Elective in Robotics

Quadrotor tracking control on SE(3)

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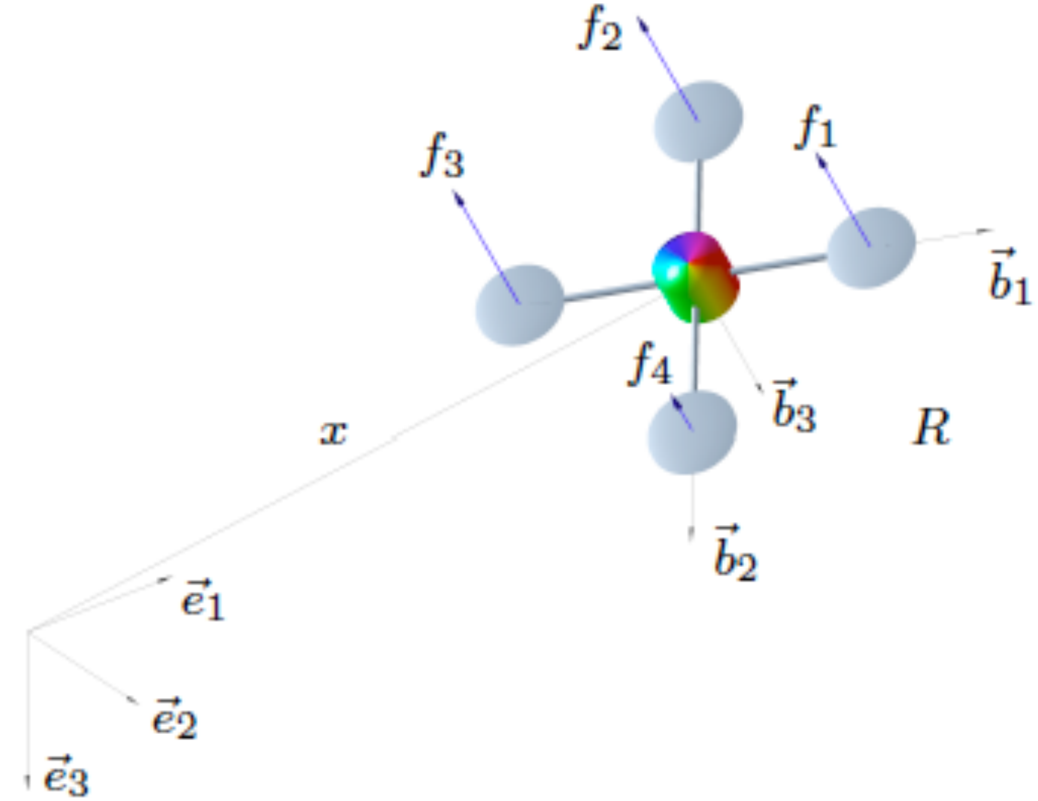
model for control design

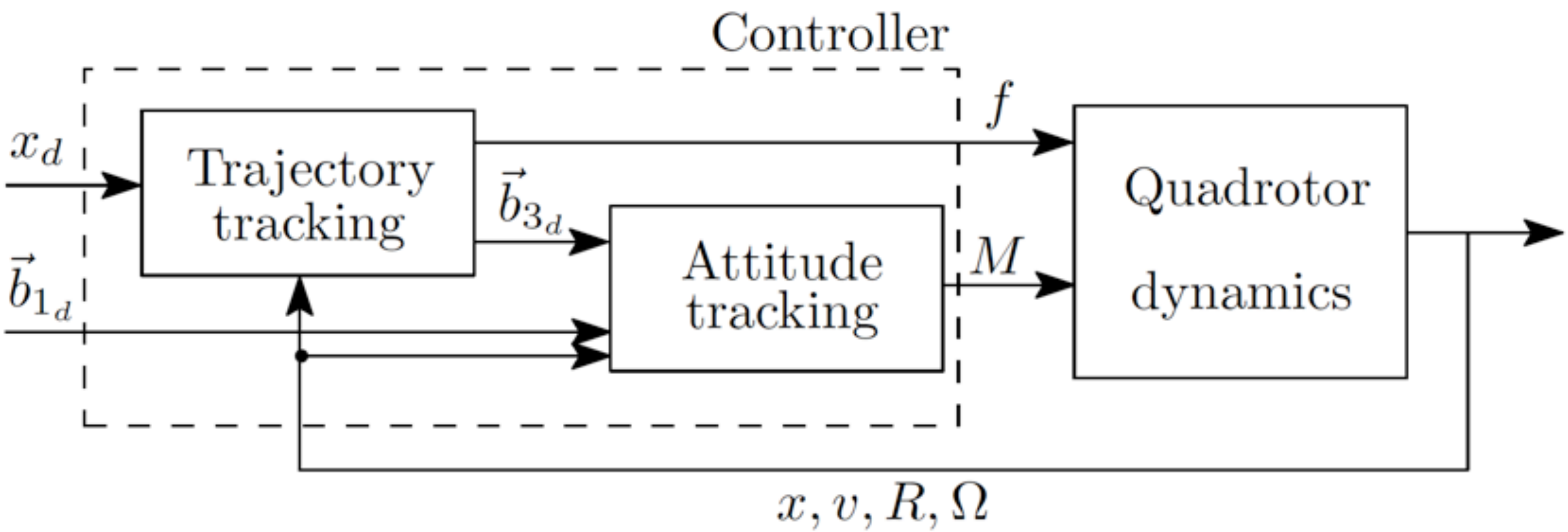
- *static* feedback controller defined on SE(3)

