Elective in Robotics

Quadrotor Modeling

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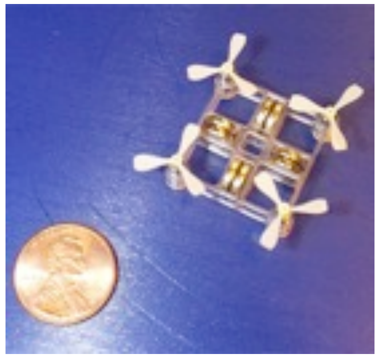
Sapienza
Università di Roma

- Introduction
- Modeling
- Control Problems
- Models for control
- Main control approaches

Hummingbird Ascending Technologies GmbH

CEA Quadrotor

DraganFlyer X4
Draganfly Innovations Inc.



Mesicopter Stanford University

applications

- surveying, maintenance
- aerial transportation, manipulation
- communication networks
- search and rescue operations

microdrones GmbH

STARMAC Stanford University

all these activities require
vertical, stationary, slow flight

a quadrotor is characterized by

- high maneuverability
- vertical take-off and landing (VTOL)
- hovering capabilities

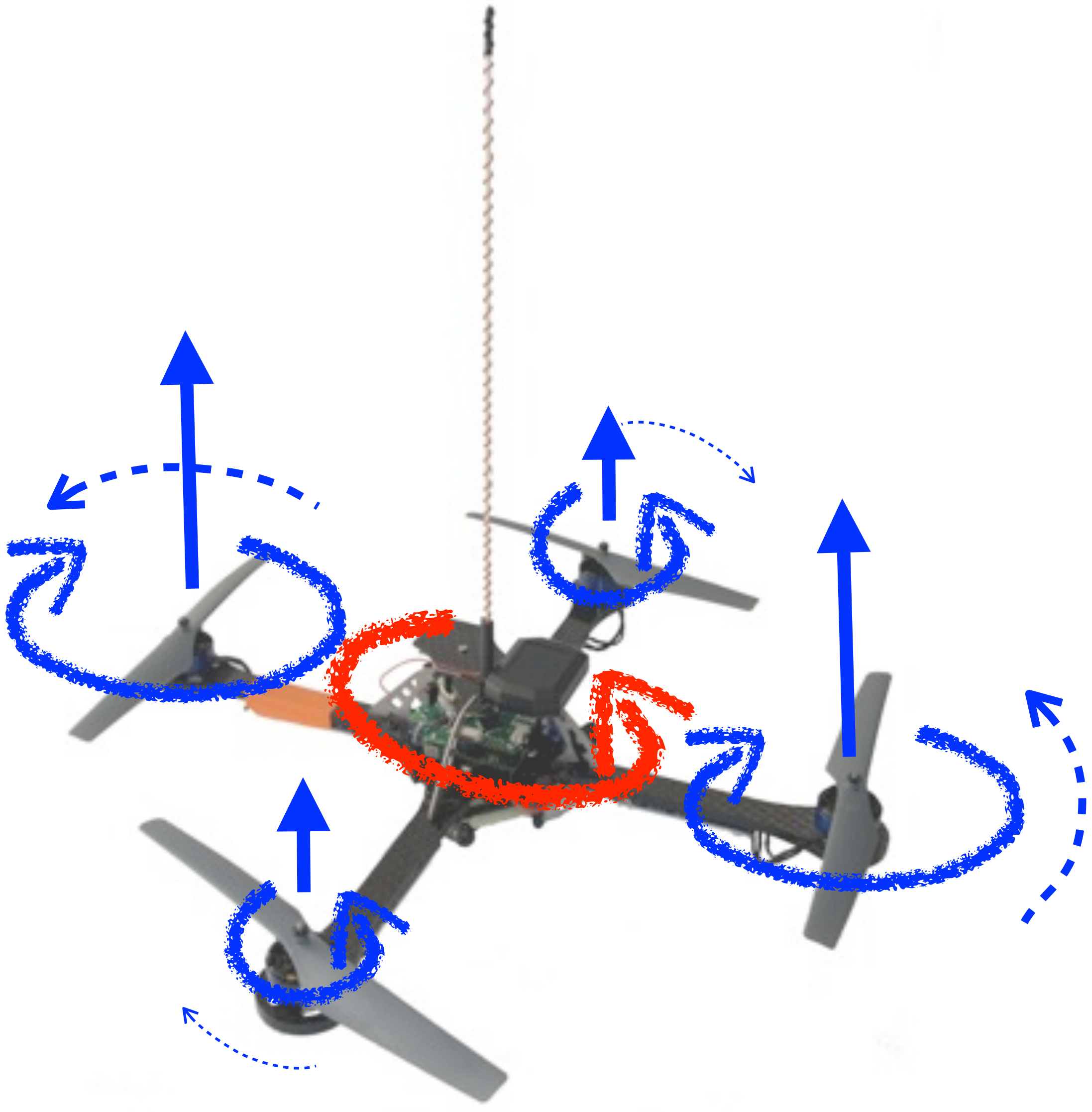
Pelican

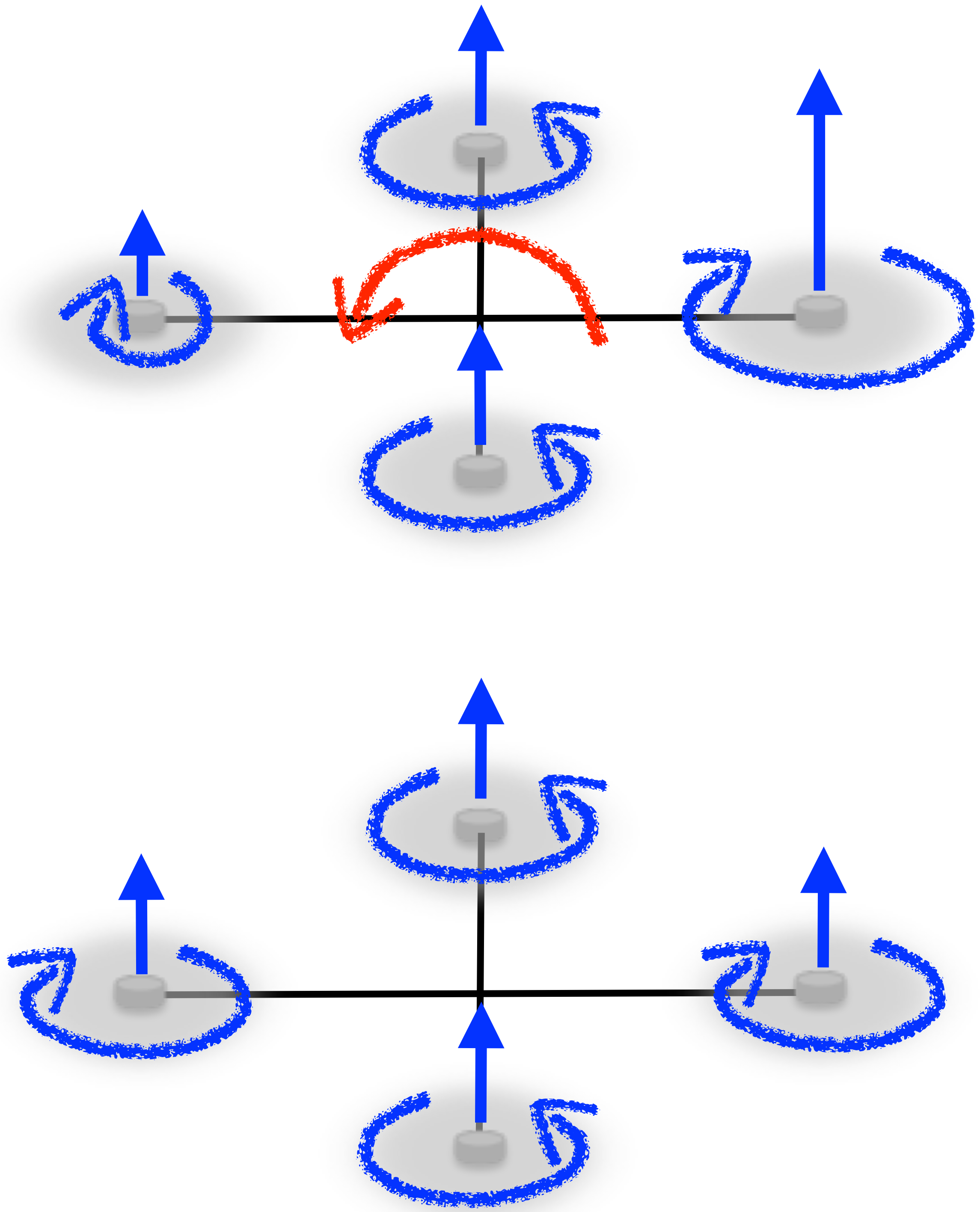
MIT in coll. with Ascending Technologies GmbH

