2nd lecture
A simple Production Problem
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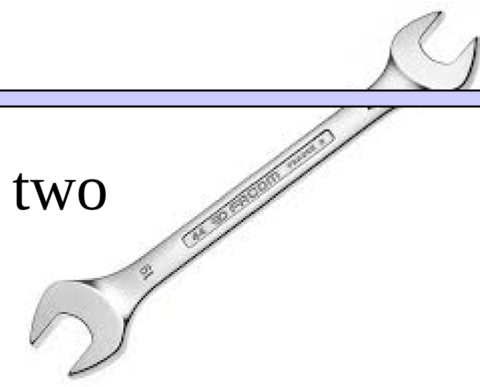
OPERATIONS RESEARCH

Production planning

An engineering factory produces two types of tools:
pliers and spanners.



Pliers cost to the production 1.5 € each
Spanners cost to the production 1 € each

Production costs

Pliers are sold at 6 € each
Spanners are sold at 5 € each

Selling price

How much should the factory produce to maximize its profit ?

Objective function

A possible scenario

Assume that we are constructing → 15000 pliers → 15000 spanners

Let us define the data of the problem

unit profit = selling price – unit cost

4,5 = (6 – 1,5) unit profit for pliers   4 = (5 – 1) unit profit for spanners

Objective function = total profit P_{TOT}

P_{TOT} = number of pliers * unit profit for pliers +

number of spanners * unit profit for spanners

A possible scenario (2)

Formally:

CH = number of spanners to be produced

PI = number of pliers to be produced

variables ⇕ output of our model

Equation of the overall profit

$$P_{TOT} = 4 \cdot CH + 4{,}5 \cdot PI = 4 * 15000 + 4{,}5 * 15000 = 127500$$

15000 (CH)

15000 (PI)

Unit profit of spanners (4)

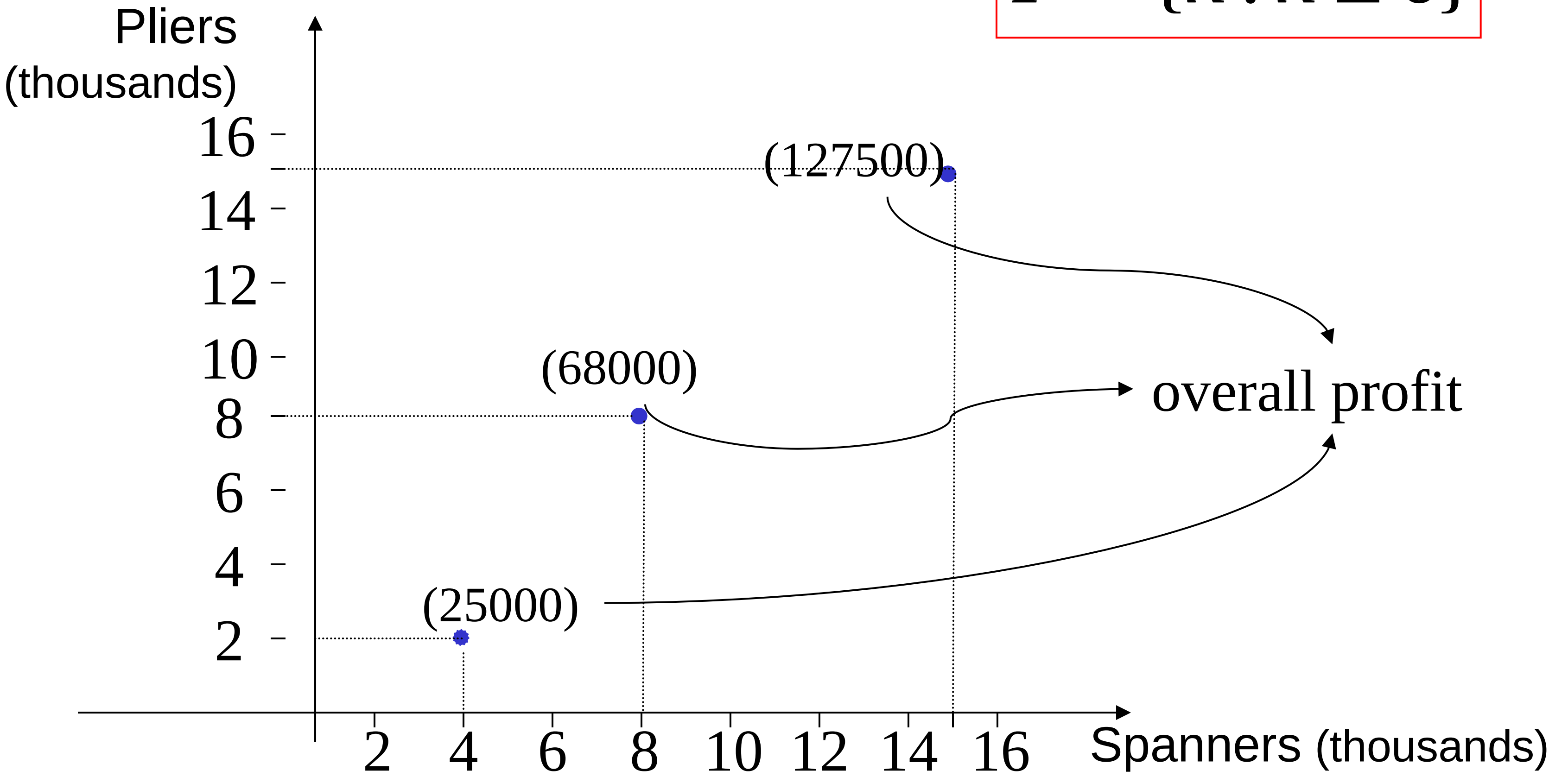
Unit profit of pliers (4,5)