$$\begin{aligned} \dot{x} &= v, \\ m\dot{v} &= mge_3 - fRe_3 \\ \dot{R} &= R\hat{\Omega} \\ J\dot{\Omega} + \Omega \times J\Omega &= M \end{aligned}$$

$$f = \sum_{i=1}^{4} f_i$$



controller structure



- the translational dynamics is controlled by the total thrust -fRe_3
- the magnitude of the total thrust f is directly controlled
- the direction of the total thrust -Re_3 is along the body fixed axis $-\vec{b}_3$
- f and $\vec{b}_{3_d}$ are selected to stabilize the translational dynamics

$$f = -(-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d) \cdot Re_3$$

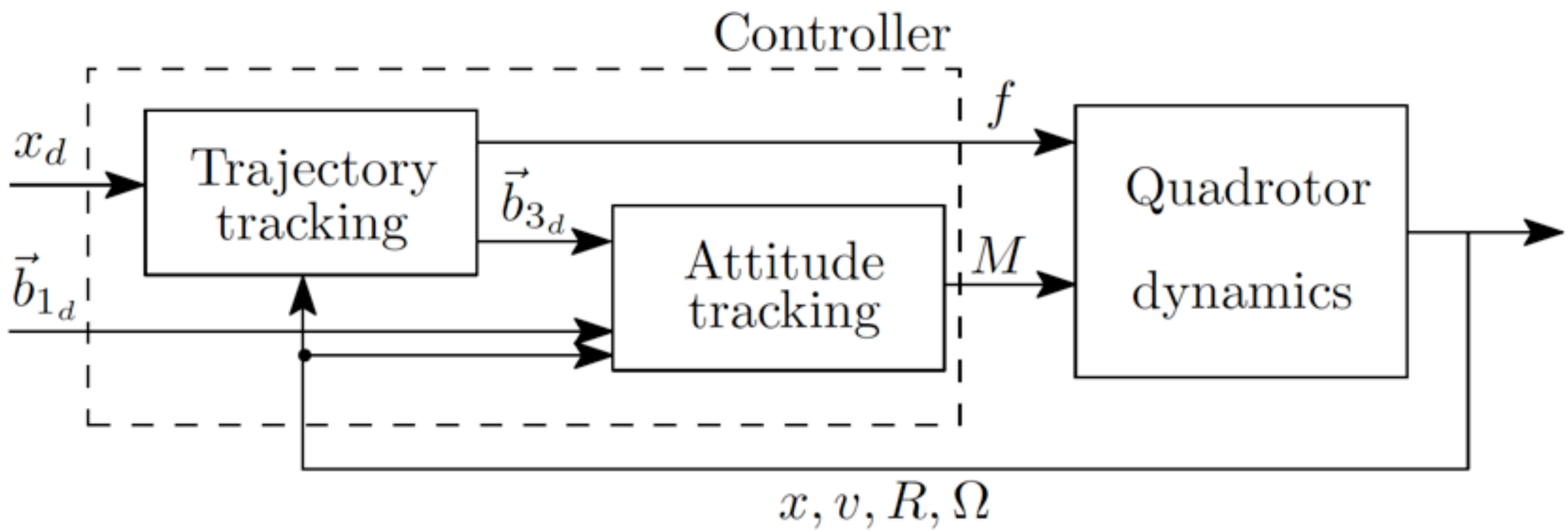
$$\vec{b}_{3_d} = -\frac{-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d}{\|-k_x e_x - k_v e_v - mge_3 + m\ddot{x}_d\|}$$

