Model checking for nonmonotonic logics: algorithms and complexity

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Abstract

We study the complexity of model checking in propositional nonmonotonic logics. Specifically, we first define the problem of model checking in such formalisms, based on the fact that several nonmonotonic logics make use of interpretation structures (i.e. default extensions, stable expansions, universal Kripke models) which are more complex than standard interpretations of propositional logic. Then, we analyze the complexity of checking whether a given interpretation structure satisfies a nonmonotonic theory. In particular, we characterize the complexity of model checking for Reiter's default logic and its restrictions, Moore's autoepistemic logic, and several nonmonotonic modal logics. The results obtained show that, in all such formalisms, model checking is computationally easier than logical inference.

1 Introduction

In recent years the problem of model checking has been widely studied in knowledge representation and AI [Levesque, 1986; Halpern and Vardi, 1991]. Informally, model checking for a logical formalism L corresponds to the following problem: given an interpretation structure I and a logical formula Σ, does I satisfy Σ according to the semantics of L?

Model checking has been convincingly advocated as an alternative to classical reasoning, i.e. logical inference. The main advantage of model checking lies in the fact that in general it is computationally easier than logical inference: For instance, it is well-known that, in first-order logic, model checking is polynomial in the size of the interpretation structure. Besides "classical" application domains (like hardware verification), model checking techniques have been recently employed in the field of planning and cognitive robotics [Cimatti *et al.*, 1997].

Lately, model checking has been studied in some propositional nonmonotonic settings [Cadoli, 1992; Liberatore and Schaerf, 1998]. In particular, [Liberatore and Schaerf, 1998] analyze the problem of checking whether a classical (propositional) interpretation "satisfies" a given default theory, in the sense that such interpretation satisfies at least one extension of the default theory.

The results obtained show that for propositional default logic this kind of model checking is in general as hard as logical inference, hence the computational advantages of model checking over theorem proving do not seem to hold in the case of default logic.

The work presented in this paper originates from a different definition of model checking for default logic and several other nonmonotonic logics. Such a notion is an immediate consequence of the fact that many nonmonotonic formalisms make use of interpretation structures (i.e. default extensions, autoepistemic expansions, universal Kripke models) which are more complex than standard interpretations of classical logic, and which can be represented in a compact way by means of logical formulas. Hence, we argue that model checking in such frameworks corresponds to verify whether a given interpretation structure of this form satisfies a nonmonotonic theory, according to the semantics of the formalism. E.g., according to this notion, a *model* of a default theory is a default extension, and model checking for propositional default logic corresponds to verify whether a given propositional *formula* represents an extension of a given default theory. Hence, the notion of model checking in such nonmonotonic formalisms is peculiar in the sense that the interpretation structure is represented by means of a logical formula.

We thus provide a computational analysis of the above notion of model checking for several propositional nonmonotonic logics. In particular, we characterize the complexity of model checking in Reiter's default logic [Reiter, 1980], *disjunctive* default logic [Gelfond *et al.*, 1991], and for several syntactic restrictions of such formalisms; we also study model checking in Moore's autoepistemic logic AEL [Moore, 1985], and in several other nonmonotonic modal logics, including McDermott and Doyle's (MDD) modal logics [Marek and Truszczyński, 1993], the modal logic of minimal knowledge $\mathsf{S5}_G$ [Halpern and Moses, 1985], and the logic of minimal knowledge and negation as failure MKNF [Lifschitz, 1991].

Our analysis shows that the problem of model checking is easier than logical inference in all the cases examined: typically, model checking for propositional nonmonotonic formalisms is complete with respect to the class Θ_2^p [Eiter and Gottlob, 1997], while logical inference is typically Π_2^p-complete in such logics. We also provide model checking algorithms for both default logic and several nonmonotonic modal logics.

In the following, we first briefly recall Reiter's default logic and Moore's autoepistemic logic. Then, in Section 3 we analyze model checking in default logic, and in Section 4 we study model checking in nonmonotonic modal logics. Finally, in Section 5 we compare our approach with recent related work, and conclude in Section 6.