Is this the best value ? (127500)

Constraints

Every point in the non negative quadrant is a possible feasible solution (a possible production planning)

$$F = \{x : x \geq 0\}$$



In principle: the more I produce the more I gain !

In practice: constraints exist that limit the production

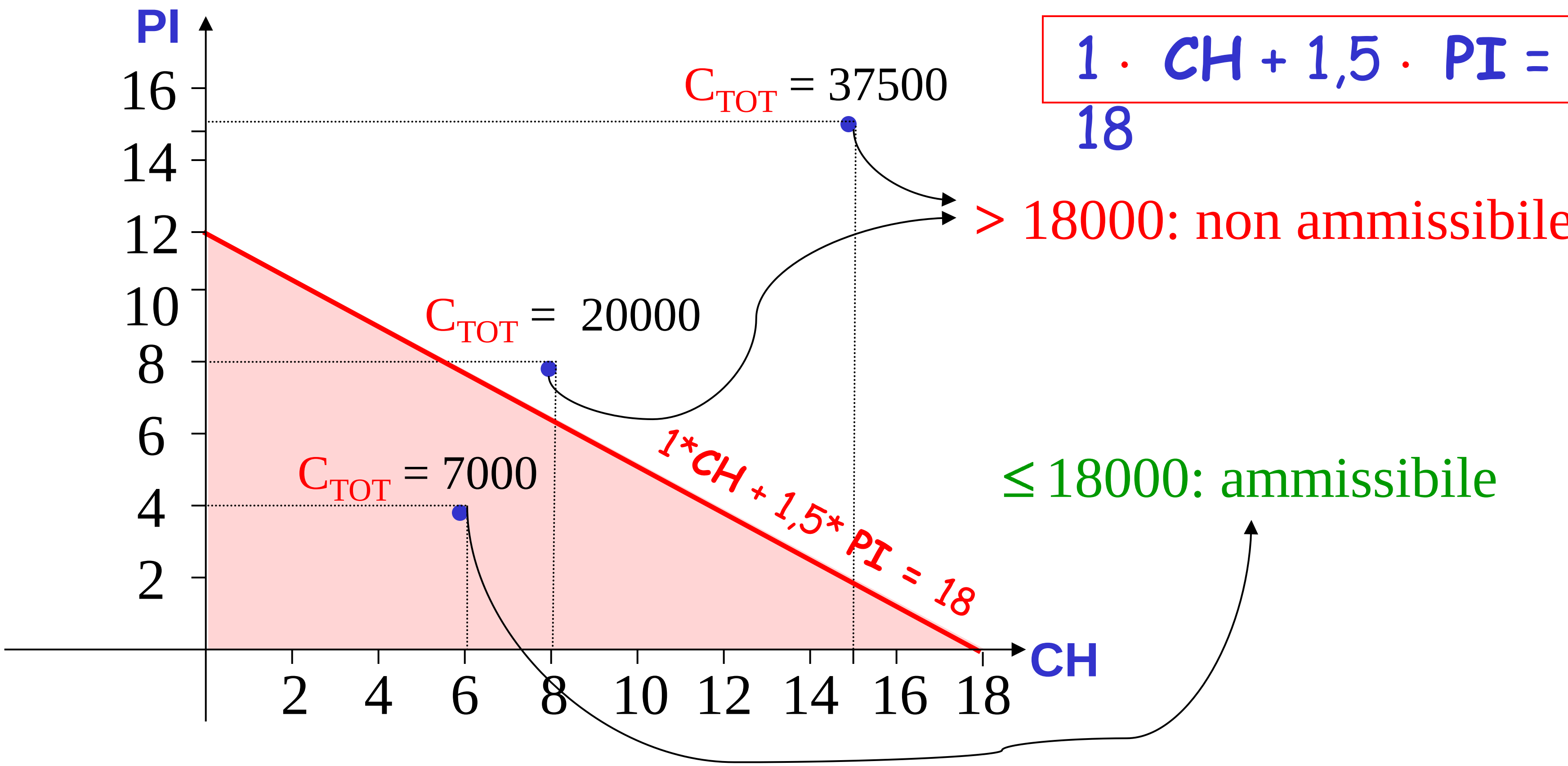
Geometric view of the constraint

Let draw the set *F* of the feasible solutions

