Elective in Robotics

Quadrotor control via dynamic feedback linearization

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simplified model for control design

negligible

- aerodynamics
- gyroscopic effects

assuming

- small φ and ϑ $\Rightarrow (\dot{\varphi}, \dot{\vartheta}, \dot{\psi}) \simeq (p, q, r)$
- symmetric shape
- negligible disturbances

$$
\begin{aligned}
\ddot{x} &= -(\cos(\psi)\sin(\vartheta)\cos(\varphi) + \sin(\psi)\sin(\varphi))\frac{T}{m} \\
\ddot{y} &= -(\sin(\psi)\sin(\vartheta)\cos(\varphi) - \sin(\varphi)\cos(\psi))\frac{T}{m} \\
\ddot{z} &= -\cos(\vartheta)\cos(\varphi)\frac{T}{m} + g \\
\ddot{\varphi} &= \frac{\tau_\varphi}{I_x} \\
\ddot{\vartheta} &= \frac{\tau_\vartheta}{I_y} \\
\ddot{\psi} &= \frac{\tau_\psi}{I_z}
\end{aligned}
$$

state

$$\xi = (x,\, y,\, z,\, v_x,\, v_y,\, v_z,\, \varphi,\, \vartheta,\, \psi,\, p,\, q,\, r)'$$

inputs

$$u = (T,\, \tau_\varphi,\, \tau_\vartheta,\, \tau_\psi)'$$

feedback linearization

- due to underactuation, the quadrotor system cannot be transformed into an equivalent linear, controllable system by static state feedback
- it is however possible to resort to dynamic state feedback to obtain full state linearization
- given the nonlinear system

$$\dot{\xi} = f(\xi) + g(\xi)\, u \qquad \xi \in R^n, \qquad u \in R^m$$

- find, if possible, a dynamic compensator of the form

$$\begin{aligned} \dot{\zeta} &= a(\xi, \zeta) + b(\xi, \zeta) v \qquad \zeta \in R^\nu,\ v \in R^m \\ u &= c(\xi, \zeta) + d(\xi, \zeta) v \end{aligned}$$

- s.t. the c.l. system, under state transformation, is equivalent to a linear system

- first define an m-dimensional output

$$\eta = h(\xi)$$

- then proceed by successively differentiating the output until the input appears in a nonsingular way

- if the sum of the output differentiation orders equals the dimension of the extended state space $n+\nu$, full input–state–output linearization is obtained

- the closed-loop system is then equivalent to a set of decoupled input–output chains of integrators from the input v_i to the output η_i $(i=1,\ldots,m)$

- the quadrotor output to be controlled is

$$\eta(\xi) = (x,\ y,\ z,\ \psi)^{\mathrm{T}}$$

- the input u appears in a nonsingular way deriving 4 times the cartesian position $P = (x,\ y,\ z)^{\mathrm{T}}$ and 2 times the yaw angle ψ

- $\Rightarrow$ it is necessary to introduce two integrators on the input channel $u_1 = T$

$$\zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} T \\ \dot{T} \end{pmatrix}$$

- the output derivatives can be written as

$$\begin{pmatrix} P^{(4)} \\ \psi^{(2)} \end{pmatrix} = l(\xi, \zeta) + J(\xi, \zeta)\tilde{u}$$

with $\tilde{u} = (\ddot{T},\ \tau_\varphi,\ \tau_\vartheta,\ \tau_\psi)^T$

- the linearizing and decoupling control law is

$$\tilde{u} = J(\xi, \zeta)^{-1}(-l(\xi, \zeta) + v)$$

- resulting in the closed loop system

$$\begin{aligned} x^{(4)} &= \eta_1^{(4)} = v_1 \\ y^{(4)} &= \eta_2^{(4)} = v_2 \\ z^{(4)} &= \eta_3^{(4)} = v_3 \\ \psi^{(2)} &= \eta_4^{(2)} = v_4 \end{aligned}$$

- trajectory tracking can be obtained by choosing

$$\begin{aligned} v_1 &= \eta_{1d}^{(4)} + \textstyle\sum_{j=1}^{4} c_{j-1}^{1} e_1^{(j-1)} \\ v_2 &= \eta_{2d}^{(4)} + \textstyle\sum_{j=1}^{4} c_{j-1}^{2} e_2^{(j-1)} \\ v_3 &= \eta_{3d}^{(4)} + \textstyle\sum_{j=1}^{4} c_{j-1}^{3} e_3^{(j-1)} \\ v_4 &= \eta_{4d}^{(2)} + \textstyle\sum_{j=1}^{2} c_{j-1}^{4} e_4^{(j-1)} \end{aligned}$$

