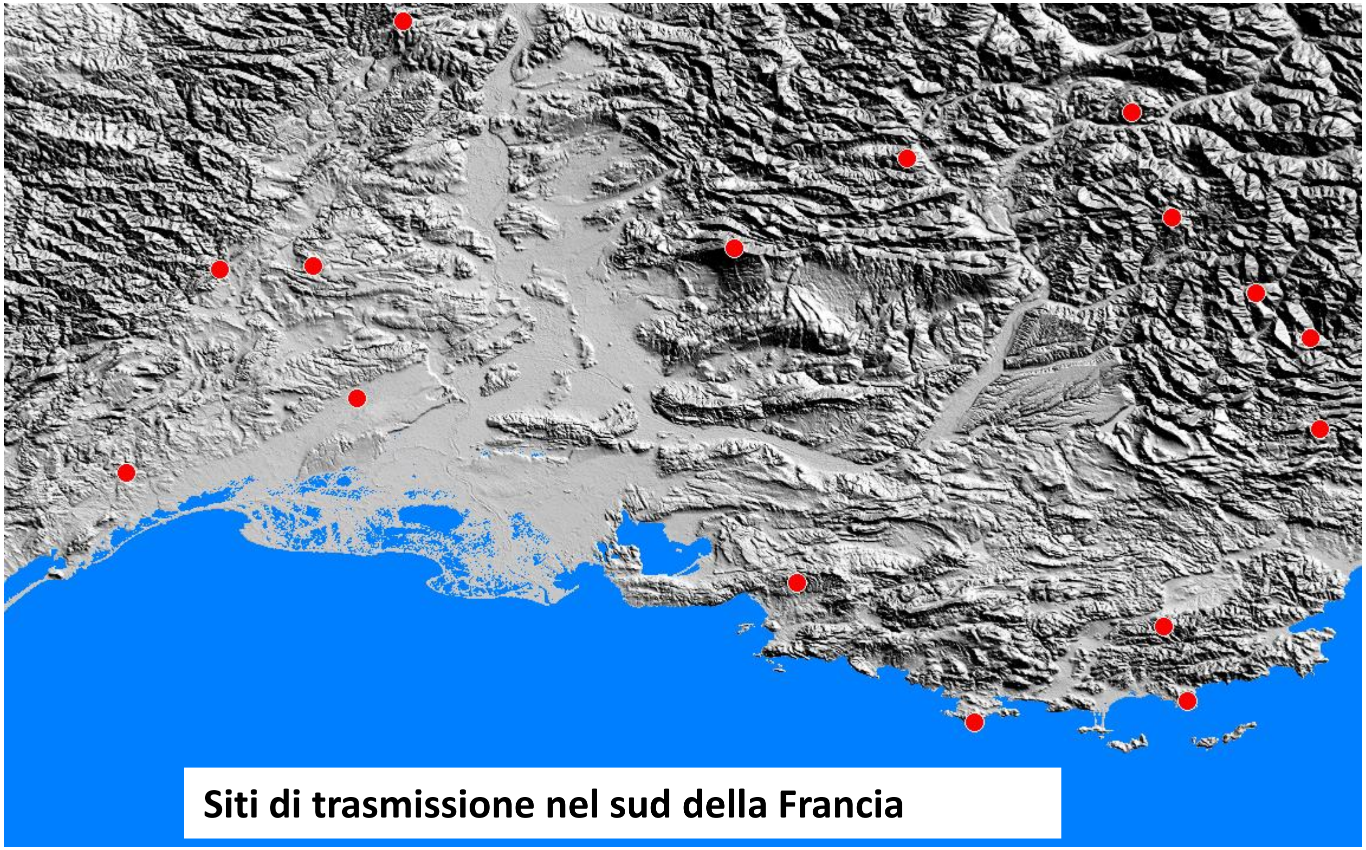
Sito candidato di Roma Monte Mario



Problema: Ritardi Ottimi di una rete OFDM

INPUT

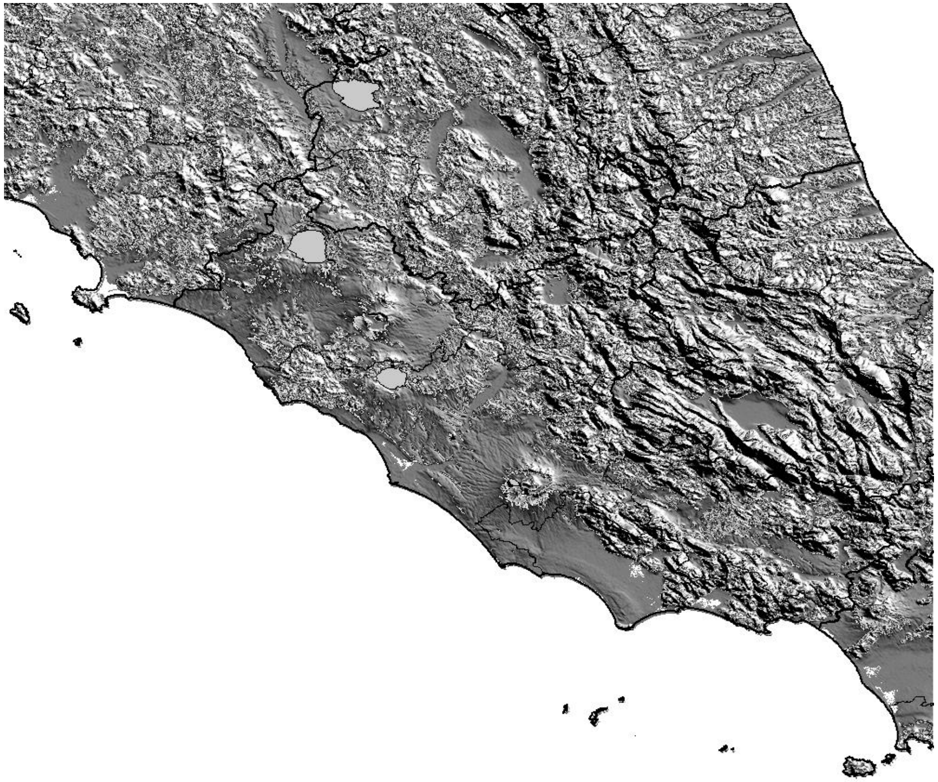
- Digital Terrain Model (**300.000 testpoint** in Europa)