NanoQuad
KMel Robotics

...

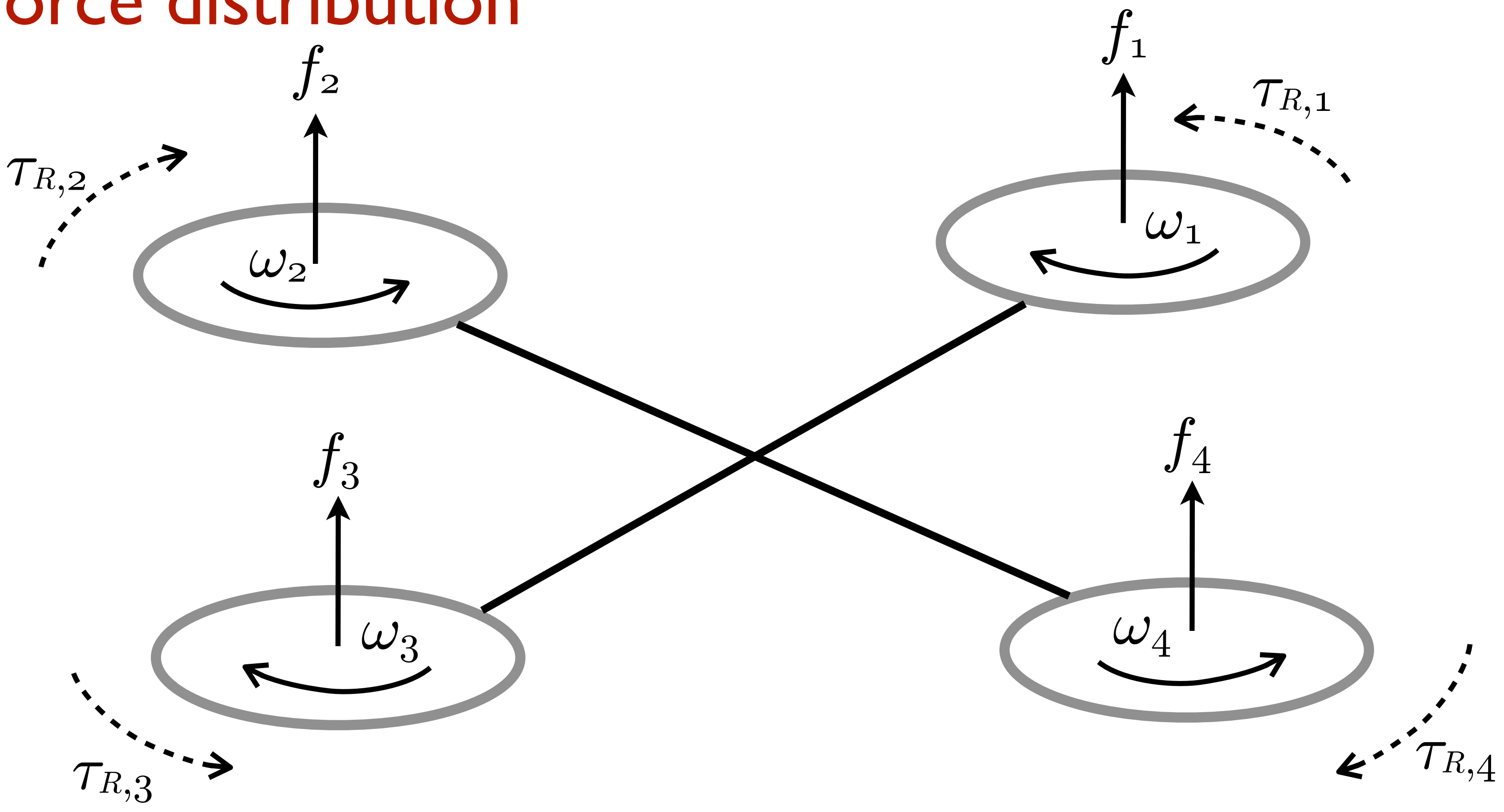
- four motors located at the extremities of a cross-shaped frame
- controlled by varying the angular speed of each rotor





actuation

force distribution



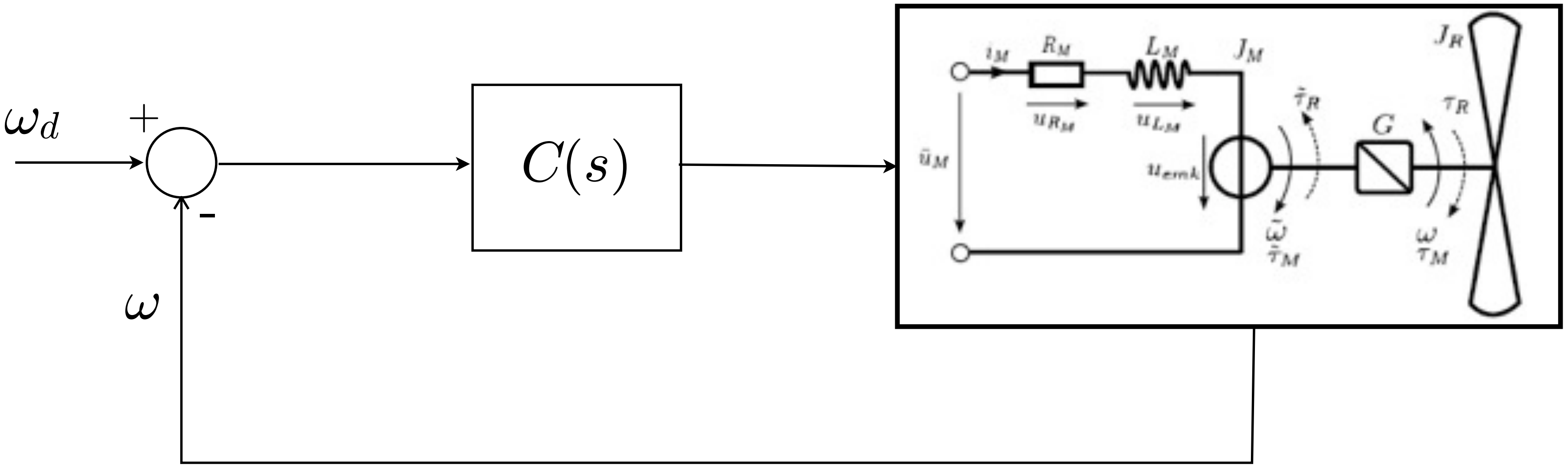
$$f_i = b\,\omega_i^2 \qquad i = 1, \ldots, 4$$