with $e_i = (\eta_d - \eta)$

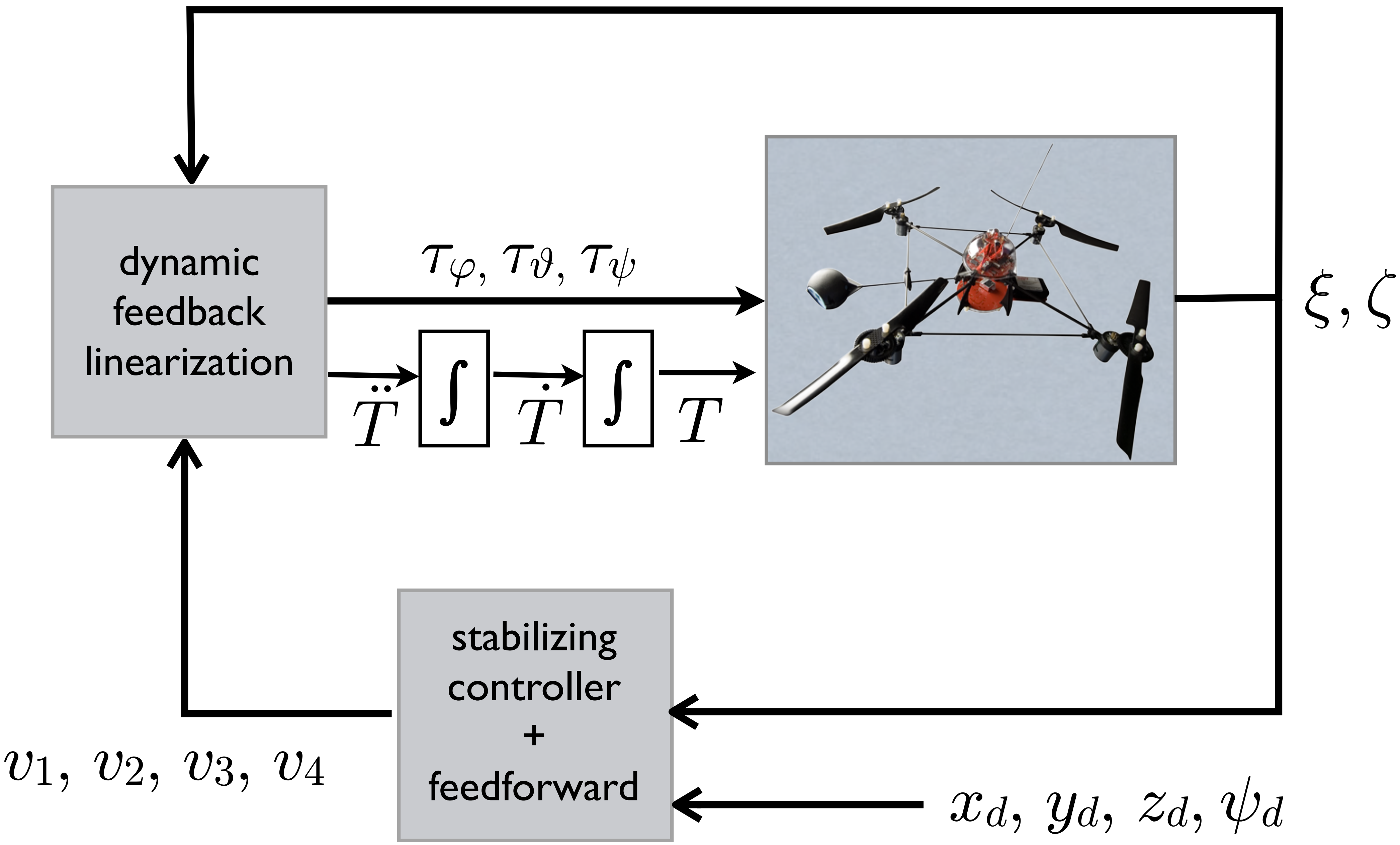
- the dynamic compensator takes the form

$$
\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= J_1(\xi,\zeta)^{-1}(-l(\xi,\zeta)+v)
\end{aligned}
$$

$$
\begin{aligned}
T &= \zeta_1 \\
\tau_\varphi &= J_2(\xi,\zeta)^{-1}(-l(\xi,\zeta)+v) \\
\tau_\vartheta &= J_3(\xi,\zeta)^{-1}(-l(\xi,\zeta)+v) \\
\tau_\psi &= J_4(\xi,\zeta)^{-1}(-l(\xi,\zeta)+v)
\end{aligned}
$$

where $J_i(\xi,\zeta)^{-1}$ (i=1,...,4) denotes the i-th row of $J_i(\xi,\zeta)^{-1}$

control system



simulation results

- **scenario 1:** the behaviour of the system is tested closing the feedback loop around the *complete model*, including aerodynamic friction

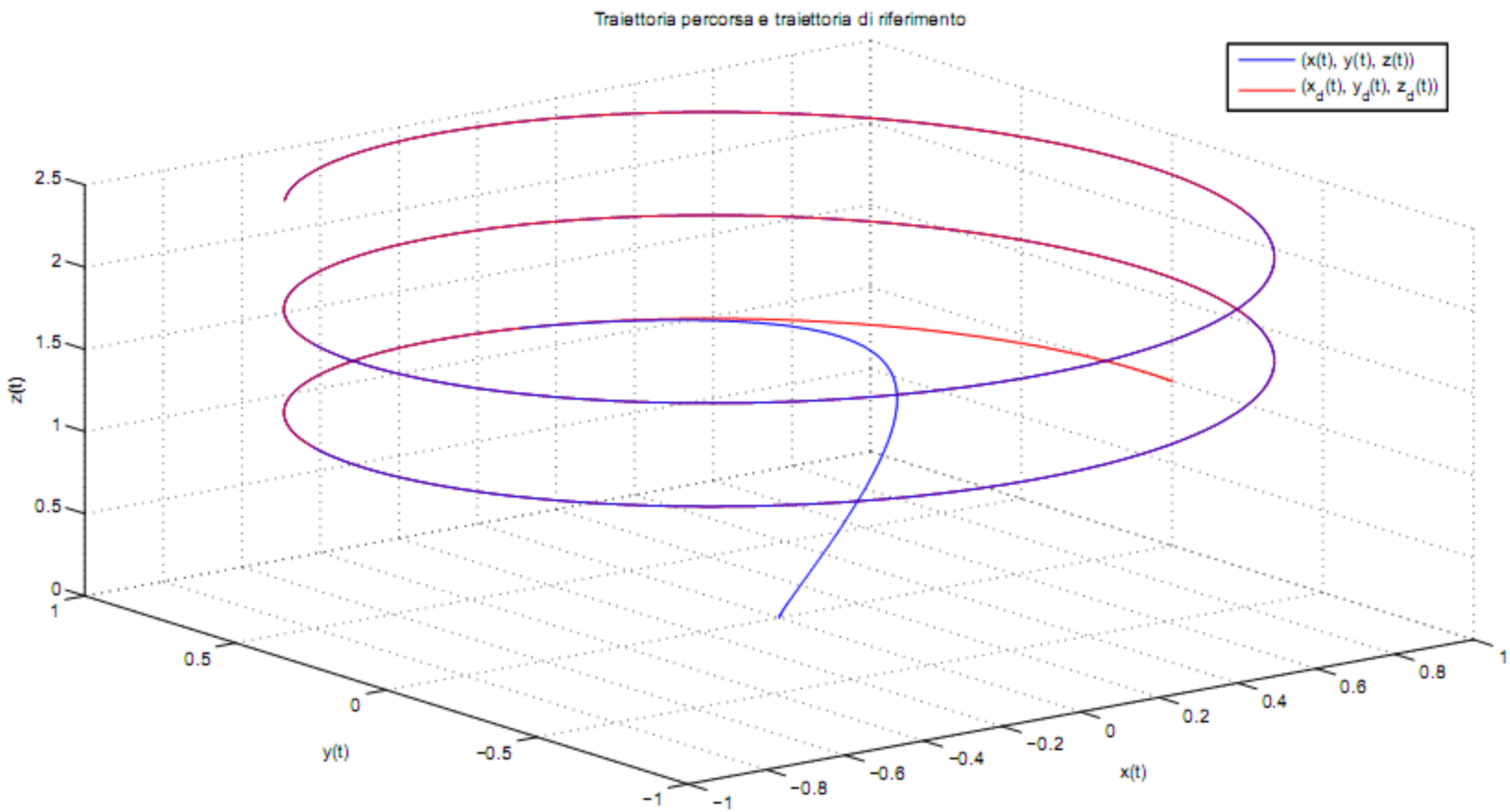
- **scenario 2:** a *wind gust* is added as a *disturbance* on the X inertial axis, to test the *robustness* and *sensitivity* of the controller