- **Informazioni su posizione, quota e tipo di vegetazione**
- **Siti dei trasmettitori (35.000 in Europa)**

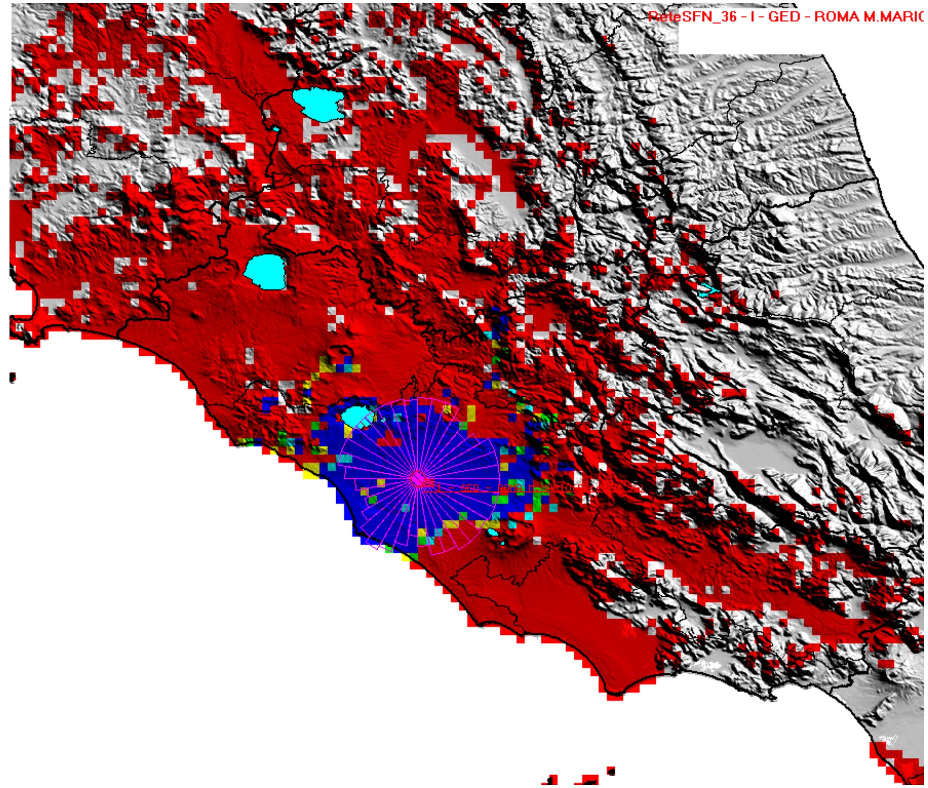


SRTM DTM (*Nasa*)