$$\tau_{R,i} = d\,\omega_i^2$$

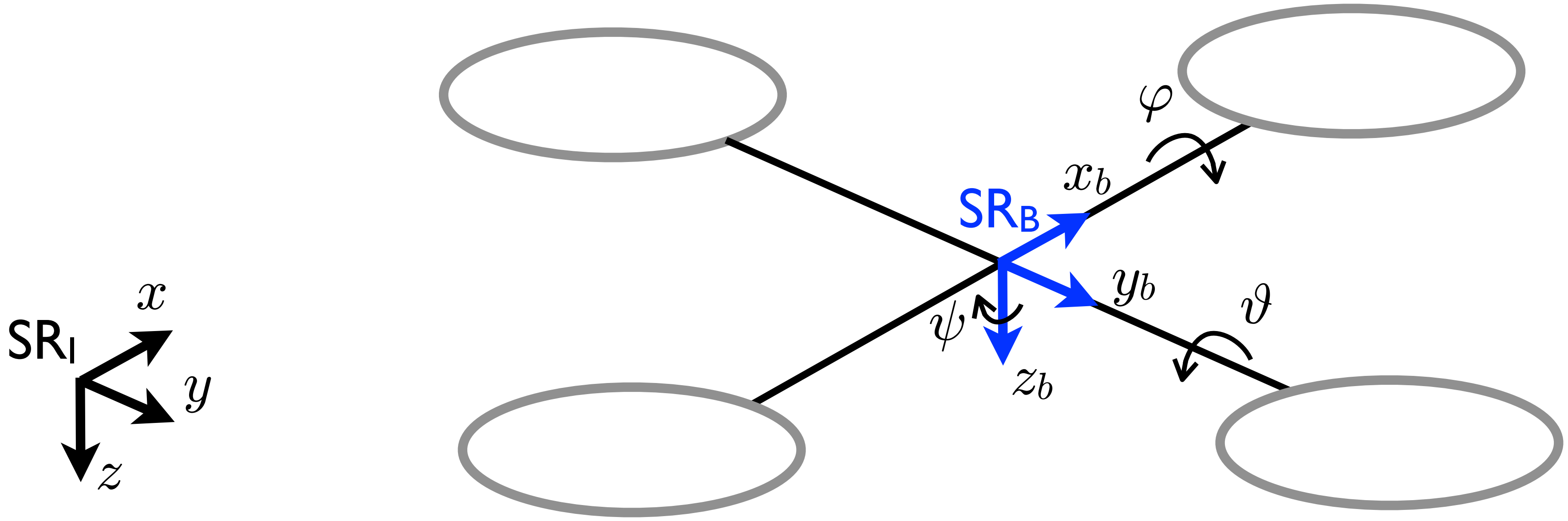
- b thrust factor, d drag factor
- both depend on the rotor geometry and profile, its disk area and radius and on air density
- can be determined by static thrust test

motor control

a low level controller stabilizes the rotational speed of each blade



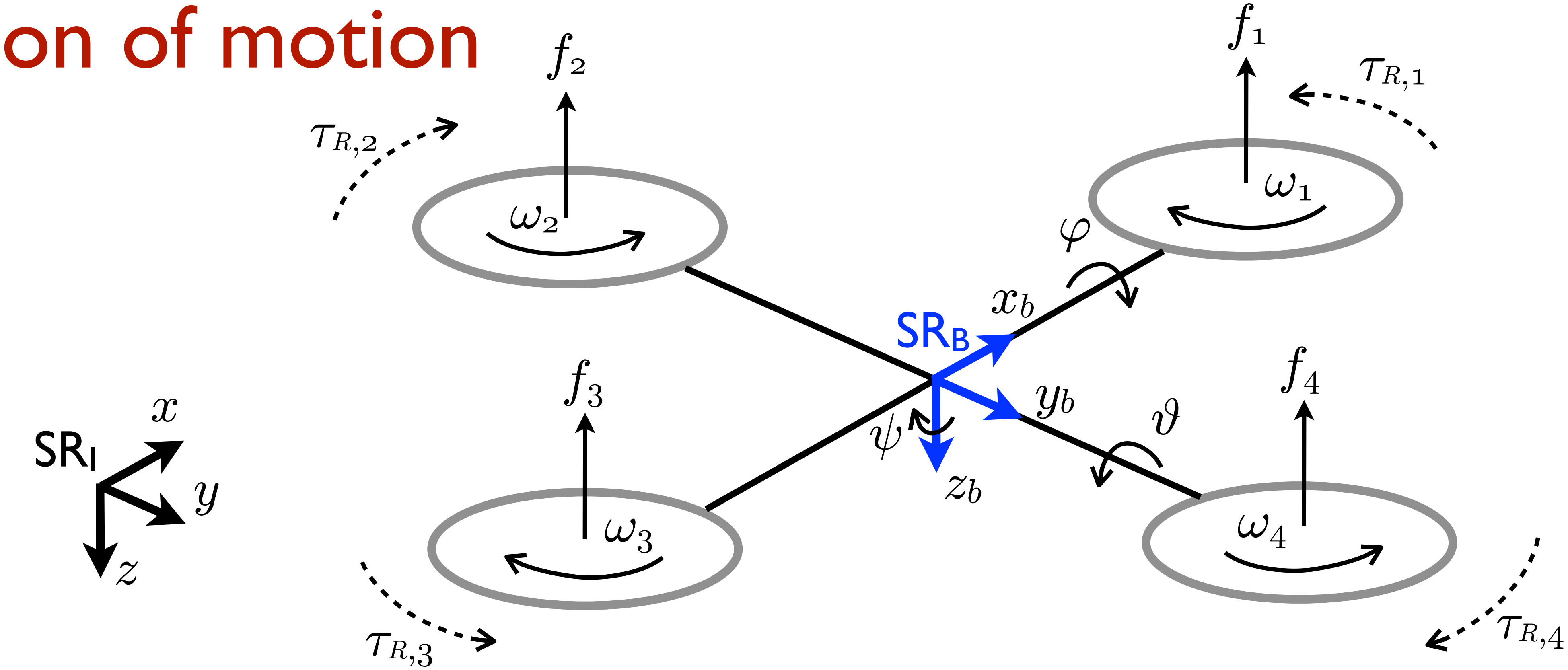
configuration



- $(x,\ y,\ z)$ position of SR$_B$ origin in SR$_I$
- $(\varphi,\ \vartheta,\ \psi)$ RPY angles expressing the orientation of SR$_B$ w.r.t. SR$_I$

$$ {}^{\mathbf{I}}\mathbf{R}_{\mathbf{B}} = \begin{pmatrix} c_\psi c_\vartheta & c_\psi s_\vartheta s_\varphi - s_\psi c_\varphi & c_\psi s_\vartheta c_\varphi + s_\psi s_\varphi \\ s_\psi c_\vartheta & s_\psi s_\vartheta s_\varphi + c_\psi c_\varphi & s_\psi s_\vartheta c_\varphi - s_\varphi c_\psi \\ -s_\vartheta & c_\vartheta s_\varphi & c_\vartheta c_\varphi \end{pmatrix} $$

equation of motion



dynamics of a rigid body with mass m subject to external forces applied to the center of mass according to Newton-Eulero formalism

translational dynamics in SR$_I$ $$\boxed{\sum F_I = m\dot{V}_I}$$

F_I external force applied to the com and expressed in SR$_I$

$V_I = (v_x,\, v_y,\, v_z)'$ velocity of the com expressed in SR$_I$

rotational dynamics in SR$_B$ $$\boxed{\sum M_B = J\dot{\Omega} + \Omega \times J\Omega}$$