- **scenario 3:** the same gust is then added also on Y and Z axes

- the reference trajectory chosen for the simulation is the spiral

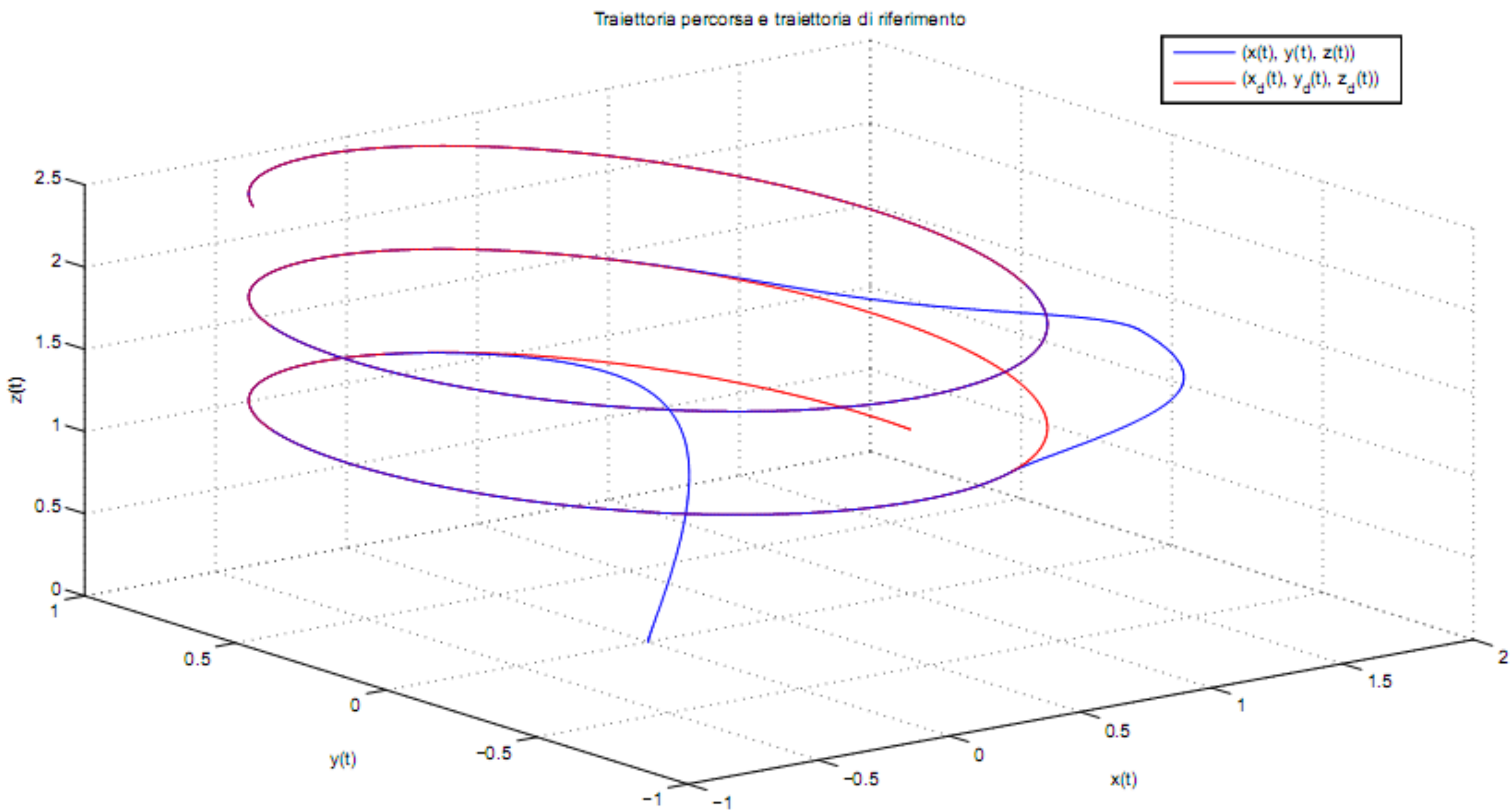
$$
\begin{aligned}
x_d(t) &= \cos t \\
y_d(t) &= \sin t \\
z_d(t) &= 1 + \frac{t}{10}
\end{aligned}
$$