2 Preliminaries

We start by briefly recalling Reiter's default logic [Reiter, 1980]. Let $\mathcal{L}$ be the usual propositional language. A *default rule* is a rule of the form

$$\frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma} \quad (1)$$

where $n \geq 0$ and α (called the *prerequisite*), $\beta_1, \ldots, \beta_n$ (called *justifications*), and γ (called the *conclusion*) are all formulas from $\mathcal{L}$. A *default theory* is a pair $\langle D, W\rangle$ where $W \in \mathcal{L}$ and D is a set of default rules.

Default theories in which each rule is of the form $\frac{\alpha:\beta}{\beta}$ are called *normal* (i.e. the justification is equal to the consequence of the default). Moreover, if each default is of the form $\frac{:\beta}{\beta}$, then the default theory is called *supernormal*.

The characterization of default theories is given through the notion of *extension*, i.e. a deductively closed set of propositional formulas. In the following, given a set of propositional formulas G, we denote with $Cn(G)$ the deductive closure of G, i.e. the set of propositional formulas logically implied by G.

Let $E \subseteq \mathcal{L}$, and let D be a set of default rules. We denote with $D(E)$ (and say that $D(E)$ is the *reduct* of D with respect to E) the set

$$\left\{ \frac{\alpha :}{\gamma} : \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma} \in D \text{ and } \neg\beta_i \notin E \text{ for each i} \right\}$$

We say that a set $E \subseteq \mathcal{L}$ is closed under a set of justification-free default rules D if, for each $\frac{\alpha:}{\gamma} \in D$, if $\alpha \in E$ then $\gamma \in E$.

Definition 1 *[Gelfond* et al.*, 1991, Theorem 2.3] Let $\langle D, W\rangle$ be a default theory, and let $E \subseteq \mathcal{L}$. E is an extension for $\langle D, W\rangle$ iff $W \in E$ and E is the minimal set closed under deduction and closed under the set $D(E)$.*

We recall that each extension is fully characterized by the set of conclusions of the default rules applied during this construction: in fact, it is easy to see that, for each extension of $\langle D, W\rangle$, there exists a subset G of the set of conclusions $\{\gamma_1, \ldots, \gamma_k\}$ of the default rules in D (which is denoted as $Con(D)$) such that $E = Cn(W \wedge \bigwedge_{\gamma_i \in G} \gamma_i)$. Hence, each extension of a given default theory can be represented in terms of a propositional formula f (or any propositional formula equivalent to f). Moreover, the propositional formula $f = W \wedge \bigwedge_{\gamma_i \in G} \gamma_i$ provides a *finite* representation of an infinite structure (i.e. the extension).

In [Gelfond *et al.*, 1991] default logic has been extended to the case of disjunctive conclusions, in the following way. A *disjunctive default rule* is a rule of the form

$$\frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma_1 | \ldots | \gamma_m}$$

where $n, m \geq 0$ and $\alpha, \beta_1, \ldots, \beta_n, \gamma_1, \ldots, \gamma_m \in \mathcal{L}$. A disjunctive default theory is a pair $\langle D, W\rangle$ where $W \in \mathcal{L}$ and D is a set of disjunctive default rules. The characterization of disjunctive default theories is given by changing (in a conservative way) the above notion of extension as follows.

The reduct $D(E)$ wrt $E \subseteq \mathcal{L}$ of a set of disjunctive defaults D is the set

$$\left\{ \frac{\alpha :}{\gamma_1 | \ldots | \gamma_m} : \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma_1 | \ldots | \gamma_m} \in D \text{ and } \neg\beta_i \notin E \text{ for each i} \right\}$$

We say that a set $E \subseteq \mathcal{L}$ is closed under a set of justification-free disjunctive default rules D if, for each $\frac{\alpha:}{\gamma_1|\ldots|\gamma_m} \in D$, if $\alpha \in E$ then $\gamma_i \in E$ for some i such that $1 \leq i \leq m$. $E \subseteq \mathcal{L}$ is an extension for a disjunctive default theory $\langle D, W\rangle$ iff $W \in E$ and E is a minimal set closed under deduction and under the set $D(E)$.