M_B, J resp. external moment around com and inertia tensor expressed in SR$_B$

$\Omega = (\,p,\, q,\, r)'$ rotational velocity expressed in SR$_B$

inertia matrix and rotational velocity

$$J = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \qquad \Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -s_\vartheta \\ 0 & c_\varphi & c_\vartheta s_\varphi \\ 0 & -s_\varphi & c_\vartheta c_\varphi \end{pmatrix} \begin{pmatrix} \dot\varphi \\ \dot\vartheta \\ \dot\psi \end{pmatrix}$$

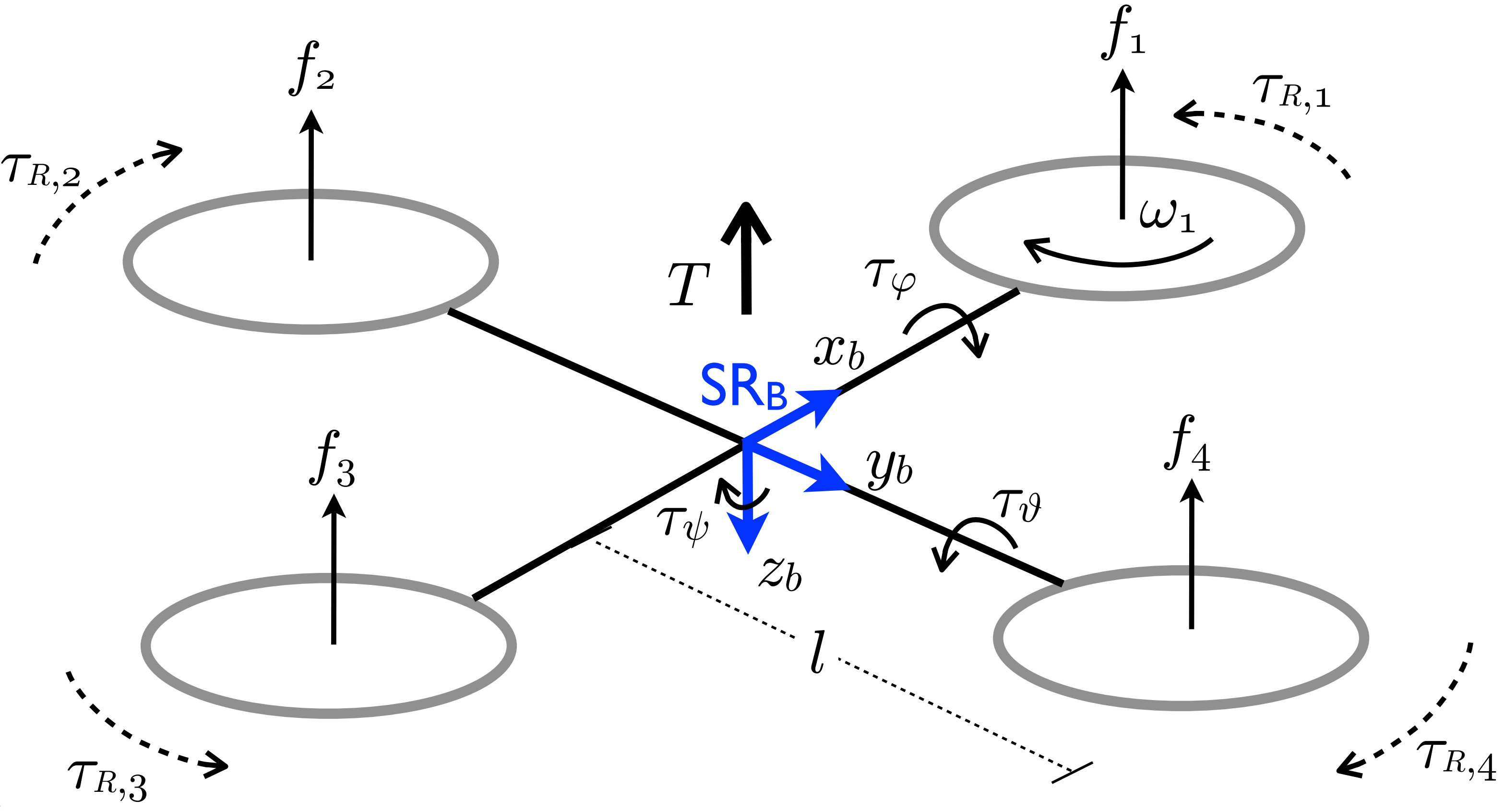
control inputs

$$T = f_1 + f_2 + f_3 + f_4$$

$$\tau_\varphi = l(f_2 - f_4)$$

$$\tau_\vartheta = l(f_1 - f_3)$$

$$\tau_\psi = -\tau_{R,1} + \tau_{R,2} - \tau_{R,3} + \tau_{R,4}$$



applied forces and moments

$$\sum F_I = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} + {}^{\mathbf{I}}\mathbf{R}_{\mathbf{B}}(\varphi, \vartheta, \psi) \begin{pmatrix} 0 \\ 0 \\ -T \end{pmatrix} + F_A + F_D$$

weight

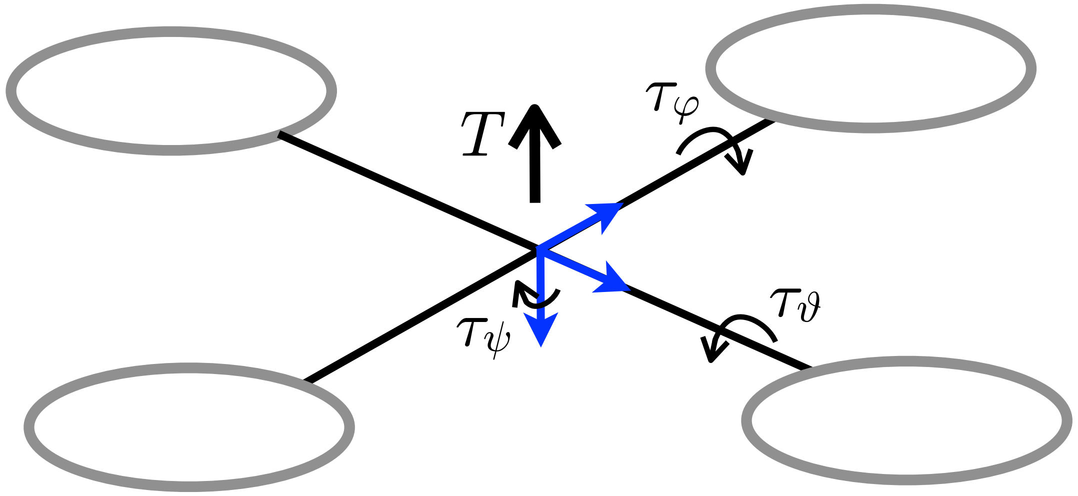
actuation

aerodynamics

disturbances

$$\sum M_B = \begin{pmatrix} L_\varphi \\ L_\vartheta \\ L_\psi \end{pmatrix} + \begin{pmatrix} \tau_\varphi \\ \tau_\vartheta \\ \tau_\psi \end{pmatrix} + \tau_A + \tau_D$$