scenario 1



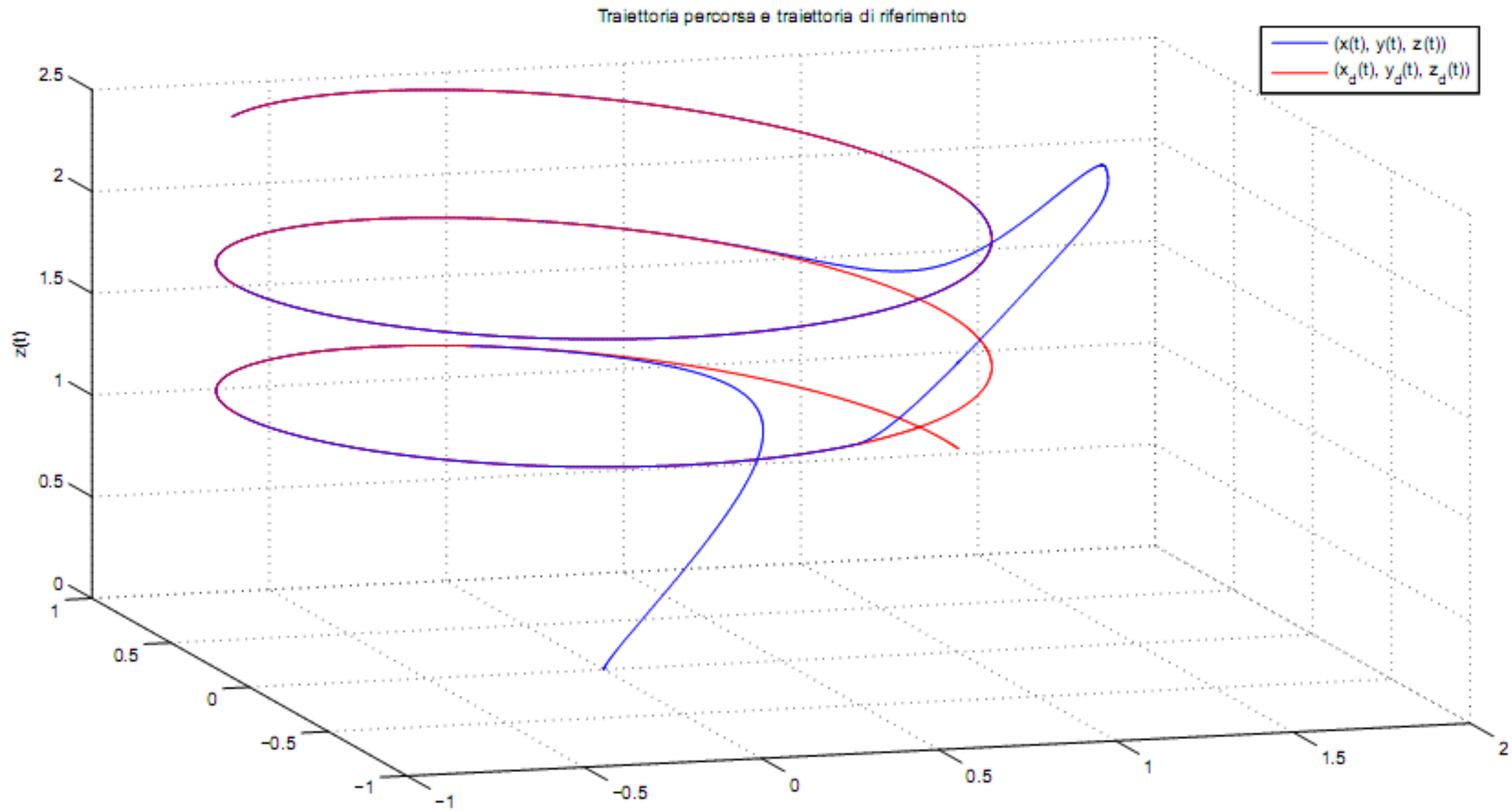
Traiettoria percorsa e traiettoria di riferimento
(x(t), y(t), z(t))
(x_d(t), y_d(t), z_d(t))
z(t)
y(t)
x(t)
2.5
2
1.5
1
0.5
0
1
0.5
0
-0.5
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

scenario 2



Traiettoria percorsa e traiettoria di riferimento
(x(t), y(t), z(t))
(x_d(t), y_d(t), z_d(t))
z(t)
2.5
2
1.5
1
0.5
0
1
0.5
0
-0.5
-1
y(t)
-1
-0.5
0
0.5
1
1.5
2
x(t)

scenario 3



Traiettoria percorsa e traiettoria di riferimento
(x(t), y(t), z(t))
(x_d(t), y_d(t), z_d(t))
z(t)
2.5
2
1.5
1
0.5
0
1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
1.5
2