We finally briefly recall Moore's autoepistemic logic (AEL). We denote with $\mathcal{L}_K$ the modal extension of $\mathcal{L}$ with the modality K. Moreover, we denote with $\mathcal{L}_K^F$ the set of *flat* modal formulas, that is the set of formulas from $\mathcal{L}_K$ in which each propositional symbol appears in the scope of exactly one modality.

Definition 2 *A consistent set of formulas T from $\mathcal{L}_K$ is a* stable expansion *for a formula $\Sigma \in \mathcal{L}_K$ if T satisfies the following equation:*

$$T = Cn_{\mathsf{KD45}}(\{\Sigma\} \cup \{\neg K\varphi \mid \varphi \notin T\})$$

where Cn_{KD45} is the logical consequence operator of modal logic KD45.

Given $\Sigma, \varphi \in \mathcal{L}_K$, $\Sigma \models_{\mathrm{AEL}} \varphi$ iff φ belongs to all the stable expansions of Σ. Notably, each stable expansion T is a *stable set*, i.e. (i) T is closed under propositional consequence; (ii) if $\varphi \in T$ then $K\varphi \in T$; (iii) if $\varphi \notin T$ then $\neg K\varphi \in T$. We recall that each stable set S corresponds to a *maximal* universal S5 model $\mathcal{M}_S$ such that S is the set of formulas satisfied by $\mathcal{M}_S$ (see e.g. [Marek and Truszczyński, 1993]).

With the term AEL *model* for Σ we will refer to an S5 model whose set of theorems corresponds to a stable expansion for Σ in AEL: without loss of generality, we will identify such a model with the set of interpretations it contains. Moreover, each S5 model corresponding to a stable expansion S of a formula Σ can be characterized by a propositional formula f such that $\mathcal{M}_S = \{I : I \models f\}$; f is called the *objective kernel* of the stable expansion S. As in the case of default logic, f provides a finite representation of an infinite structure.

Finally, notice that, as in e.g. [Marek and Truszczyński, 1993], we have adopted the notion of *consistent* autoepistemic logic, i.e. we do not allow the inconsistent theory consisting of all modal formulas to be a (possible) stable expansion. The results we present can be easily extended to this case (corresponding to Moore's original proposal).

We finally briefly introduce the complexity classes mentioned throughout the paper (we refer to [Johnson, 1990] for further details). All the classes we use reside in the *polynomial hierarchy*. In particular, the complexity class Σ_2^p is the class of problems that are solved in polynomial time by a nondeterministic Turing machine that uses an NP-oracle (i.e., that solves in constant time any problem in NP), and Π_2^p is the class of problems that are complement of a problem in Σ_2^p. The class Θ_2^p [Eiter and Gottlob, 1997] (also known as $\Delta_2^p[O(log\ n)]$) is the class of problems that are solved in polynomial time by a *deterministic* Turing machine that makes a number of calls to an NP-oracle which is logarithmic in the size of the input. Hence, the class Θ_2^p is "mildly" harder than the class NP, since a problem in Θ_2^p can be solved by solving "few" (i.e. a logarithmic number of) instances of problems in NP. It is generally assumed that the polynomial hierarchy does not collapse, and that a problem in the class Θ_2^p is computationally easier than a Σ_2^p-hard or Π_2^p-hard problem.

3 Model checking in default logic

In this section we analyze the complexity of model checking for propositional default logic. We start by proving that such a problem belongs to the complexity class Θ_2^p. To this aim, we define the algorithm DL-Check (reported in Figure 1) for checking whether a propositional formula f represents an extension of a default theory $\langle D, W\rangle$. The algorithm first computes D', the reduct of D with respect to f, then computes a formula representing the extension of $\langle D', W\rangle$, and finally checks whether such a formula is equivalent to f.

In the algorithm, we make use of the well-known fact that a justification-free default theory $\langle D', W\rangle$ has exactly one extension. We denote as $Ext(\langle D', W\rangle)$ the propositional formula representing such an extension, which can be naively computed through a quadratic (in the cardinality of D') number of NP-calls, starting from $Ext(\langle D', W\rangle) = W$ and conjoining to $Ext(\langle D', W\rangle)$ the conclusions γ_i of each default rule $d_i = \frac{\alpha_i}{\gamma_i}$ in D' such that α_i is logically implied by $Ext(\langle D', W\rangle)$.