gyroscopic effects



mathematical model of the system

$$
\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{z} &= v_z \\
\dot{v}_x &= F_{A,x} - (\cos(\psi)\sin(\vartheta)\cos(\varphi) + \sin(\psi)\sin(\varphi))\frac{T}{m} \\
\dot{v}_y &= F_{A,y} - (\sin(\psi)\sin(\vartheta)\cos(\varphi) - \sin(\varphi)\cos(\psi))\frac{T}{m} \\
\dot{v}_z &= F_{A,z} + g - \cos(\vartheta)\cos(\varphi)\frac{T}{m} \\
\dot{\varphi} &= p + \sin(\varphi)\tan(\vartheta)q + \cos(\varphi)\tan(\vartheta)r \\
\dot{\vartheta} &= \cos(\varphi)q - \sin(\varphi)r \\
\dot{\psi} &= \sin(\varphi)\sec(\vartheta)q + \cos(\varphi)\sec(\vartheta)r \\
\dot{p} &= \tau_{A,x} + \frac{I_r}{I_x}q\Omega_r + \frac{I_y - I_z}{I_x}qr + \frac{\tau_\varphi}{I_x} \\
\dot{q} &= \tau_{A,y} + \frac{I_r}{I_y}p\Omega_r + \frac{I_z - I_x}{I_y}pr + \frac{\tau_\vartheta}{I_y} \\
\dot{r} &= \tau_{A,z} + \frac{I_x - I_y}{I_z}pq + \frac{\tau_\psi}{I_z}
\end{aligned}
$$

$$\dot{\xi} = f(\xi) + g(\xi)\,u$$

state

$$\xi = (x,\, y,\, z,\, v_x,\, v_y,\, v_z,\, \varphi,\, \vartheta,\, \psi,\, p,\, q,\, r)'$$

inputs

$$u = (T,\, \tau_\varphi,\, \tau_\vartheta,\, \tau_\psi)'$$

Ω_r average blades rotation velocity

I_r blades inertia

simplified model for control design

negligible

- aerodynamics
- gyroscopic effects

assuming

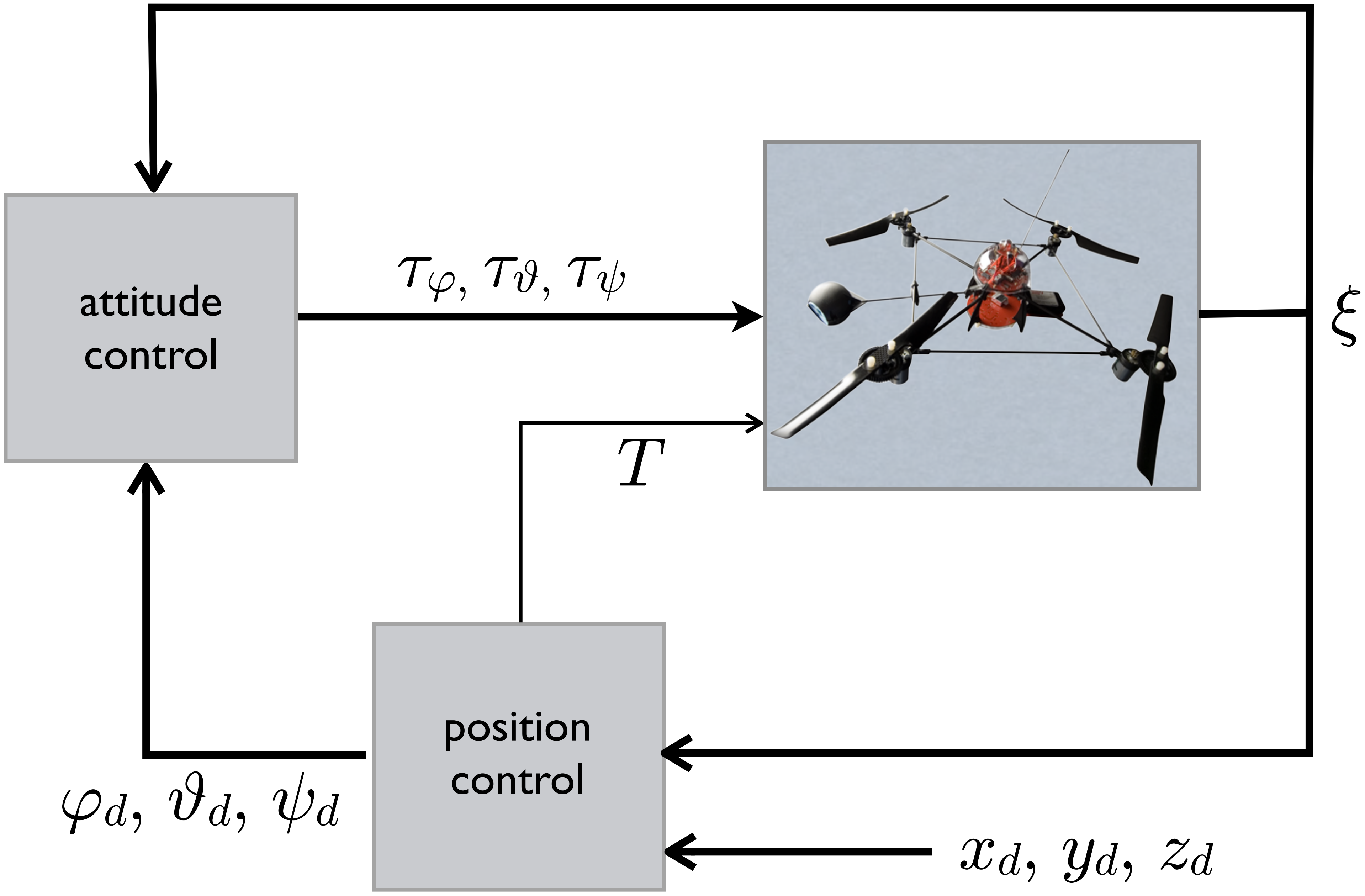
- small φ and $\vartheta \Rightarrow (\dot{\varphi}, \dot{\vartheta}, \dot{\psi}) \simeq (p, q, r)$
- symmetric shape
- negligible disturbances

$$\boxed{\begin{aligned}
\ddot{x} &= -(\cos(\psi)\sin(\vartheta)\cos(\varphi) + \sin(\psi)\sin(\varphi))\frac{T}{m} \\
\ddot{y} &= -(\sin(\psi)\sin(\vartheta)\cos(\varphi) - \sin(\varphi)\cos(\psi))\frac{T}{m} \\
\ddot{z} &= -\cos(\vartheta)\cos(\varphi)\frac{T}{m} + g \\
\ddot{\varphi} &= \frac{\tau_\varphi}{I_x} \\
\ddot{\vartheta} &= \frac{\tau_\vartheta}{I_y} \\
\ddot{\psi} &= \frac{\tau_\psi}{I_z}
\end{aligned}}$$

control and planning problems

- attitude control
- eight control
- position control
- trajectory planning
- trajectory tracking
- sensor-based control

control system



attitude control

$\tau_\varphi,\ \tau_\vartheta,\ \tau_\psi$

ξ

T

position control

$\varphi_d,\ \vartheta_d,\ \psi_d$

$x_d,\ y_d,\ z_d$

attitude control

determine the torques $\tau_\varphi, \tau_\vartheta, \tau_\psi$ necessary to obtain a stable desired attitude $\varphi_d,\ \vartheta_d,\ \psi_d$