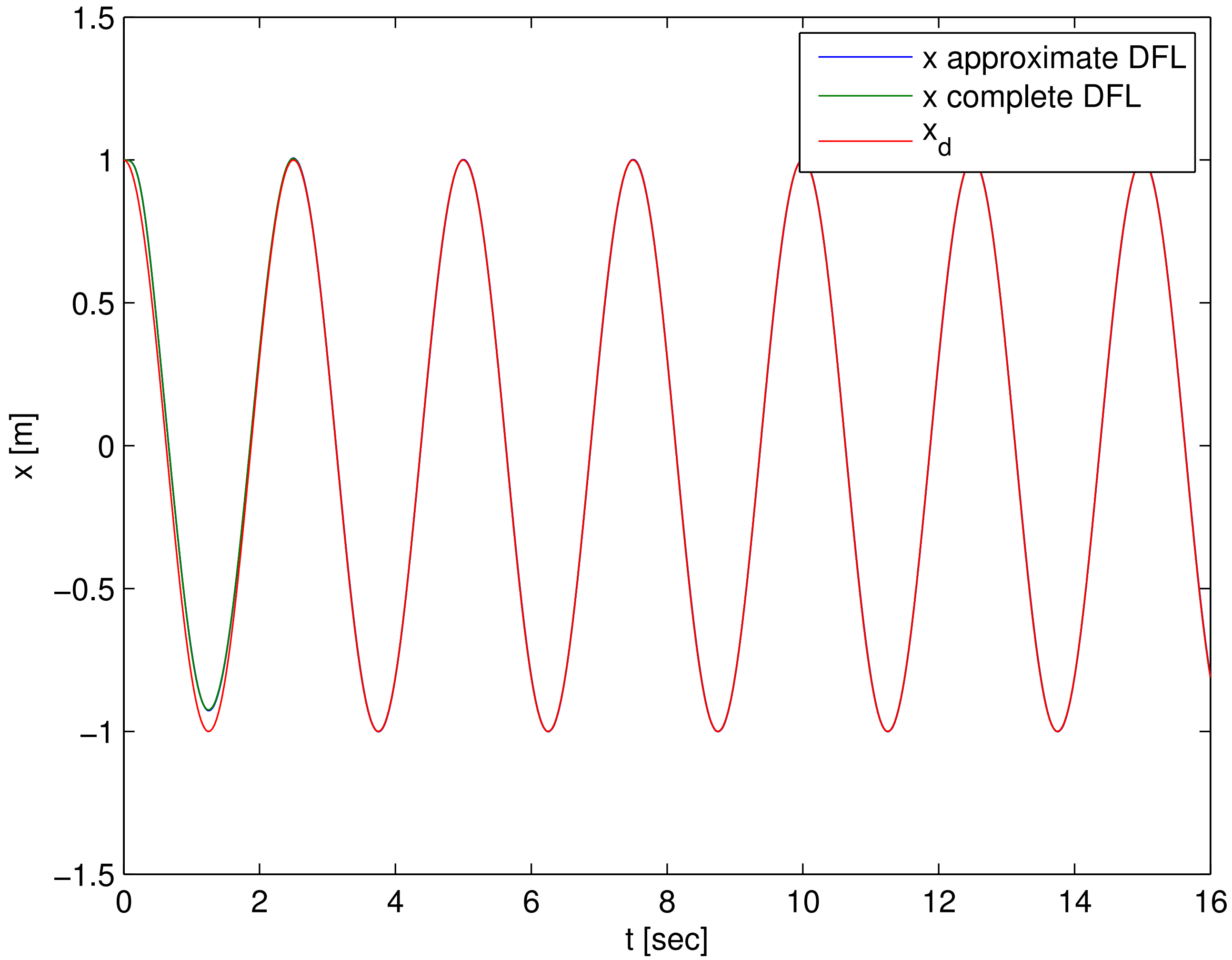
removing the 'small angles' assumption

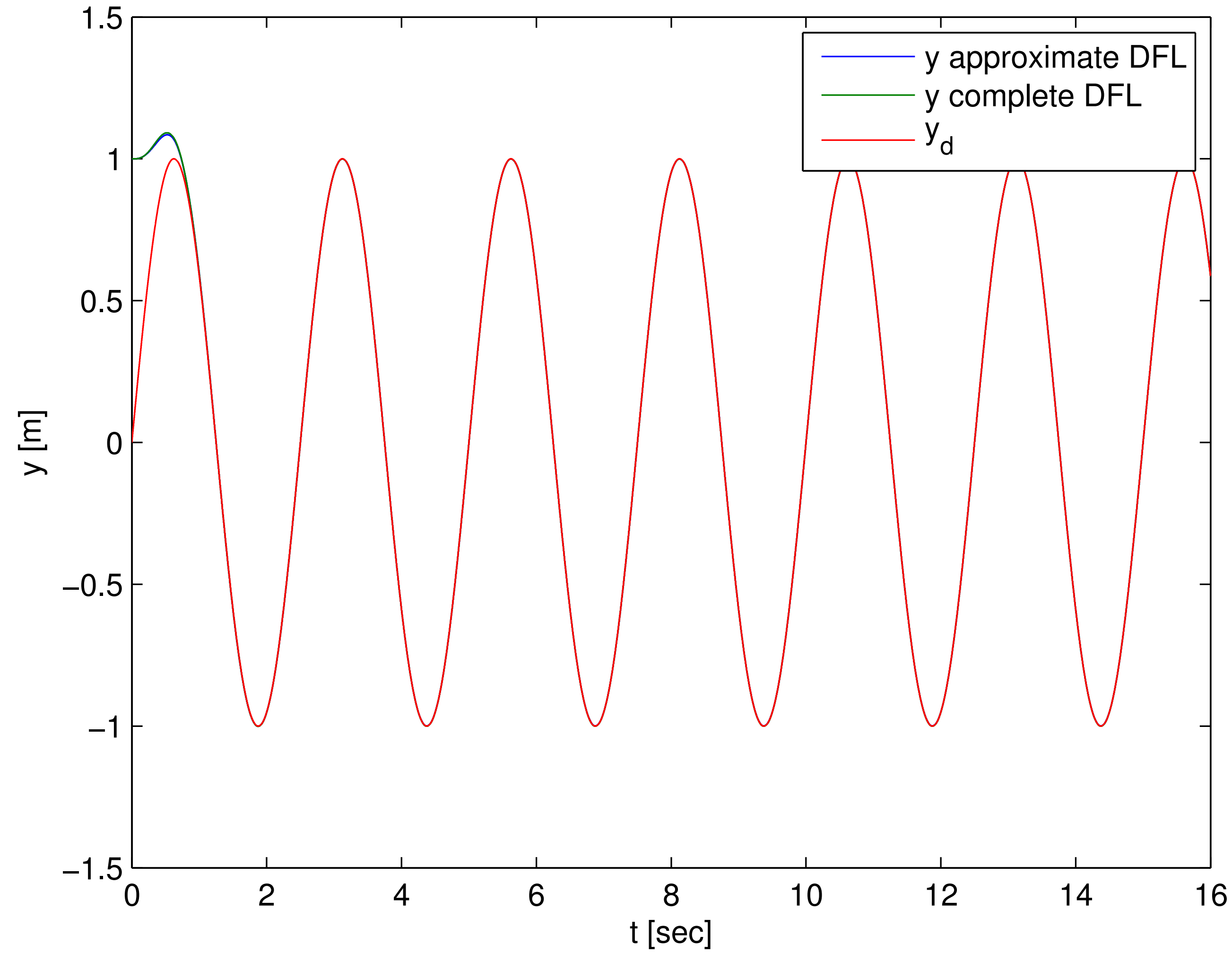
$$
\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{z} &= v_z \\
\dot{v}_x &= \cancel{F_{A,x}} - (\cos(\psi)\sin(\vartheta)\cos(\varphi) + \sin(\psi)\sin(\varphi))\frac{T}{m} \\
\dot{v}_y &= \cancel{F_{A,y}} - (\sin(\psi)\sin(\vartheta)\cos(\varphi) - \sin(\varphi)\cos(\psi))\frac{T}{m} \\
\dot{v}_z &= \cancel{F_{A,z}} + g - \cos(\vartheta)\cos(\varphi)\frac{T}{m} \\
\dot{\varphi} &= p + \sin(\varphi)\tan(\vartheta)q + \cos(\varphi)\tan(\vartheta)r \\
\dot{\vartheta} &= \cos(\varphi)q - \sin(\varphi)r \\
\dot{\psi} &= \sin(\varphi)\sec(\vartheta)q + \cos(\varphi)\sec(\vartheta)r \\
\dot{p} &= \cancel{\tau_{A,x}} + \cancel{\frac{I_r}{I_x}q\Omega_r} + \frac{I_y - I_z}{I_x}qr + \frac{\tau_\varphi}{I_x} \\
\dot{q} &= \cancel{\tau_{A,y}} + \cancel{\frac{I_r}{I_y}p\Omega_r} + \frac{I_z - I_x}{I_y}pr + \frac{\tau_\vartheta}{I_y} \\
\dot{r} &= \cancel{\tau_{A,z}} + \frac{I_x - I_y}{I_z}pq + \frac{\tau_\psi}{I_z}
\end{aligned}
$$