Correctness of the algorithm follows immediately from Definition 1.

Lemma 3 *Let $\langle D, W\rangle$ be a default theory, and let $f \in \mathcal{L}$. Then, $Cn(f)$ is an extension of $\langle D, W\rangle$ iff DL-Check($\langle D, W\rangle, f$) returns* true.

The computational analysis of the algorithm DL-Check provides an upper bound for the model checking problem in default logic.

Algorithm DL-Check($\langle D, W\rangle, f$)
Input: default theory $\langle D, W\rangle$, formula $f \in \mathcal{L}$;
Output: true if $Cn(f)$ is an extension of $\langle D, W\rangle$,
false otherwise
begin
$D' = \emptyset$;
for each $d = \frac{\alpha:\beta_1,\ldots,\beta_n}{\gamma} \in D$ **do**
if $f \not\models \neg\beta_i$ for each $i = 1, \ldots, n$
then add $\frac{\alpha:}{\gamma}$ to D';
compute $Ext(\langle D', W\rangle)$;
if $\models f \equiv Ext(\langle D', W\rangle)$
then return true
else return false
end

Figure 1: Algorithm DL-Check.

Theorem 4 *Let $\langle D, W\rangle$ be a default theory, and let $f \in \mathcal{L}$. Then, the problem of establishing whether $Cn(f)$ is an extension of $\langle D, W\rangle$ is in Θ_2^p.*

Proof sketch. First, we prove that it is possible to compute the formula $Ext(\langle D', W\rangle)$ through a *linear* number (in the cardinality of D') of calls to an NP-oracle, by using the following procedure:

$Ext(\langle D', W\rangle) := W$;
for each $\gamma_i \in Con(D')$ **do**
if for each partition (P, N) of $Con(D')$
($\gamma_i \in P$) **or**
(there exists $\gamma_j \in N$ s. t. $(W \wedge \bigwedge_{\gamma\in P} \gamma) \models \alpha_j$)
then $Ext(\langle D', W\rangle) := Ext(\langle D', W\rangle) \wedge \gamma_i$

Then, based on the use of the above procedure for computing $Ext(\langle D', W\rangle)$, we show that the algorithm DL-Check can be reduced to an *NP-tree* [Eiter and Gottlob, 1997], which immediately implies an upper bound of Θ_2^p for the problem of model checking in propositional default logic. □

We now turn our attention to establishing lower bounds for model checking in default logic. We first prove that such a problem is Θ_2^p-hard even if default rules are normal.

Theorem 5 *Let $\langle D, W\rangle$ be a normal default theory, and let $f \in \mathcal{L}$. Then, the problem of establishing whether $Cn(f)$ is an extension of $\langle D, W\rangle$ is Θ_2^p-hard.*

Proof sketch. We reduce the Θ_2^p-complete problem PARITY(SAT) [Eiter and Gottlob, 1997] to model checking for a normal default theory. Informally, an instance of PARITY(SAT) is a set of propositional formulas $\varphi_1, \ldots, \varphi_n$, such that if φ_i is not satisfiable then, for each $j \geq i$, φ_j is not satisfiable. The problem is to establish if the number of satisfiable formulas is odd.

Given an instance of such a problem, in which we assume n odd without loss of generality, we construct the normal default theory $\langle D, W\rangle$, in which $W =$ true and

$$D = \left\{ \frac{\neg\varphi_1 : p'}{p'}, \frac{\neg\varphi_3 : \varphi_2 \wedge p'}{\varphi_2 \wedge p'}, \ldots, \frac{\neg\varphi_n : \varphi_{n-1} \wedge p'}{\varphi_{n-1} \wedge p'} \right\}$$

where p' is a propositional symbol not appearing in $\varphi_1, \ldots, \varphi_n$.