with

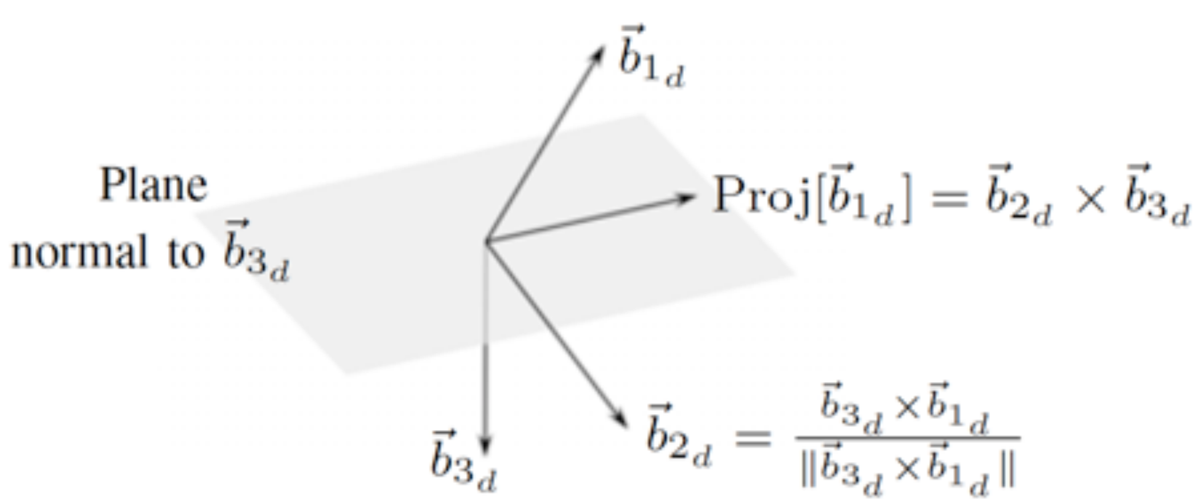
$$e_x = x - x_d$$

$$e_v = v - v_d$$

controller structure



- given $\vec{b}_{1_d}$ and determined $\vec{b}_{3_d}$ the desired attitude $R_d = [\vec{b}_{2_d} \times \vec{b}_{3_d}, \vec{b}_{2_d}, \vec{b}_{3_d}] \in$ SO(3) is selected as



- attitude and angular velocity error definition

$$e_R = \frac{1}{2}(R_d^T R - R^T R_d)^\vee$$

$$e_\Omega = \Omega - R^T R_d \Omega_d$$

- tracking controller on SO(3)

$$M = -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega - J(\hat{\Omega} R^T R_d \Omega_d - R^T R_d \dot{\Omega}_d)$$

controller stability properties

- the position tracking error converges to zero when there is no attitude tracking error