- 1 tp = **6**x**6** secondi
- 1 tp ≈ **180**x**180** mt

La propagazione elettromagnetica



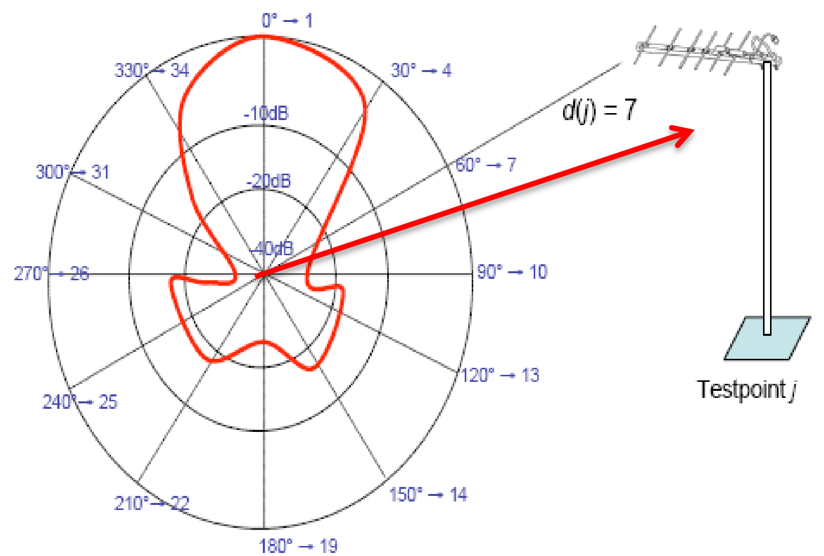


- In ogni testpoint possiamo calcolare la potenza dei segnali trasmessi
- Il calcolo viene effettuato utilizzando modelli di simulazione certificati dalla ***International Telecommunication Union*** (**ITU**)
- **Il modello di simulazione tiene conto della distanza e dell'orografia**
- **La potenza ricevuta dipende alla potenza emessa**

Come verificare se un PdV è “servito”

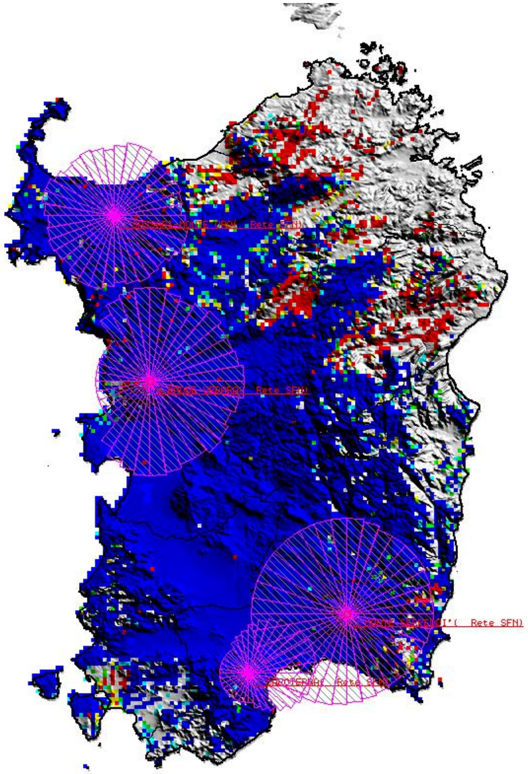
- In ciascun PdV la qualità del servizio è una funzione della:
  - Potenza ricevuta dai segnali **serventi**
  - Potenza ricevuta dai segnali **interferenti**