We prove that there is an odd number of satisfiable formulas in $\varphi_1, \ldots, \varphi_n$ iff true is an extension of $\langle D, W \rangle$. Informally, this is due to the fact that the number of satisfiable formulas is even if and only if either all formulas are not satisfiable (i.e. φ_1 is not satsfiable) or, for some even i, it holds that φ_i is satisfiable and φ_{i+1} is not satisfiable. Now, the rules in D are built in such a way that, if this situation occurs, then there is a default rule which is applied, thus forcing knowledge of p' in the extension. Therefore, in this case true is not an extension for $\langle D, W \rangle$. □

The above property, together with Theorem 4, immediately implies that model checking is Θ_2^p-complete both for general propositional default theories and for normal default theories.

Then, with a proof similar to the previous one, it is possible to show that model checking is Θ_2^p-hard also in the case of prerequisite-free default theories.

Theorem 6 *Let $\langle D, W \rangle$ be a prerequisite-free default theory, and let $f \in \mathcal{L}$. Then, the problem of establishing whether $Cn(f)$ is an extension of $\langle D, W \rangle$ is Θ_2^p-hard.*

Again, the above theorem and Theorem 4 prove that model checking is Θ_2^p-complete for prerequisite-free default theories.

We now turn our attention to supernormal (i.e. both normal and prerequisite-free) default theories, and prove that in this case model checking is computationally easier than for unrestricted default theories.

Theorem 7 *Let $\langle D, W \rangle$ be a supernormal default theory, and let $f \in \mathcal{L}$. The problem of establishing whether $Cn(f)$ is an extension of $\langle D, W \rangle$ is coNP-complete.*

Proof sketch. As for membership in coNP, we reduce the problem to a propositional validity problem. The key property is the fact that, given

$$D = \left\{ \frac{:\beta_1}{\beta_1}, \ldots, \frac{:\beta_n}{\beta_n} \right\},$$

$Cn(f)$ is an extension of $\langle D, W \rangle$ iff the following two conditions hold:

1. for each i, either $f \models \beta_i$ or $f \models \neg\beta_i$;
2. $W \wedge \bigwedge_{f \models \beta_i} \beta_i$ is equivalent to f.

We prove that it is possible to encode each of the two above conditions in terms of a propositional validity problem, through two polynomial transformation of the input. We thus obtain two propositional formulas $\tau_1(\langle D, W \rangle, f)$ and $\tau_2(\langle D, W \rangle, f)$ such that condition 1. holds iff $\tau_1(\langle D, W \rangle, f)$ is valid and condition 2. holds iff $\tau_1(\langle D, W \rangle, f)$ is valid. Then, by simply using two distinct alphabets for the two formulas, it is possible to reduce the two problems to a single validity problem.

Hardness with respect to coNP follows from the fact that propositional validity of a formula f can be reduced to the problem of establishing whether f is an extension of $\langle \emptyset, \mathsf{true} \rangle$. □