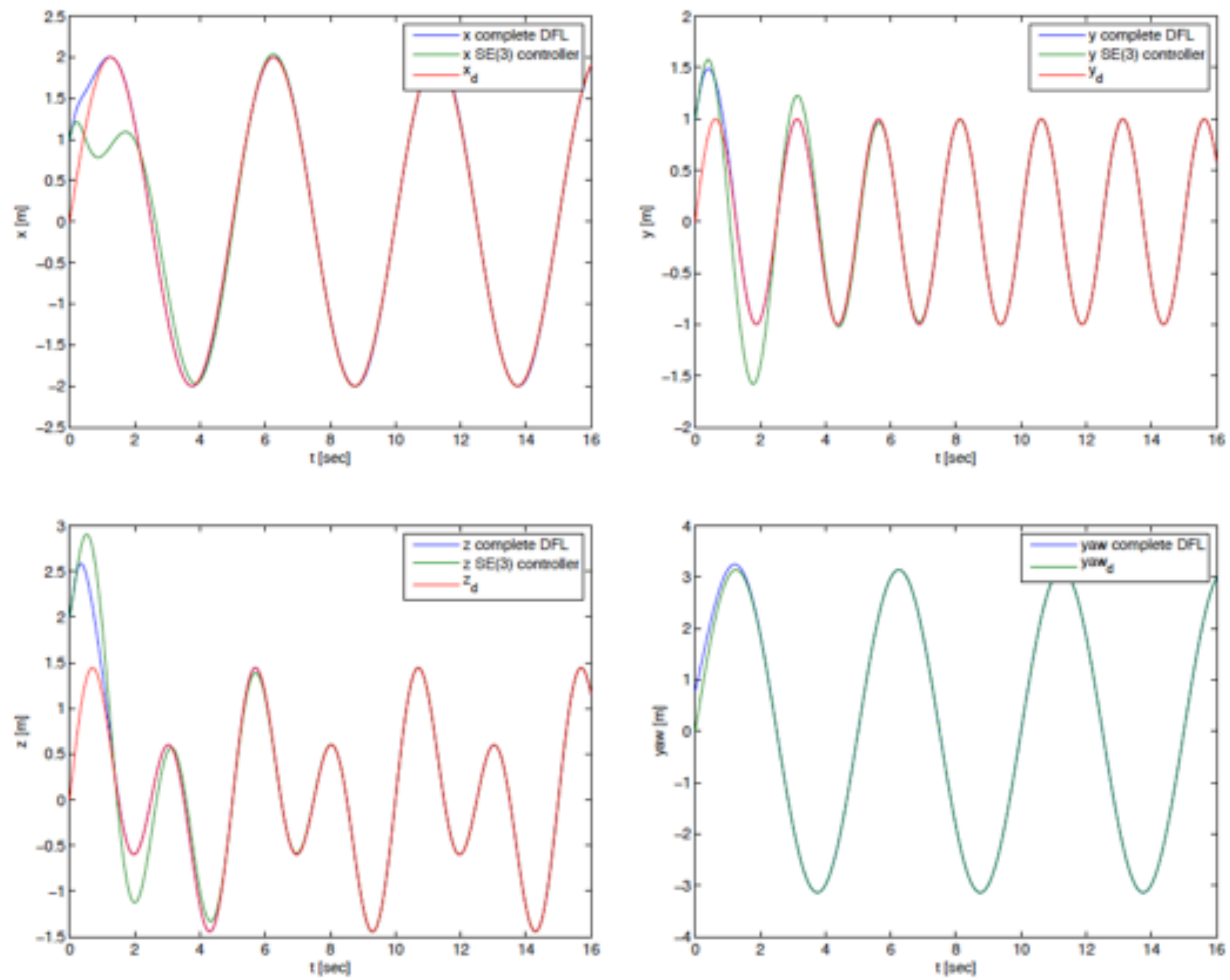
- it is properly adjusted for non-zero attitude tracking errors to achieve asymptotic stability of the complete dynamics: the dot product between the desired third body axis $\vec{b}_{3_d} = R_d e_3$ and the actual one $\vec{b}_3 = R e_3$ in the definition of f reduces the magnitude of f when the angle between the two axes becomes bigger than zero

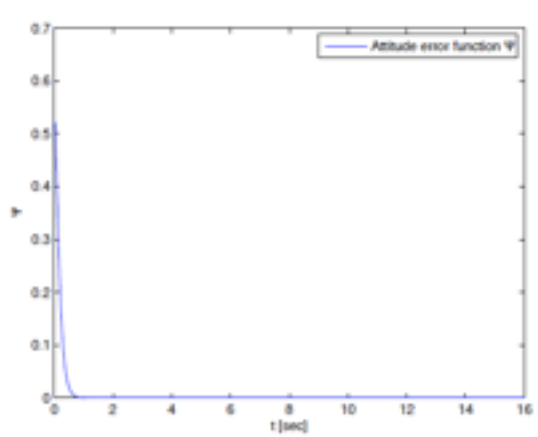
- the attitude tracking controller exponentially stabilizes the zero equilibrium of the attitude tracking error

