CONTROL SYSTEMS - 5/2/2019 (B)

(time 3 hours; no textbooks; no programmable calculators)

1) Given \( P(s) = \frac{2s+1}{s^2} \) design a 1-dimensional controller \( G(s) \), if any, such that
   (i) the feedback system \( W(s) = \frac{PG(s)}{1+PG(s)} \) is asymptotically stable (use the Nyquist criterion with reasonable approximations for the Bode plots) and its steady state output response \( y_{ss} \) to constant disturbances \( d \) is 0,
   (ii) \( \int \omega G(i\omega) dB < -\infty \) \( \forall \omega \)
   (iii) the open loop system \( PG \) has phase margin \( m_{ph} \geq 30^\circ \) and crossover frequency \( \omega_c \in [0.5, 1] \) rad/sec.

2) Given
   \[
   \begin{align*}
   &\text{with } P(s) = \frac{2s+3}{s^2-3s+3}, \text{ determine, if any, a 2-dimensional controller } G(s) \\
   &\text{such that the given feedback system has the following properties:} \\
   &\text{i) it is asymptotically stable with poles having negative real part} \leq -1 \\
   &\text{ii) the steady state output response to constant disturbances } d_1 \text{ is 0} \\
   &\text{ii) the absolute value of the steady state output response to unit ramp disturbances } d_2 \text{ (i.e. } d_2 = t) \text{ is } \leq 0.1.
   \end{align*}
   \]

3) Given \( P(s) = \frac{(s+1)^2}{(s+1)^2} \) draw the root locus of \( P \) and design, if any, a controller \( G_1(s) = K \) such that the closed-loop system \( W(s) = \frac{PG_1(s)}{1+PG_1(s)} \) is asymptotically stable. With \( G_1(s) = \frac{1}{s+1} \) draw the root locus of \( PG_1 \) (help: the positive root locus has the singular points \( s \approx 0.2 \pm 0.6j \) for \( k \approx 0.2 \) and \( s \approx 0.4 \) for \( k \approx 0.1 \); the negative root locus has the singular point \( s \approx -2.4 \) for \( k \approx -28 \)). Design, if any, a controller \( G_2(s) = K \) such that the closed-loop system \( W(s) = \frac{PG_2(s)}{1+PG_2(s)} \) is asymptotically stable.

4) Given the system
   \[
   \dot{x} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} d + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \quad y = \begin{pmatrix} 0 & 1 \end{pmatrix} x
   \]

   with state \( x \in \mathbb{R}^2 \), control \( u \), controlled output \( y \), disturbance \( d = Dt \), with unknown \( D \in \mathbb{R} \), and reference input \( v = e^{-t} \), find, if possible, a controller such that the closed loop system is asymptotically stable and its steady state output response \( y_{ss} \) is \( v \).