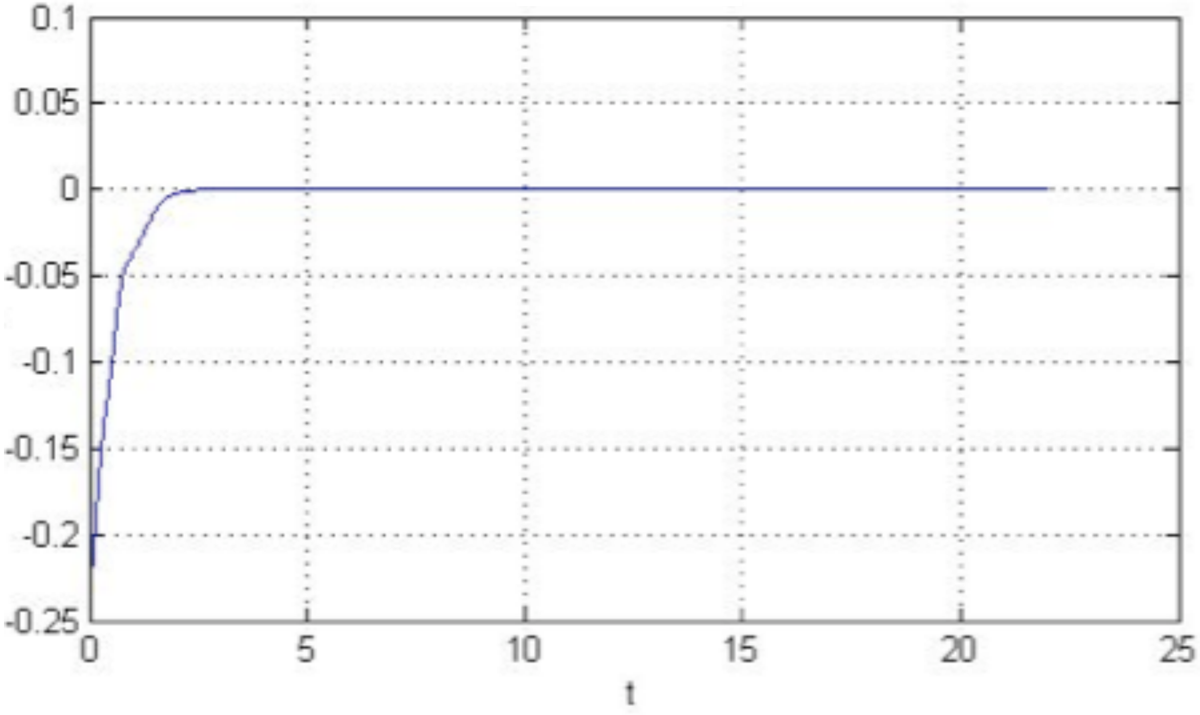
$$\begin{aligned}
\tau_\varphi &= [K_{\varphi p}(\varphi_d - \varphi) + K_{\varphi d}(\dot{\varphi}_d - \dot{\varphi})] \\
\tau_\vartheta &= [K_{\vartheta p}(\vartheta_d - \vartheta) + K_{\vartheta d}(\dot{\vartheta}_d - \dot{\vartheta})] \\
\tau_\psi &= [K_{\psi p}(\psi_d - \psi) + K_{\psi d}(\dot{\psi}_d - \dot{\psi})]
\end{aligned}$$

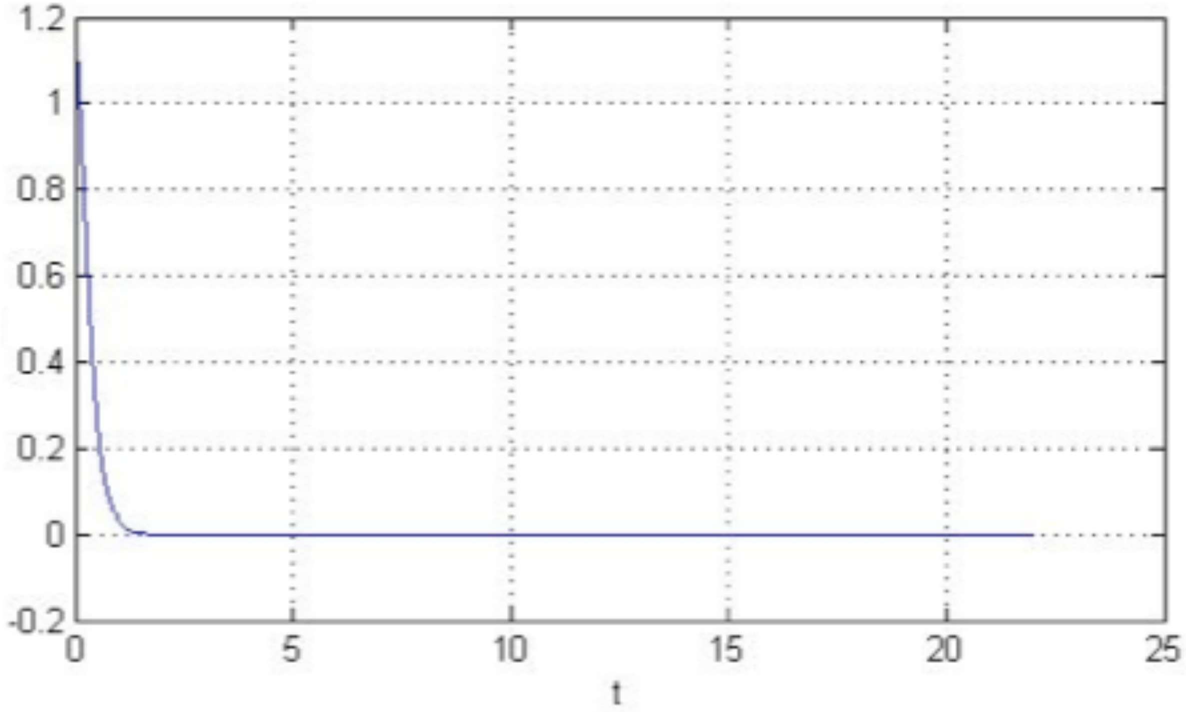
height control

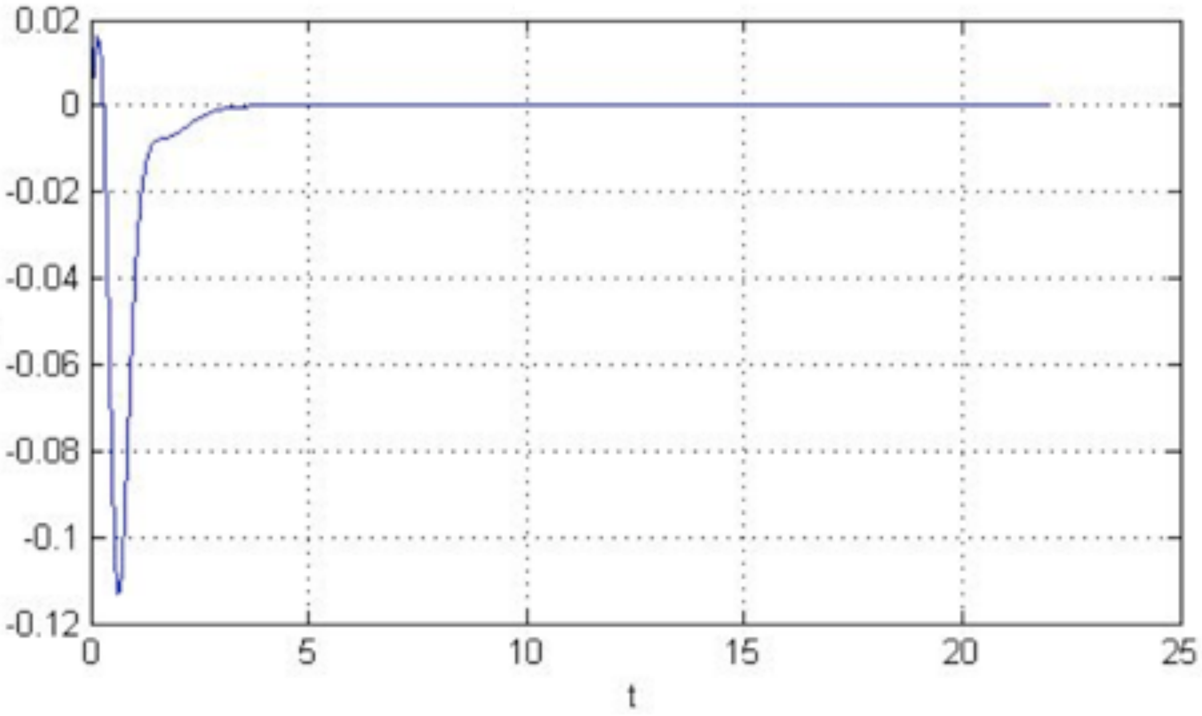
determine the thrust T necessary to bring and keep the quadrotor to a desired height $z_d \Rightarrow$ from the z dynamics:

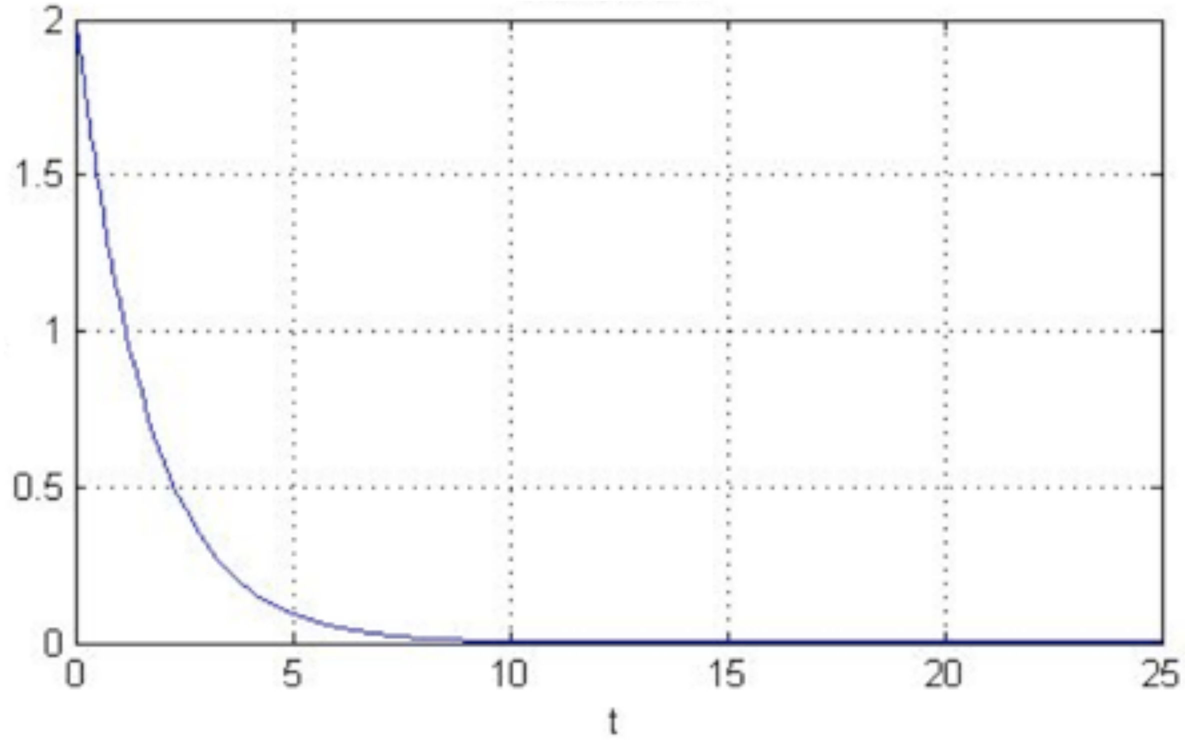
$$T = \frac{m}{\cos(\vartheta)\cos(\varphi)}[g + \ddot{z}_d + K_{zp}(z_d - z) + K_{zd}(\dot{z}_d - \dot{z})]$$