comparative simulation with the DFL controller

- simulation 1: tracking an 8-shaped trajectory while spinning

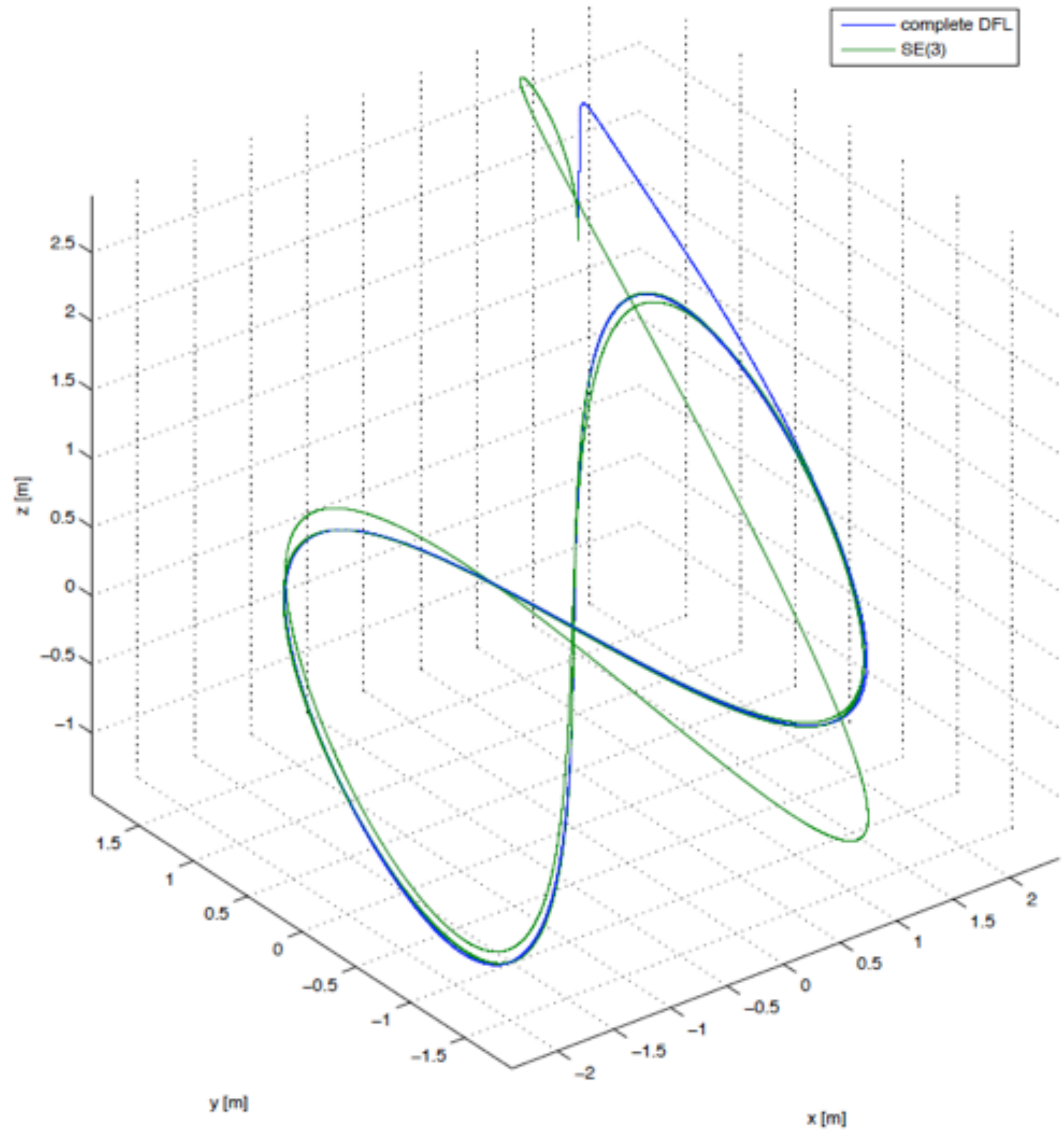
$$x_d = \left(2\,\sin\frac{2\pi}{T}t \quad \sin 2\,\frac{2\pi}{T}t \quad 0.6\,\sin\frac{2\pi}{T}t + \sin 2\,\frac{2\pi}{T}t\right)^T \;, \quad \psi_d = \pi\,\sin\frac{2\pi}{T}t$$