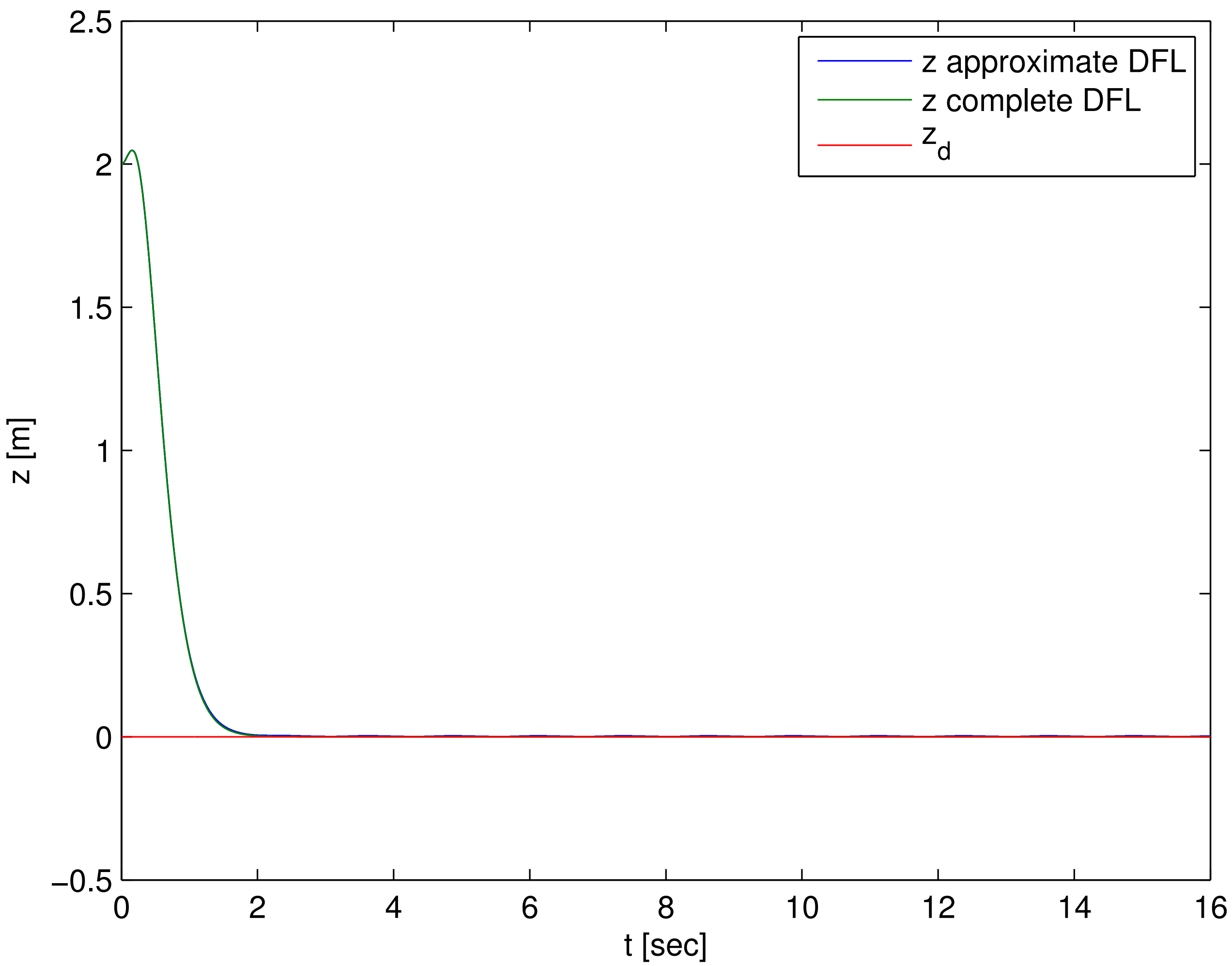
- consider the complete model neglecting aerodynamics, gyroscopic effects, disturbances
- compute DFL

