Operations Research

Master in Mechanical Engineering

Primal and dual pairs of transportation problems

Laura Palagi

http://www.dis.uniroma1.it/∼palagi

Dipartimento di Ingegneria informatica automatica e gestionale A. Ruberti
Sapienza Università di Roma

Via Ariosto 25

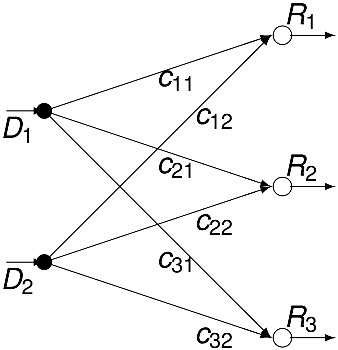
Consider a water transport problem from two production plant to three bottling plants. The daily availability of water (in

hectoliters) at the two plants is respectively $D_1 = 50, D_2 = 55$, while daily requests of water (in hl) at the three plants are respectively $R_1 = 30, R_2 = 40, R_2 = 35$ The shipping costs c_{ij} by each production plant P_i to each bottling plant S_j are reported in the table

	S_1	S_2	S_3
P_1	250	100	85
P_2	120	80	150

Formulate a model that allows you to schedule transportation from the production plant to the bottling facilities so as to minimize the overall cost.

Scheme



Decision variables: x_{ij} the quantities of water (in hl) to be transported from the factory i to the bottling plant j.

The LP model

$$
\begin{array}{lrrrrrrrl}
\min & 250x_{11} & +100x_{12} & +85x_{13} & +120x_{21} & +80x_{22} & +150x_{23} & & \\
 & x_{11} & +x_{12} & +x_{13} & & & & = & 50 \\
 & & & & x_{21} & +x_{22} & +x_{23} & = & 55 \\
 & x_{11} & & & +x_{21} & & & = & 30 \\
 & x_{12} & & & +x_{22} & & & = & 40 \\
 & & & x_{13} & & & +x_{23} & = & 35
\end{array}
$$

$$x_{ij} \geq 0$$

The point of view of a transport company

Suppose that a company that is specialized in the transport of water (outside the industry) wants to present itself to the industry of mineral waters to make the transportation of water by factories to plants.

Of course, the transportation company needs to propose not disadvantageous transport rates so that the water company will use them for transportation.

The transport company wants to choose the prices to apply so to maximize its overall profit.

LP model for the transport company

The prices to apply are the decision of the transportation company. The transport company offers the industry to take a hectolitre of water from each of two plants for a price (in thousands of €/ hl) U_1 and U_2 respectively and to deliver a hectolitre of water to each of the three plants for a price (in thousands of EUR / hl) v_1, and v_2 v_3 respectively.

So its overall profit is obtained by the product of the quantities collected/delivered *times* Price

$$50u_1 + 55u_2 + 30v_1 + 40v_2 + 35v_3.$$

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LP model for the transport company

However, in order that the offer of the transport company is convenient for the industry of mineral waters, the proposed transportation prices must not be higher than those that the industry would face

$$
\begin{aligned}
u_1 + v_1 &\leq 250 = c_{11} \\
u_1 + v_2 &\leq 100 = c_{12} \\
u_1 + v_3 &\leq 85 = c_{13} \\
u_2 + v_1 &\leq 120 = c_{21} \\
u_2 + v_2 &\leq 80 = c_{22} \\
u_2 + v_3 &\leq 150 = c_{23}.
\end{aligned}
$$

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The problem of the transport company is

$$
\begin{array}{lrrrrrl}
\max & 50u_1 & +55u_2 & +30v_1 & +40v_2 & +35v_3 & \\
 & u_1 & & +v_1 & & & \leq 250 \\
 & u_1 & & & +v_2 & & \leq 100 \\
 & u_1 & & & & +v_3 & \leq 85 \\
 & & u_2 & +v_1 & & & \leq 120 \\
 & & u_2 & & +v_2 & & \leq 80 \\
 & & u_2 & & & +v_3 & \leq 150
\end{array}
$$

It is a primal dual pair

$$
\begin{array}{llllllllll}
\min & 250x_{11} & +100x_{12} & +85x_{13} & +120x_{21} & +80x_{22} & +150x_{23} & & \\
(u_1) & x_{11} & +x_{12} & +x_{13} & & & & = & 50 \\
(u_2) & & & & x_{21} & +x_{22} & +x_{23} & = & 55 \\
(v_1) & x_{11} & + & & x_{21} & & & = & 30 \\
(v_2) & x_{12} & + & & x_{22} & & & = & 40 \\
(v_3) & & & x_{13} & + & & x_{23} & = & 35
\end{array}
$$

$$x_{ij} \geq 0$$

$$
\begin{array}{llllllll}
\max & 50u_1 & +55u_2 & +30v_1 & +40v_2 & +35v_3 & & \\
(x_{11}) & u_1 & & +v_1 & & & \leq & 250 \\
(x_{12}) & u_1 & & & +v_2 & & \leq & 100 \\
(x_{13}) & u_1 & & & & +v_3 & \leq & 85 \\
(x_{21}) & & u_2 & +v_1 & & & \leq & 120 \\
(x_{22}) & & u_2 & & +v_2 & & \leq & 80 \\
(x_{23}) & & u_2 & & & +v_3 & \leq & 150
\end{array}
$$