CONTROL SYSTEMS (A) - 4/6/2019

1) Given

\[ \frac{d}{dt} + \frac{e}{G(s)} + \frac{u}{P(s)} + y \]

with \( P(s) = \frac{1}{s^3} \) design a controller \( G(s) = K \left( \frac{1 + s}{1 + r s} \right)^2 \) such that

(i) the feedback system \( W(s) = \frac{PG(s)}{1 + PG(s)} \) is asymptotically stable (use the Nyquist criterion with reasonable approximations for the Bode plots) and its steady state output \( y_{ss} \) to disturbances \( d(t) = \sin \omega t \) is such that \( |y_{ss}| \leq 0.11 \) for all \( \omega \in [0, 0.1] \) rad/sec,

(ii) \( |G(j\omega)|_{dB} \leq 0 \) dB for all \( \omega \),

(iii) the open loop system \( PG \) has as large as possible phase margin.

2) Given

\[ \frac{d}{dt} + \frac{e}{K} + \frac{u}{P(s)} + y \]

with \( P(s) = \frac{1}{(s + a)^3} \), determine for which real values of \( K \) and \( a \) the feedback system \( W(s) = \frac{KP(s)}{1 + KP(s)} \) has the following properties:

i) it is asymptotically stable

ii) all its poles are real and negative.

Draw the root locus of \( P(s) \). Choose any value of \( K \) and \( a \) for which (i) and (ii) are satisfied and calculate the steady state response \( y_{ss}(t) \) to disturbances \( d(t) = \sin(at) \).

3) Given the system \( \dot{x} = Ax + Bu \) with

\[ A = \begin{pmatrix} 0 & 1 \\ -\alpha & -(1 + \alpha) \end{pmatrix}, \quad B = \begin{pmatrix} \beta \\ -\beta \end{pmatrix} \]

discuss the values of \( \alpha, \beta \in \mathbb{R} \) and \( \gamma > 0 \) for which there exists a control law \( u = Fx \) such that the eigenvalues of \( A + BF \) have real part \( \leq -\gamma \) and determine \( F \).