- attitude error function

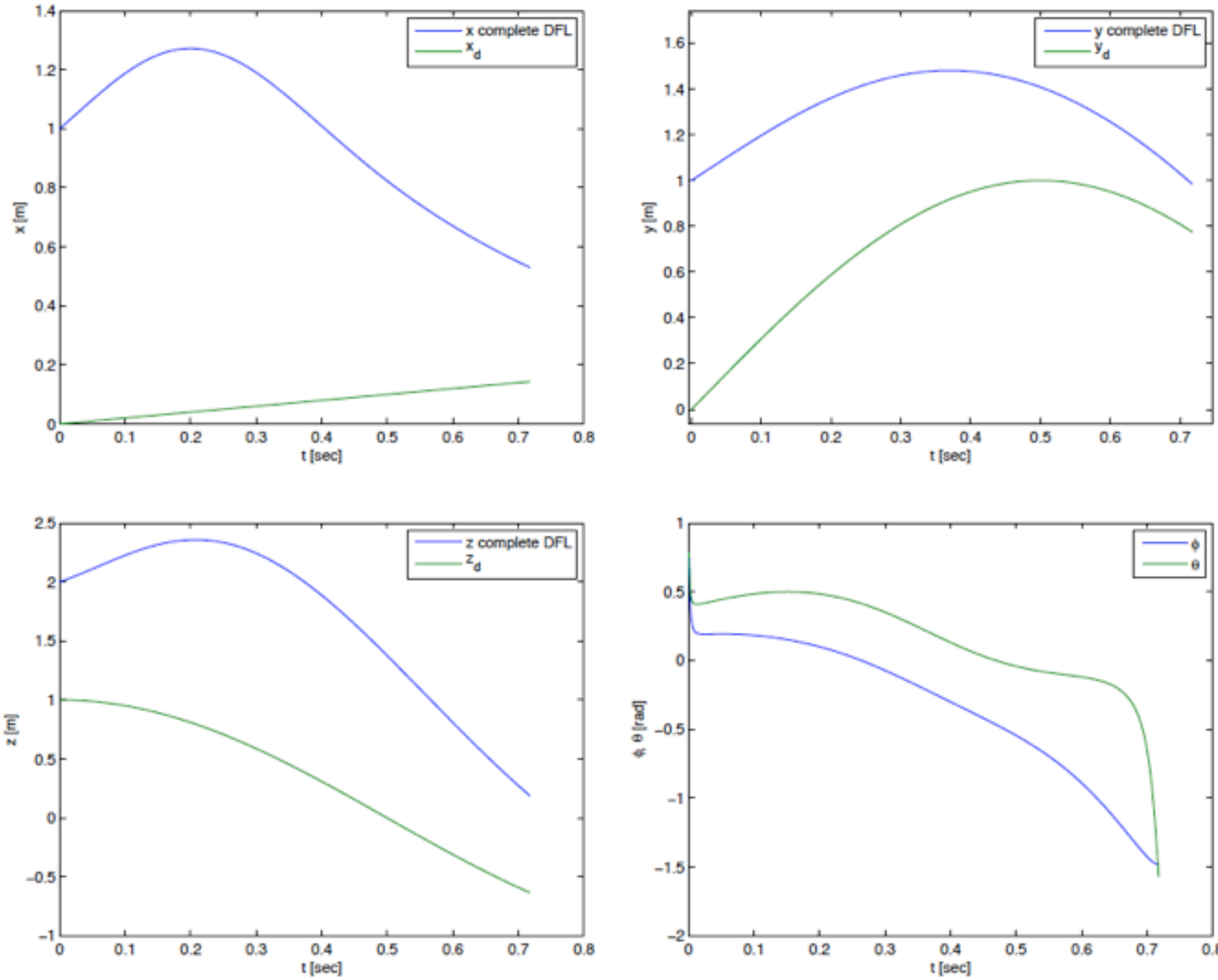
$$\Psi(R, R_d) = \frac{1}{2}\mathrm{tr}\left[I - R_d^T R\right]$$