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Algorithm AEL-Check(Σ, f)
Input: formula Σ ∈ L_K, formula f ∈ L;
Output: true if M = {I : I ⊨ f} is AEL model for Σ,
        false otherwise
begin
  while Σ ∉ L do begin
     choose a subformula Kφ from Σ
           such that φ ∈ L;
     if f ⊨ φ
     then Σ := Σ(Kφ → true)
     else Σ := Σ(Kφ → false)
  end;
  if ⊨ f ≡ Σ
  then return true
  else return false
end
```

Figure 2: Algorithm AEL-Check.

As for disjunctive default logic, the easiest way to characterize model checking is to exploit known correspondences between such a formalism and nonmonotonic modal logic MKNF [Lifschitz, 1991]. In particular, the existence of a polynomial embedding of disjunctive default theories in the flat fragment of the logic MKNF makes it possible to show that model checking is in Θ_2^p. Moreover, Θ_2^p-hardness follows from Theorem 5 and from the fact that disjunctive default logic is a conservative generalization of default logic. Hence, the following property holds.

Theorem 8 *Let $\langle D, W \rangle$ be a disjunctive default theory, and let $f \in \mathcal{L}$. Then, the problem of establishing whether $Cn(f)$ is an extension of $\langle D, W \rangle$ is Θ_2^p-complete.*

The above property and Theorem 6 also imply Θ_2^p-completeness of model checking for prerequisite-free disjunctive default theories.

In Table 1 we summarize the complexity results described in this section. Each column of the table corresponds to a different condition on the conclusion part of default rules.

The results reported in the table, together with known complexity characterizations of the inference problem in default logic (for a survey see [Cadoli and Schaerf, 1993]), show that model checking is easier than logical inference in all the cases considered. In fact, logical inference is already Π_2^p-hard (skeptical reasoning) or Σ_2^p-hard (credulous reasoning) for supernormal default theories, while model checking is always in Θ_2^p.

4 Model checking in nonmonotonic modal logics

In this section we analyze model checking for nonmonotonic modal logics. Due to lack of space, in the following we only sketch our complexity analysis, and refer to [Marek and Truszczyński, 1993; Lifschitz, 1991] for a formal definition of MDD logics and MKNF: all the results obtained are summarized in Table 2.

	General	Normal	Disjunctive
General	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete
Prerequisite-free	Θ_2^p-complete	coNP-complete	Θ_2^p-complete

Table 1: Complexity of model checking for default logic

	AEL	$\mathsf{S4F}_{\mathrm{MDD}}$	$\mathsf{SW5}_{\mathrm{MDD}}$	$\mathsf{S5}_G$	MKNF
General	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete	Σ_2^p-complete	Σ_2^p-complete
Flat	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete	Θ_2^p-complete

Table 2: Complexity of model checking for nonmonotonic modal logics

We start by examining the case of autoepistemic logic. In Figure 2 we report the algorithm AEL-Check for checking whether a propositional formula represents an autoepistemic model of a modal formula. In the algorithm, $\Sigma(K\varphi \rightarrow \mathsf{true})$ represents the formula obtained from Σ by replacing each occurrence of the subformula $K\varphi$ with true, while $\Sigma(K\varphi \rightarrow \mathsf{false})$ represents the formula obtained from Σ by replacing each occurrence of the subformula $K\varphi$ with false.

Informally, the algorithm iteratively computes the value of all modal subformulas (without nested occurrences of the modality) in Σ according to f, until all modal subformulas have been replaced by a truth value. The resulting propositional formula is compared with f, and the algorithm returns true if and only if the two formulas are equivalent.

Correctness of the algorithm can be established by means of previous results on reasoning in autoepistemic logic [Marek and Truszczyński, 1993].

Lemma 9 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, $M = \{I : I \models f\}$ is an* AEL *model of Σ iff* AEL-*Check(Σ, f) returns* true.

The above property allows us to prove Θ_2^p-completeness of model checking in AEL.

Theorem 10 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an* AEL *model of Σ is Θ_2^p-complete.*

Proof sketch. Membership in Θ_2^p follows from Lemma 9 and from the fact that the algorithm AEL-Check can be polynomially reduced to an NP-tree. Hardness follows from the fact that it is possible to reduce an instance of the problem of model checking for prerequisite-free default theories to model checking in AEL: the reduction is based on the correspondence between the prerequisite-free default $\frac{:\beta}{\gamma}$ and the modal formula $\neg K\neg\beta \supset \gamma$ in autoepistemic logic. □

It can actually be shown that model checking for AEL is Θ_2^p-hard (and thus, from the above theorem, Θ_2^p-complete) even under the restriction that the formula Σ is *flat*, i.e. each propositional symbol in Σ lies within the scope of exactly one modality. The proof of this property can be obtained through a reduction from PARITY(SAT).

