CONTROL SYSTEMS - 23/3/2019

1) Given

\[ \begin{align*}
\dot{x} &= \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} m + \begin{pmatrix} 0 \\ 1 \end{pmatrix} d, \\
w &= \begin{pmatrix} 1 & 0 \end{pmatrix} x,
\end{align*} \tag{1} \]

and \( P_2(s) = \frac{s^2}{s^3 + 1} \), determine, if any, a 2-dimensional controller \( G(s) \) such that the given feedback system has the following properties:

i) it is asymptotically stable with poles having negative real part \( \leq -2 \)

ii) the steady state output response \( y_{ss} \) to constant disturbances \( d(t) \) is 0.

Set \( d(t) = 0 \). Let \( PG \) the series interconnection of \( P_1, P_2 \) and \( G \). Draw the root locus of \( PG \) and determine for which values of \( K \in \mathbb{R} \) the feedback system \( W(s) = \frac{KPG(s)}{1 + KPG(s)} \) has all real poles.

3) Given

\[ \begin{align*}
P_1 : \dot{x}_1 &= \begin{pmatrix} 0 & -10 \\ 1 & -11 \end{pmatrix} x_1 + \begin{pmatrix} 2 \\ 1 \end{pmatrix} m_1, \\
y_1 &= \begin{pmatrix} 0 & 1 \end{pmatrix} x_1,
\end{align*} \]

\[ P_2 : \dot{x}_2 = -2x_2 + m_2, \quad y_2 = x_2, \]

determine, if any, controllers \( G(s) \) and \( K(s) \) such that the given feedback system has the following properties:

i) the input-output (from \( v \) to \( y \) transfer function \( W(s) \) has two complex poles \( \pm j \)

ii) the disturbance-output (from \( d \) to \( y \) transfer function \( W_d(s) \) is such that \( |W_d(j\omega)| \leq 0.1 \) for all \( \omega \in [0,10] \text{ rad/sec} \).