- La potenza ricevuta in un testpoint è una funzione dei seguenti fattori:
  - **distanza**
  - **terreno** (orografia, palazzi, vegetazione…)
  - **guadagno dell’antenna** del settore “visto” dall’antenna ricevente



La funzione obiettivo

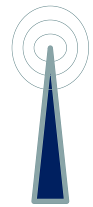
Massimizzare l'estensione del servizio ottenibile dai siti candidati **calcolando** opportunamente i **ritardi di trasmissione di tutti i trasmettitori**



Simulazione del Servizio

MODELLO DIGITALE TERRITORIO

DATI NOMINALI DEI TRASMETTITORI



PREDIZIONE DEL CAMPO ELETTROMAGNETICO

- **Interferenza nominale**
  - *in ciascun punto di verifica* j
  - *da ciascun trasmettitore* h

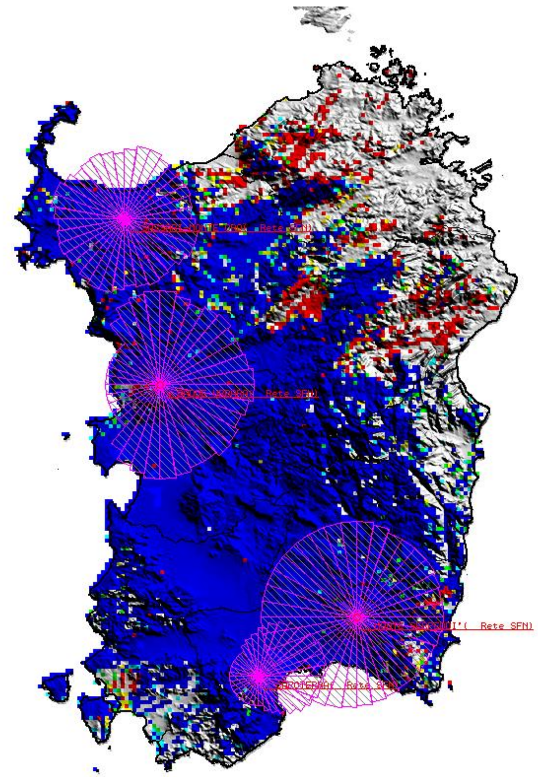
OTTIMIZZA I RITARDI

t_k

Interferenza effettiva

in ciascun punto di verifica j

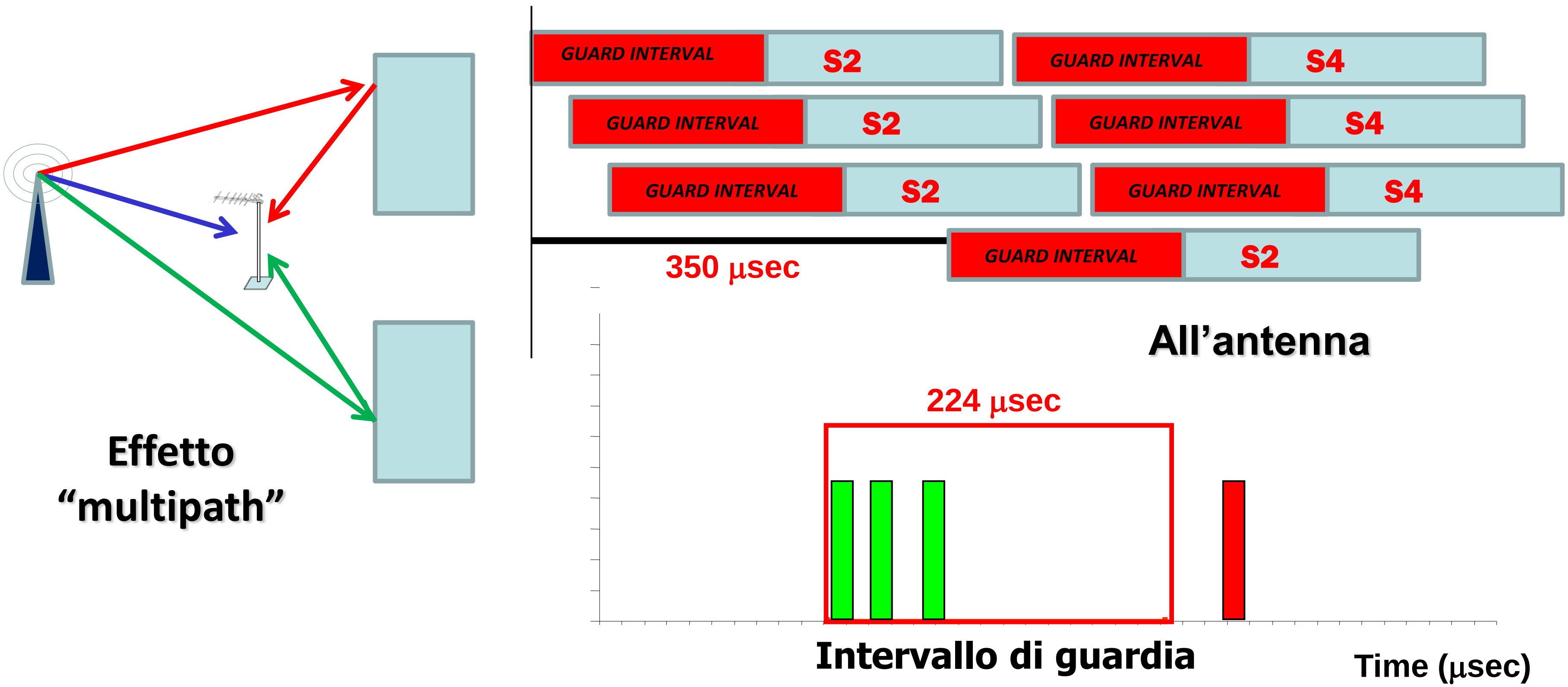
ALGORITMO DI PREDIZIONE DEL SERVIZIO OFDM



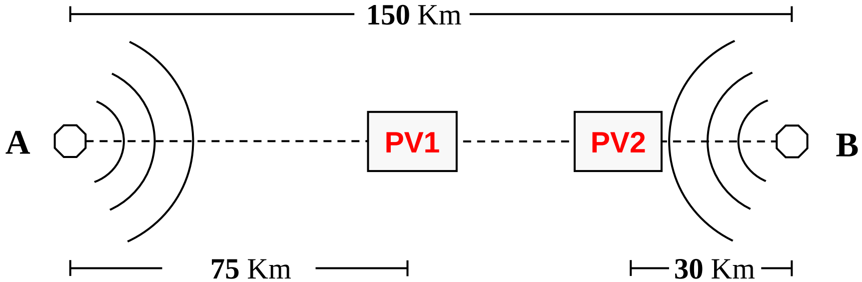
Come ottimizzare i ritardi artificiali

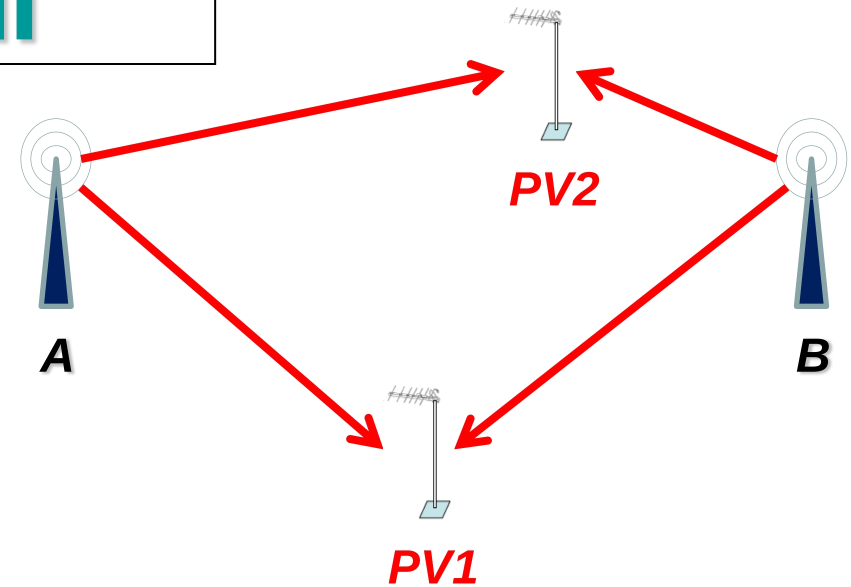
Intervallo di guardia