In the plane (PI, CH), first draw the equation of the budget constraint

Feasible region ⇕ All non negative points "*below*" the line

$1 \cdot CH + 1{,}5 \cdot PI = 18$

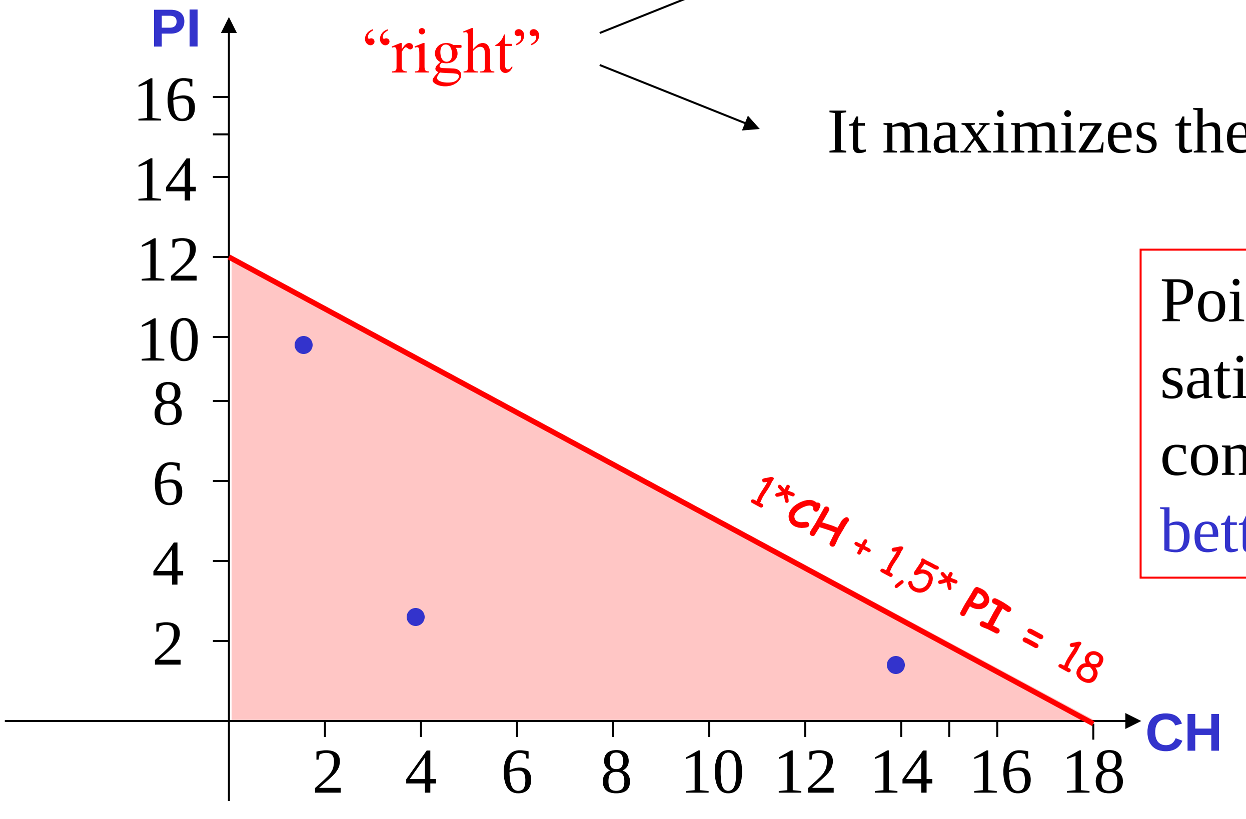


Geometric view of the problem

We need to find the "right" solution among the feasible ones

"right" → It satisfies the budget constraint

"right" → It maximizes the profit



Points in the red area satisfies the budget constraint, which is the better one ?