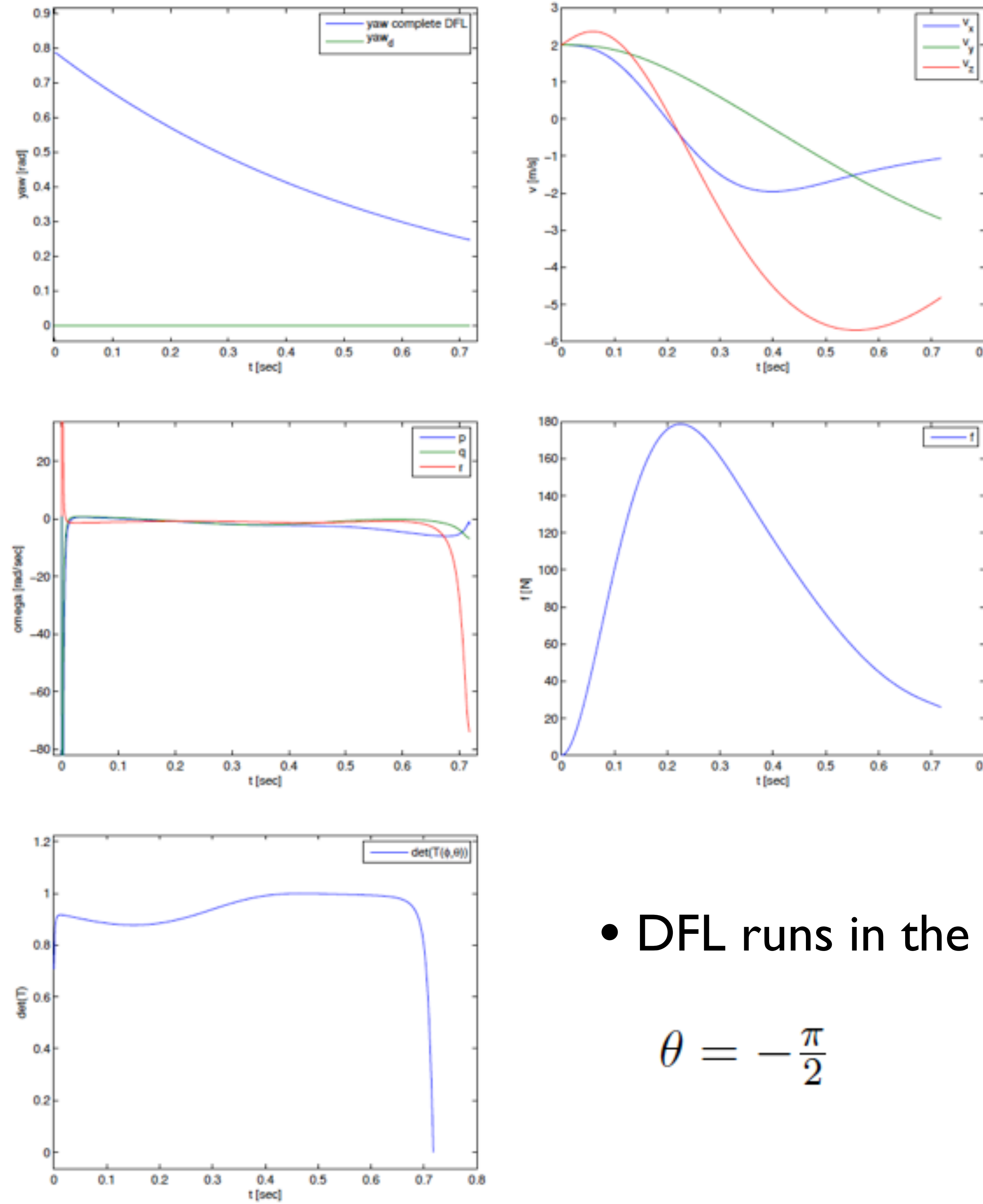
- different transient behaviors in position tracking: faster DFL convergence

- simulation 2: tracking a helix with constant yaw angle

$$x_d = \left(0.2\,t \quad \sin\frac{2\pi}{T}t \quad \cos\frac{2\pi}{T}t\right)^T \;, \quad \psi_d = 0$$