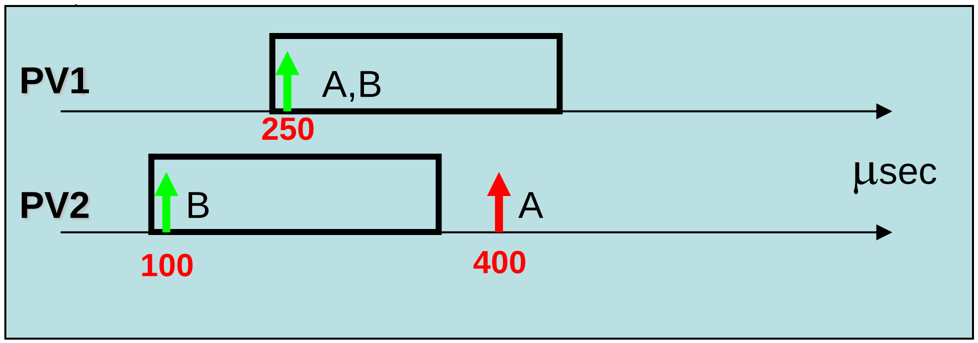
- *L'OFDM aggiunge un **prefisso** ad ogni simbolo*
- *Se il **prefisso** è **più lungo del ritardo***
- *Si può evitare **l'interferenza intersimbolo***



Ritardi artificiali

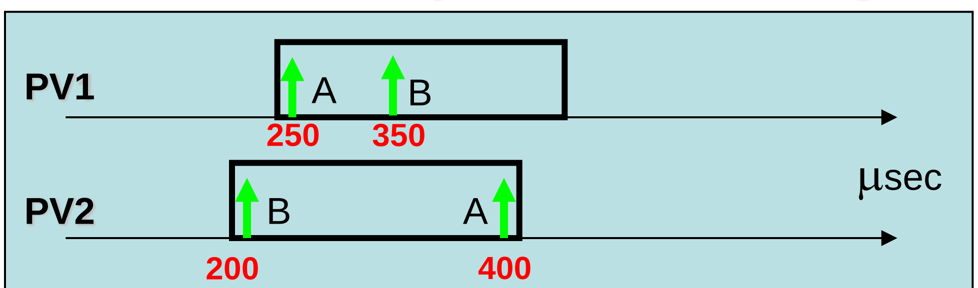




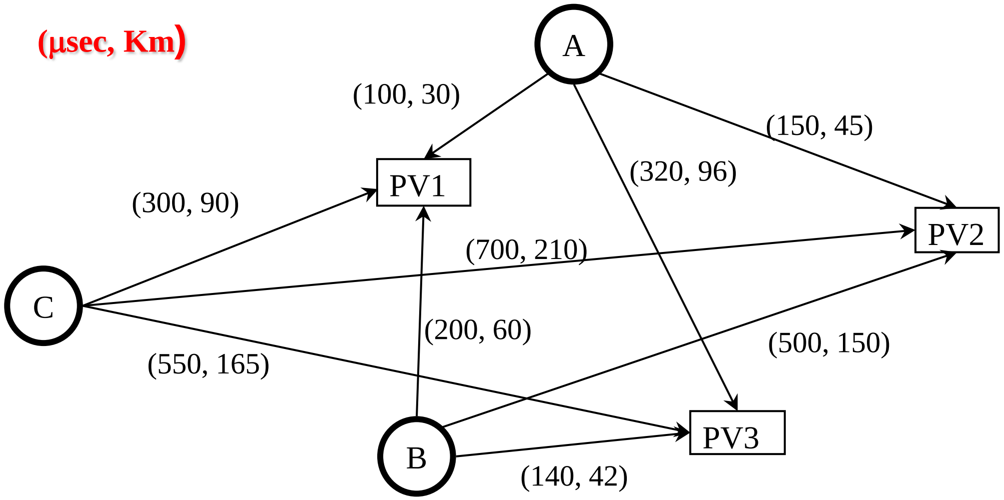


- *Due trasmettitori collocati in* ***siti diversi***
- *.. arrivano con ritardi diversi in* ***punti diversi del territorio***
- *.. in* ***PV2*** ***non c'è servizio*** *(interferenza) … che fare?*

