Exercise 1 Bode plots of $P(s)$ are drawn in Fig. 1. For (i) we must have $G(s) = G_1(s)G_2(s)$ with $G_1(s) = K_{G,1}$ and $G_2(0) = 1$ such that the steady state error $e_0$ to inputs $v(t) = \delta_1(t)$ satisfies

$$|e_0| = |W_e(0)| = \frac{1}{1 + P(0)K_{G,1}} \leq 1 \Rightarrow |K_{G,1}| \geq 2$$

Set $K_{G,1} = 2$. For maximizing the phase margin since $G(s)$ must be one-dimensional and since the phase of $G_1(s)P(s)$ is $-180^\circ$ for all $\omega$ (see Bode plot of $G_1(s)P(s)$ in Fig. 2), we use an anticipative action $R_a(s)$ with $m_a = 16$ at $\omega_t^* = 4$ rad/sec and $\omega_N = 4$ rad/sec (maximum phase increase $\approx 62^\circ$ with magnitude increase $\approx 12dB$). Notice that in this way we maximize the phase margin, having at hand only a one-dimensional $G(s)$, with a certain crossover frequency $\omega_t^*$. Notice that $\omega_t^* = 4$ rad/sec has been chosen on account of the fact that the magnitude of $|G_1P(j\omega_t^*)|_{dB} \leq -12dB$ which is the magnitude increase given by the anticipative action at $\omega_N$ but $|G_1P(j\omega_t^*)|_{dB} \geq -24dB$ where 24 dB is the maximum magnitude increase which can be given by a single anticipative action ($m_a = 16$). We get $\tau_a = 1$ and

$$G_2(s) = K_{G,2}R_a(s) = K_{G,2}\frac{1+s}{1 + \frac{s}{16}}$$

where $K_{G,2}$ is to be selected. In order to place the crossover frequency exactly at $\omega_t^* = 5$ rad/sec, notice that $|G_1P(j\omega_t^*)|_{dB} \approx -18dB$ and $|G_1R_aP(j\omega_t^*)|_{dB} \approx -5dB$ ($R-a$ provides a magnitude increase $\approx 12dB$). Therefore, we choose $K_{G,2} = 6dB \approx 2$ and finally the controller $G(s)$ is

$$G(s) = 4 \frac{1+s}{1 + \frac{s}{16}}$$

The Bode plots of $G(s)P(s)$ and its Nyquist plot are drawn in Fig. 3 and 4. The Nyquist plot shows that the closed-loop system is asymptotically stable (we have 1 counterclockwise tour around the point $-1+0j$ and the number of poles with positive real part of $G(s)P(s)$ is 1).

Exercise 2 We have for the output $w$ with inputs $d$ and $m$ in Laplace domain

$$w(s) = \begin{pmatrix} 0 & 1 \\ 0 & s + 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} m(s) + \begin{pmatrix} 0 & 1 \\ 0 & s + 2 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \end{pmatrix} d(s) = \frac{1}{s}m(s)$$

For the output $y(s)$ we have with inputs $v$ and $d$

$$y(s) = \frac{G(s)P_2(s)\frac{w(s)}{m(s)}v(s)}{1 + G(s)P_2(s)\frac{w(s)}{m(s)}v(s)} = \frac{G(s)F(s)}{1 + G(s)F(s)}v(s)$$

where $F(s) = \frac{s^{-1}}{s(s+1)}$. Therefore, $y$ is not influenced by $d$ and (ii) is trivially satisfied. Next, design $G(s)$ so that to satisfy (i). It is possible to satisfy (i) and (iii) at the same time.
Consider $G(s)$ of the form

$$G(s) = K \frac{s + z}{s + p}$$

By comparison, requiring that the closed-loop poles are all in $-2$,

$$(s + 2)^3 = s^3 + 6s^2 + 12s + 8 = NUM(1 + G(s)F(s)) = (s + p)(s^2 + s) + K[s^2 + (z - 1)s - z] = s^3 + (p + 1 + K)s^2 + (z - 1 + p)s - zK$$

we get $zK = -8$, $p + 1 + K = 6$, $z - 1 + p = 12$ $\Rightarrow$ $z = 4 + \sqrt{8}$, $p = 5 + 8/(4 + \sqrt{8})$ and $K = -8/(4 + \sqrt{8})$.

The root locus of $G(s)F(s)$ is drawn is Figs. 5 and 6.
Exercise 3. We have for the transfer functions \( P_1 \) and \( P_2 \):

\[
P_1(s) = (0 \ 1) \begin{pmatrix} s & 10 \\ -1 & s + 11 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \frac{s + 2}{s^2 + 11s + 10}
\]

\[
R_2(s) = \frac{1}{s + 2}
\]

Moreover, the output \( Y(s) \) in Laplace domain is

\[
Y(s) = \frac{(1 + K(s)G(s)P_1(s))P_2(s)}{1 + L(s)}d(s) + \frac{L(s)}{1 + L(s)}v(s)
\]

(1)

where \( L(s) = G(s)P_1(s)P_2(s) \). The point is to design \( G(s) \) and \( K(s) \) as required with the constraint that they must be realizable (number of poles greater or equal to the number of zeroes).

(i) Design \( G(s) \) first. The closed-loop input-output transfer function is \( W(s) = \frac{L(s)}{1 + L(s)} \) (see
(1). We choose for $W(s)$ a form

$$W(s) = \frac{2}{(s + 1 - j)(s + 1 + j)} = \frac{2}{s^2 + 2s + 2}$$

(two closed-loop poles at $-1 \pm j$). Since

$$G(s)P_1(s)P_2(s) = L(s) = \frac{W(s)}{1 - W(s)} = \frac{2}{s^2 + 2s + 2} := F(s)$$

then

$$G(s) = \frac{F(s)}{P_1(s)P_2(s)} = 2 \frac{s^2 + 11s + 10}{s^2 + 2s}$$

(ii) The closed-loop disturbance-output transfer function is $W_d(s) = \frac{(1 + K(s)G(s)P_1(s))P_2(s)}{1 + L(s)}$ (see (1)). We design $K(s)$ in such a way that

$$|W_d(j\omega)| \leq 0.1, \forall \omega \in [0, 10] \text{ rad/sec.}$$
Since

\[ K(s) = \frac{(1 + L(s))W_d(s) - P_2(s)}{L(s)} = \frac{1}{2} \left[ (s^2 + 2s + 2)W_d(s) - s \right] \]

choose

\[ W_d(s) = \frac{1}{s + 10} \]

Clearly, \( W_d(s) \) satisfies (2) since

\[ |W_d(j\omega)| = \frac{1}{\sqrt{100 + \omega^2}} \leq 0.1 \]

for all \( \omega \in [0, 10] \) rad/sec and, moreover,

\[ K(s) = \frac{1}{2} \left[ \frac{2 - 8s}{s + 10} \right] \]