- DFL runs in the singularity $\theta = -\frac{\pi}{2}$



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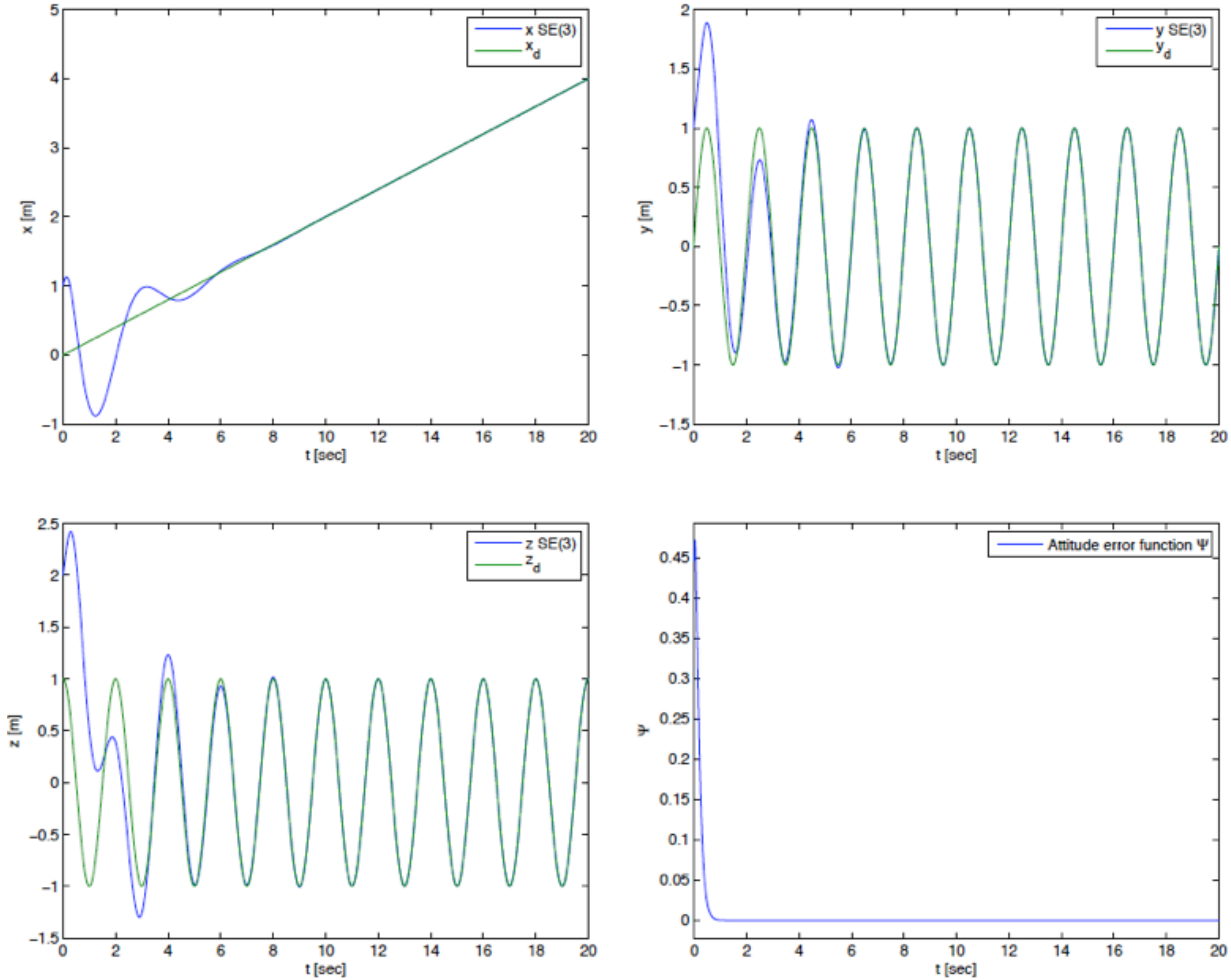

- tracking controller on SE(3)

References

- T. Lee, M. Leoky, and N. Harris McClamroch, "Geometric Tracking Control of a Quadrotor UAV on SE(3)," 49th IEEE Conference on Decision and Control, pp. 5420-5425, 2012

- T. Lee, M. Leok, and N. McClamroch, "Control of Complex Maneuvers for a Quadrotor UAV using Geometric Methods on SE(3)," [Online]. Available: http://arxiv.org/abs/1003.2005v4