- ***Ritardiamo artificialmente il segnale di B di 100 μsec !!***



Come assegnare i ritardi in modo ottimo?



$$t_k - t_h \le 224 - \Delta_{kj} + \Delta_{hj} = d^j_{hk}$$

$$t_h - t_k \le 224 - \Delta_{hj} + \Delta_{kj} = d^j_{kh}$$

PV1	$t_A - t_B \le 224 + 200 - 100 = 324$ $t_B - t_A \le 224 + 100 - 200 = 124$ $t_A - t_C \le 224 + 300 - 100 = 424$ $t_C - t_A \le 224 + 100 - 300 = 24$ $t_B - t_C \le 224 + 300 - 200 = 324$ $t_C - t_B \le 224 + 200 - 300 = 124$
PV2	$t_A - t_B \le 224 + 500 - 150 = 574$ $\mathbf{t_B - t_A \le 224 + 150 - 500 = -126}$ $t_A - t_C \le 224 + 700 - 150 = 774$ $t_C - t_A \le 224 + 150 - 700 = -326$ $t_B - t_C \le 224 + 700 - 500 = 424$ $t_C - t_B \le 224 + 500 - 700 = 24$
PV3	$\mathbf{t_A - t_B \le 224 + 140 - 320 = 44}$ $t_B - t_A \le 224 + 320 - 140 = 404$ $t_A - t_C \le 224 + 550 - 320 = 454$ $t_C - t_A \le 224 + 320 - 550 = -6$ $t_B - t_C \le 224 + 550 - 140 = 634$ $t_C - t_B \le 224 + 140 - 550 = -186$

TR = insieme dei **trasmettitori**

TP = insieme dei **punti di verifica** (testpoint)

τ_{kj} = **istante di arrivo** del segnale del trx k nel punto di verifica j

t_k = **ritardo del trx** k; Δ_{kj} = **distanza k - j (sec)**

$$\tau_{kj} = t_k + \Delta_{kj}$$

Nessuna interferenza

$$|\tau_{kj} - \tau_{hj}| \le 224 \qquad j \in TP \quad k, h \in TR$$

$$\tau_{kj} - \tau_{hj} = t_k + \Delta_{kj} - t_h - \Delta_{hj} \le 224$$

$$\tau_{hj} - \tau_{kj} = t_h + \Delta_{hj} - t_k - \Delta_{kj} \le 224 \qquad j \in TP \quad k, h \in TR$$

Per avere i ritardi basta risolvere questo sistema!

PV1	$t_A - t_B \le 224 + 200 - 100 = 324$
	$t_B - t_A \le 224 + 100 - 200 = 124$
	$t_A - t_C \le 224 + 300 - 100 = 424$
	$t_C - t_A \le 224 + 100 - 300 = 24$
	$t_B - t_C \le 224 + 300 - 200 = 324$
	$t_C - t_B \le 224 + 200 - 300 = 124$
PV2	$t_A - t_B \le 224 + 500 - 150 = 574$
	$\mathbf{t_B - t_A \le 224 + 150 - 500 = -126}$
	$t_A - t_C \le 224 + 700 - 150 = 774$
	$t_C - t_A \le 224 + 150 - 700 = -326$
	$t_B - t_C \le 224 + 700 - 500 = 424$
	$t_C - t_B \le 224 + 500 - 700 = 24$
PV3	$\mathbf{t_A - t_B \le 224 + 140 - 320 = 44}$
	$t_B - t_A \le 224 + 320 - 140 = 404$
	$t_A - t_C \le 224 + 550 - 320 = 454$
	$t_C - t_A \le 224 + 320 - 550 = -6$
	$t_B - t_C \le 224 + 550 - 140 = 634$
	$t_C - t_B \le 224 + 140 - 550 = -186$

E se non ha soluzione?

$$t_A \ge t_B + 126$$

$$t_A \le t_B + 44$$

- **Eliminare una disequazione**

→ **Due trasmettitori non sincronizzati sul PdV PV3**

→ **Popolazione (forse) non servita nel PdV PV3**

Problema: Trovare un insieme di disequazioni di peso minimo da rimuovere per rendere il sistema ammissibile

= **massimo sottosistema ammissibile**