simulation results: error

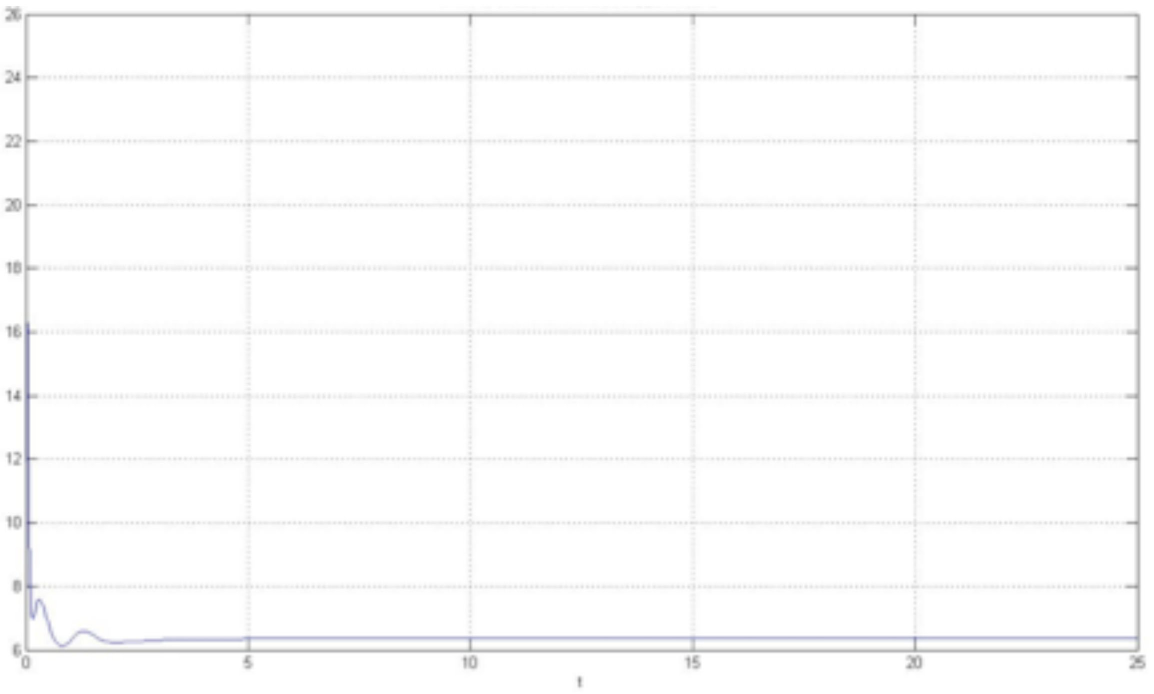

φ error


ϑ error

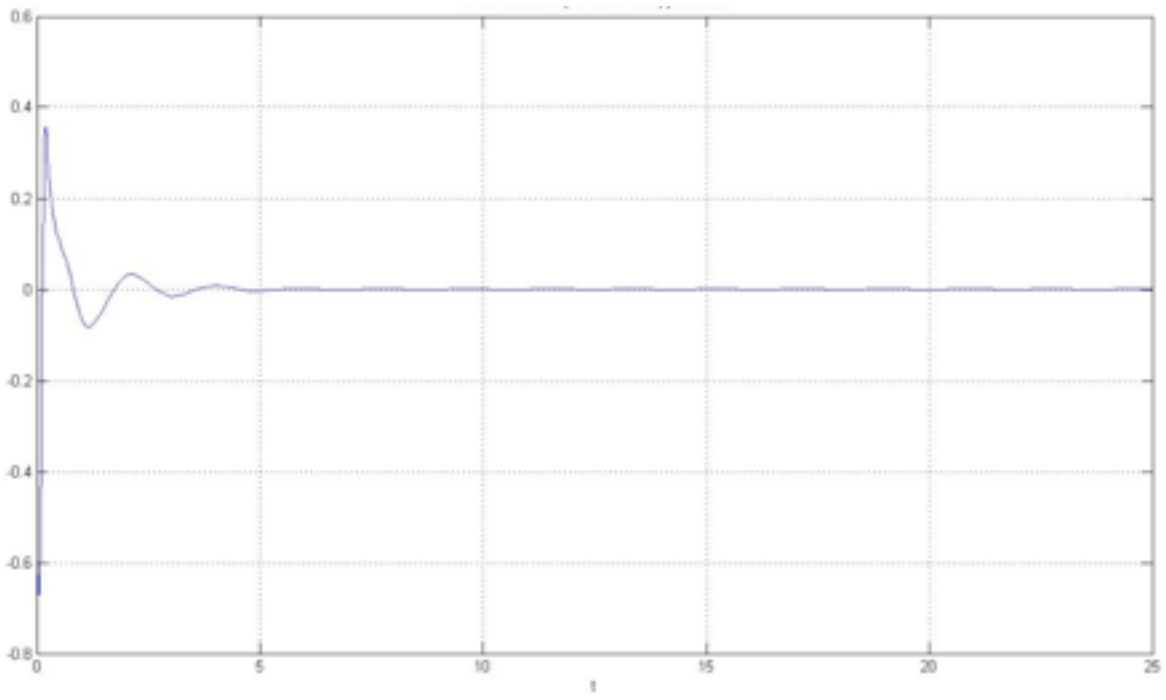

ψ error


z error

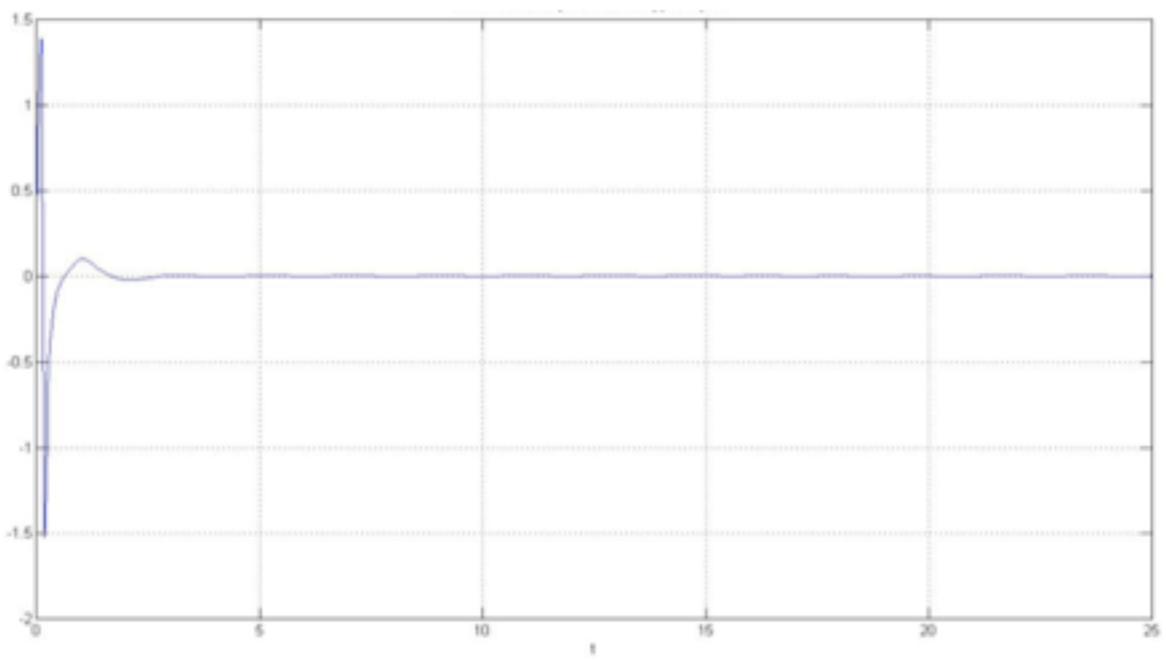
simulation results: control inputs



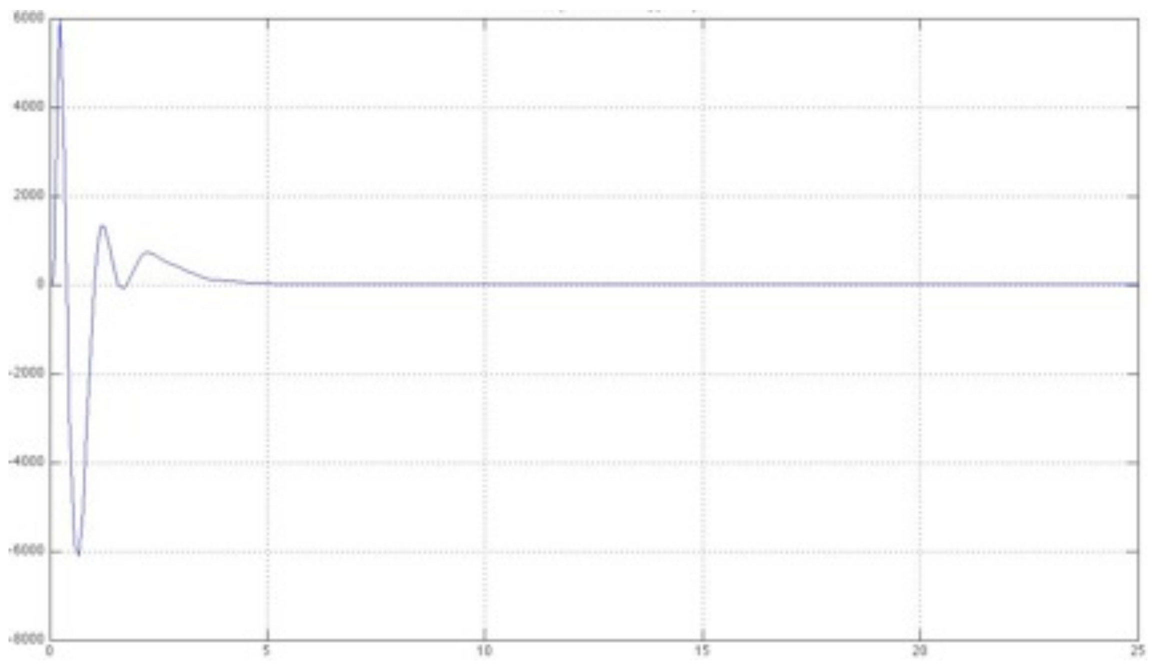
T



τ_φ



τ_ϑ



τ_ψ

- advanced nonlinear control techniques guarantee better convergence and robustness performance
- exteroceptive sensors (camera, laser, sonar) allow indoor flight and can be used to obtain an accurate estimation of the system state

References

R. Mahony, V. Kumar, P. Corke, "Multirotor Aerial Vehicles: Modeling, Estimation, and Control of Quadrotor," IEEE Robotics & Automation Magazine, vol.19, no.3, pp. 20-32, Sept. 2012.