STUDENT NUMBER

CONTROL SYSTEMS - 3/7/2018
(time 2 hours; no textbooks; no programmable pocket calculator)

1) Given

\[ P(s) = \frac{s + 1}{s^3} \]

design a controller \( G(s) \) such that

(i) \( |G(j\omega)|_{dB} \leq 36dB \) for all \( \omega \geq 0 \),

(ii) the feedback system \( W(s) = \frac{PG(s)}{1 + PG(s)} \) is asymptotically stable (use
the Nyquist criterion)

(iii) the open loop system \( PG(s) \) has crossover frequency \( \omega_c^* \geq 5 \) rad/sec
and phase margin \( \phi^* \geq 30^\circ \)

(approximated Bode plots must be used for the design).

2) Given

\[
\begin{align*}
A_1 &= \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}, & B_1 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & C_1 &= \begin{pmatrix} 1 & 0 \end{pmatrix}, \\
A_2 &= -3, & B_2 &= 1, & C_2 &= 1, \\
\end{align*}
\]

with

\[ \dot{x} = A_1 x_1 + B_1 u, \quad y_1 = C_1 x_1 \]

\[ y_2 = C_2 x_2 \]

\[ d \]

\[ + \rightarrow \]

\[ G(s) \]

\[ u \]

\[ + \rightarrow \]

\[ y \]

design a controller \( G(s) \) such that the closed-loop system is asymptotically
stable and the steady state output response to constant \( d(t) \) is zero.

Draw the root locus of \( PG(s) \) using the Routh criterion to determine the
exact picture on the imaginary axis.

3) Given

\[
\frac{y(s)}{u(s)} = P(s) = \frac{s + 2}{s(s + 1)(s - 3)}
\]

find the state response \( x(t) \) with input \( u(t) = 0 \) ensuing from the initial
state \( x_0 = (1, 1, 0)^T \). For which initial conditions \( x_0 \) the state response
\( x(t) \) with input \( u(t) = 0 \) tends asymptotically to zero? Which is the
output response \( y(t) \) with \( x_0 = (0, 0, 0)^T \) to a constant input \( u(t) \)?