A similar analysis allows for establishing the same complexity characterization for the problem of model checking in two well-known nonmonotonic modal formalisms of the McDermott and Doyle's (MDD) family, i.e. the nonmonotonic logics based on the modal systems $\mathsf{SW5}$ and $\mathsf{S4F}$ [Marek and Truszczyński, 1993].

Theorem 11 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an $\mathsf{S4F}_{\mathrm{MDD}}$ model (or an $\mathsf{SW5}_{\mathrm{MDD}}$ model) of Σ is Θ_2^p-complete.*

As in the case of autoepistemic logic, the above property also holds if we restrict to flat formulas.

For modal logics based on the *minimal knowledge* paradigm, we prove that model checking is harder than for the above presented nonmonotonic logics. In particular, it is a Σ_2^p-complete problem. However, logical inference in such logics of minimal knowledge is harder than in default logic and autoepistemic logic, since it is a Π_3^p-complete problem both in MKNF and in $\mathsf{S5}_G$ [Donini *et al.*, 1997; Rosati, 1997]. Hence, also in such formalisms model checking is easier than logical inference. We first analyze modal logic $\mathsf{S5}_G$, i.e. the logic of minimal knowledge introduced in [Halpern and Moses, 1985].

Theorem 12 *Let $\Sigma \in \mathcal{L}_K$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an $\mathsf{S5}_G$ model of Σ is Σ_2^p-complete.*

Interestingly, if we impose that the modal formula Σ is flat, then model checking in $\mathsf{S5}_G$ becomes easier.

Theorem 13 *Let $\Sigma \in \mathcal{L}_K^F$, $f \in \mathcal{L}$. Then, the problem of establishing whether $M = \{I : I \models f\}$ is an $\mathsf{S5}_G$ model of Σ is Θ_2^p-complete.*

The same computational characterization of model checking can be shown for the logic MKNF, i.e. the logic of minimal knowledge and negation as failure introduced in [Lifschitz, 1991], which extends $\mathsf{S5}_G$ with a second modal operator interpreted in terms of negation as failure.

Comparing the above results with known computational characterizations of the inference problem in nonmonotonic modal logics, it turns out that model checking is easier than logical inference in all the cases considered. Moreover, we remark that logical inference in the flat fragment of $\mathsf{S5}_G$ and MKNF is Π_2^p-complete. This implies that, for each of the cases reported in the table, if logical inference is Π_2^p-complete, then model checking is Θ_2^p-complete, and if logical inference is Π_3^p-complete, then model checking is Σ_2^p-complete.

5 Related work

Model checking has been recently studied in some non-monotonic settings (see e.g. [Cadoli, 1992; Liberatore and Schaerf, 1998]). In particular, the work reported in [Liberatore and Schaerf, 1998] is the closest to the approach presented in this paper, since it deals with the model checking problem for propositional default logic.

The notion of model checking introduced in [Liberatore and Schaerf, 1998] for default logic corresponds to check whether a propositional interpretation I "satisfies" a given default theory $\langle D, W \rangle$, in the sense that I satisfies at least one extension of $\langle D, W \rangle$. Such a notion of model checking relies on the usage of standard propositional interpretations, thus avoiding the need to resort to the representation of an interpretation structure in terms of a logical formula. On the other hand, a propositional interpretation cannot be considered as a "model" of a default theory: in fact, model-theoretic characterizations of default logic are based on possible-world structures analogous to universal S5 models introduced for autoepistemic logic. Hence, a propositional interpretation is a *component* of an interpretation structure of a default theory. Instead, our formulation of the model checking problem is based on the idea of checking a whole interpretation structure of this form against a nonmonotonic theory: in this sense, our notion is a more natural extension to nonmonotonic logics of the "classical" notion of model checking.

From the computational viewpoint, it turns out that Liberatore and Schaerf's notion of model checking is harder than the one presented in this paper. In fact, comparing Table 1 with the results reported in [Liberatore and Schaerf, 1998], it can be seen that our formulation of model checking is computationally easier in almost all the cases examined, with the exception of normal and supernormal default theories, for which the complexity of the two versions of model checking is the same.

6 Conclusions

In this paper we have studied the complexity of model checking in several nonmonotonic logics. Our results show that, as in classical logic, model checking is computationally easier than logical inference in many nonmonotonic formalisms. We have also provided algorithms for model checking in default logic and nonmonotonic modal logics.

Our results provide a positive answer to the question whether it is convenient to use "model-based" representations of knowledge in the case of nonmonotonic logics. It therefore appears possible to use the analysis presented in this paper as the basis for the development of model checking techniques in knowledge representation systems with nonmonotonic abilities.

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