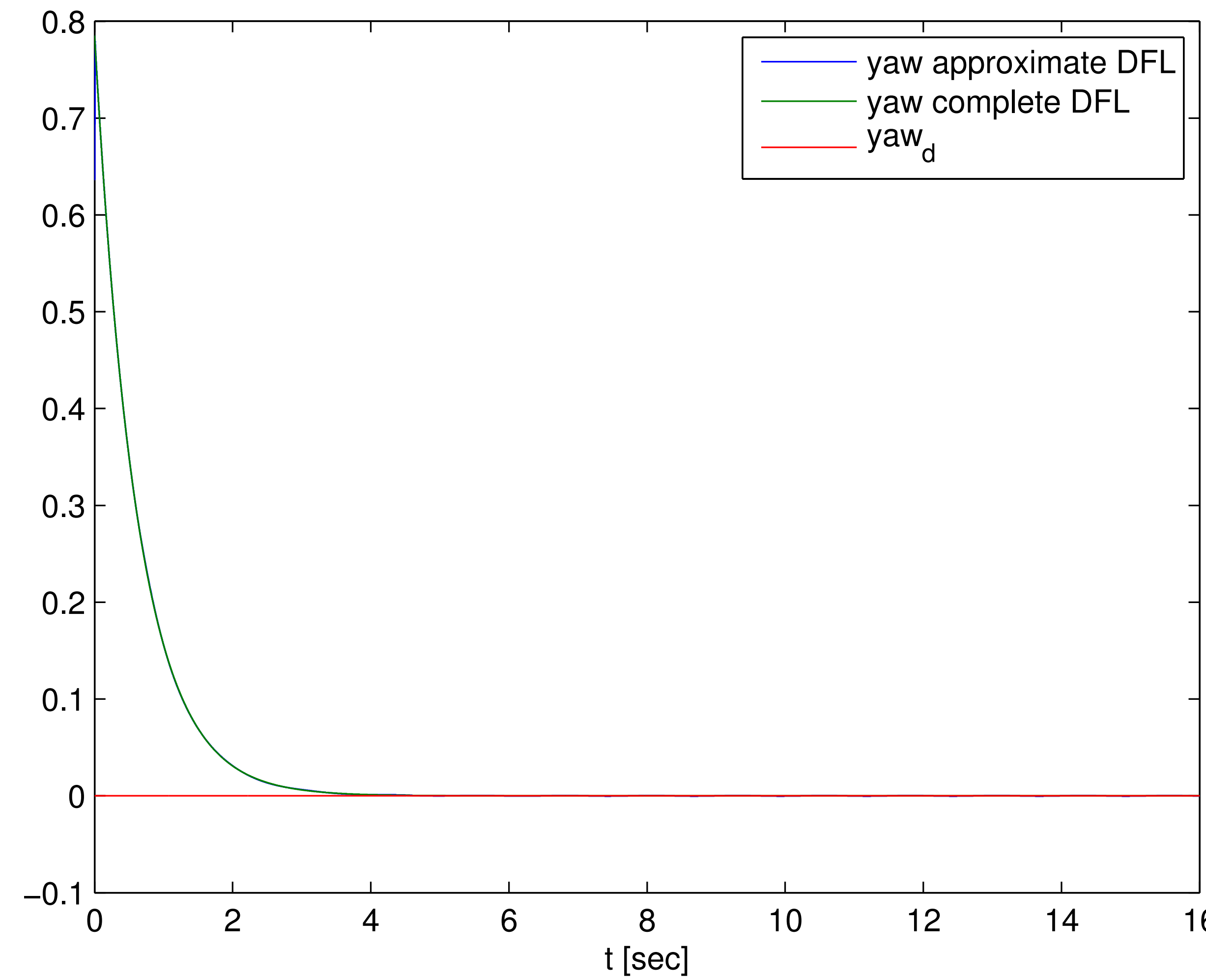
comparative simulations

- simulation 1: circle with period $T{=}2.5s$ and radius $R{=}1m$

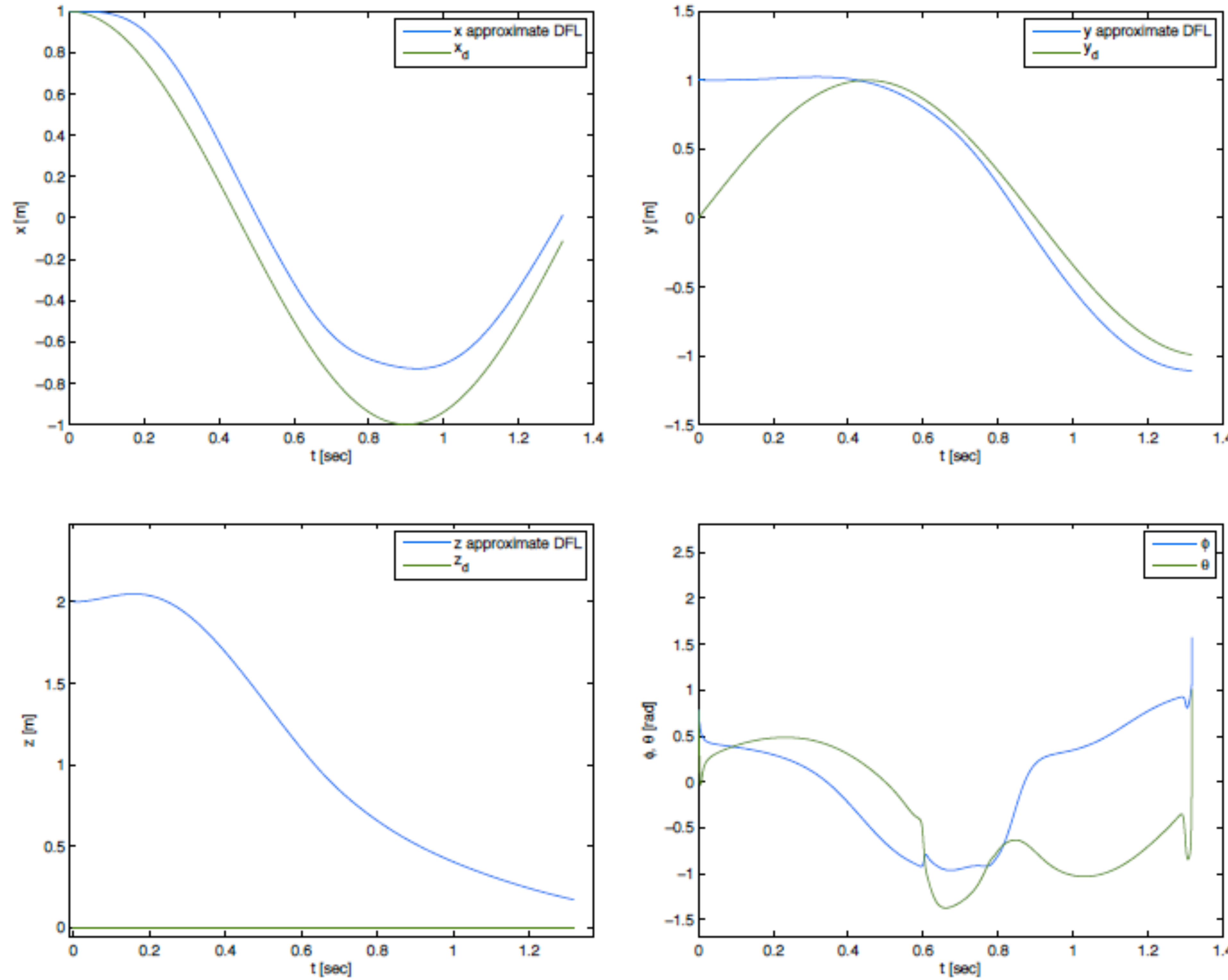


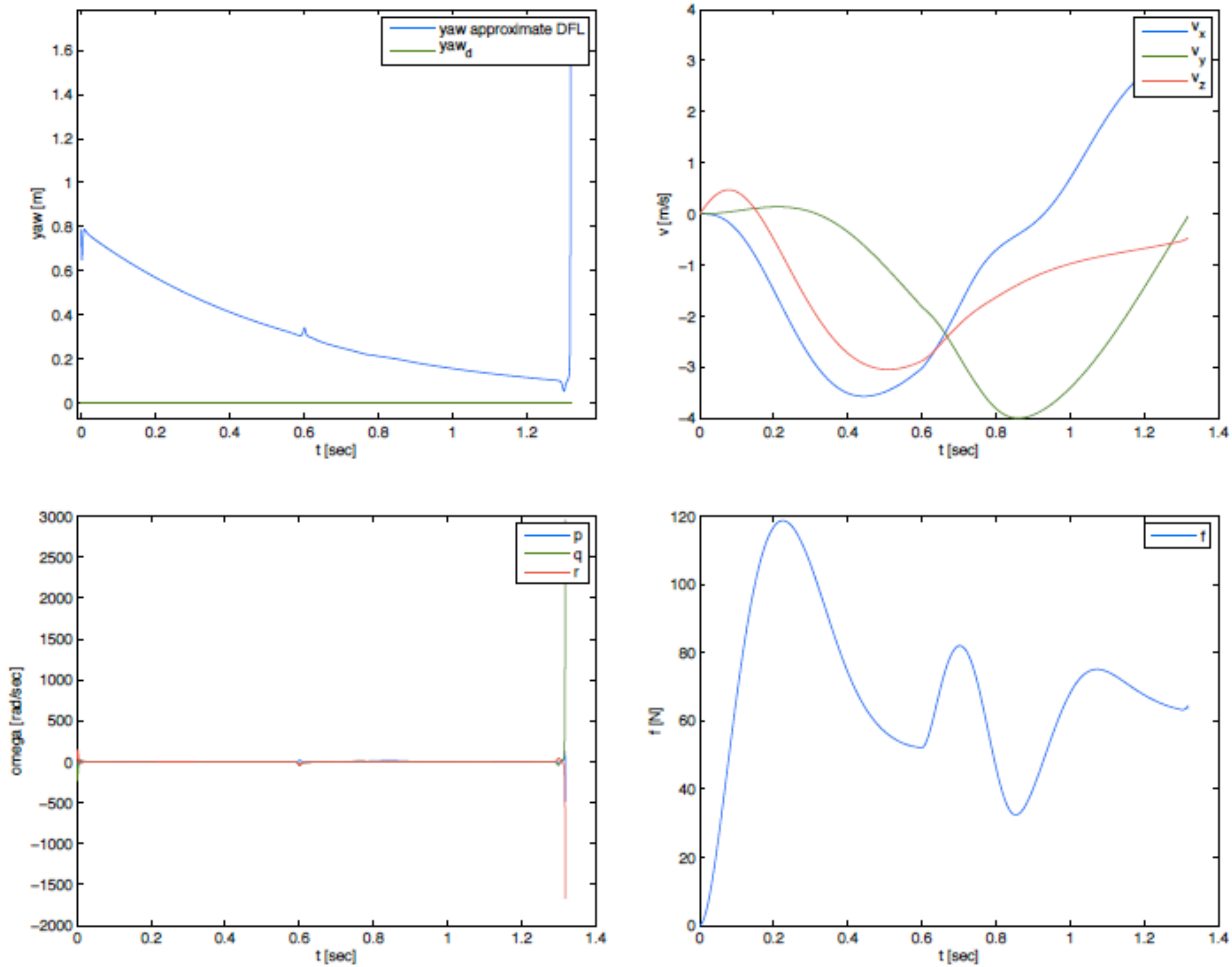



z approximate DFL
z complete DFL
z_d
z [m]
t [sec]

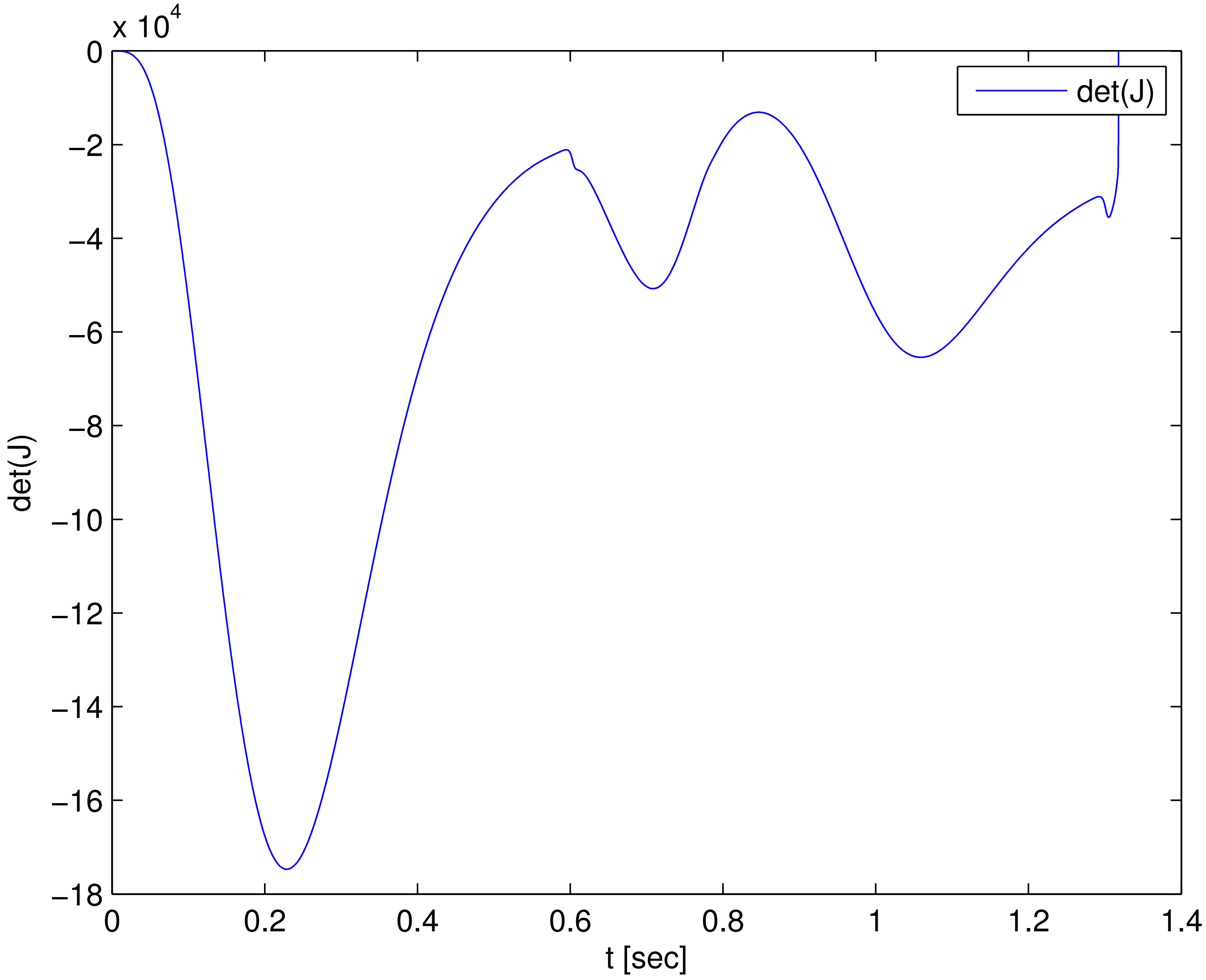

yaw approximate DFL
yaw complete DFL
yaw_d
t [sec]

- simulation 2: circle with period $T=1.8s$ and radius $R=1m$
  - approximate model





yaw approximate DFL
yaw_d
yaw [m]
t [sec]
v_x
v_y
v_z
v [m/s]
t [sec]
p
q
r
omega [rad/sec]
t [sec]
f
f [N]
t [sec]


x 10^4
0
−2
−4
−6
−8
−10
−12
−14
−16
−18
det(J)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
t [sec]
det(J)

- **simulation 2**: circle with period $T{=}1.8s$ and radius $R{=}1m$
  - complete model