Limit of the "What If" approach

The scenarios are too many: infinite solutions !

It is not possible to look over all of them !!

We may wonder if may be better when the number of solutions is finite

This is not always true, let see with an example

Geometric view of the production problem

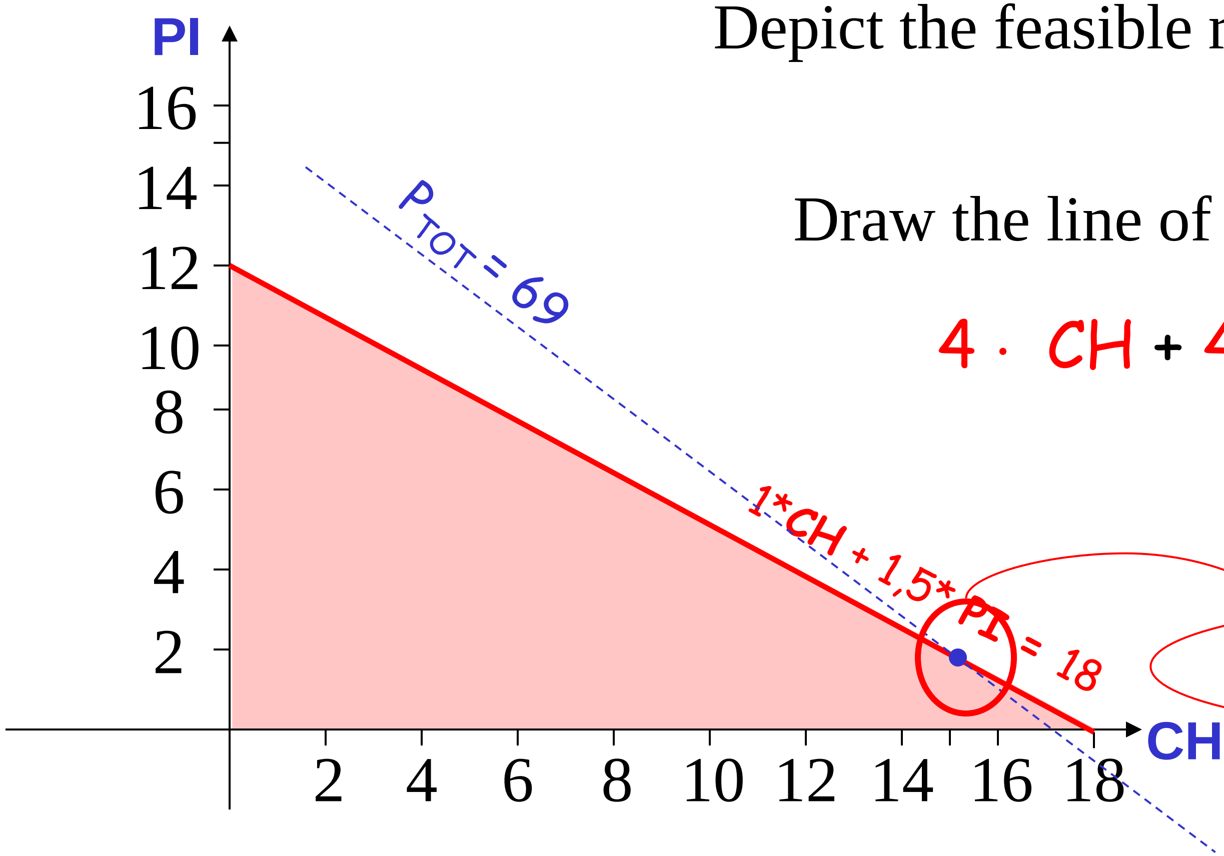
The scenarios are too many

We must find another method

Depict the feasible region F (red area)

Draw the line of the profit P_{TOT}

$$4 \cdot CH + 4,5 \cdot PI = 69$$



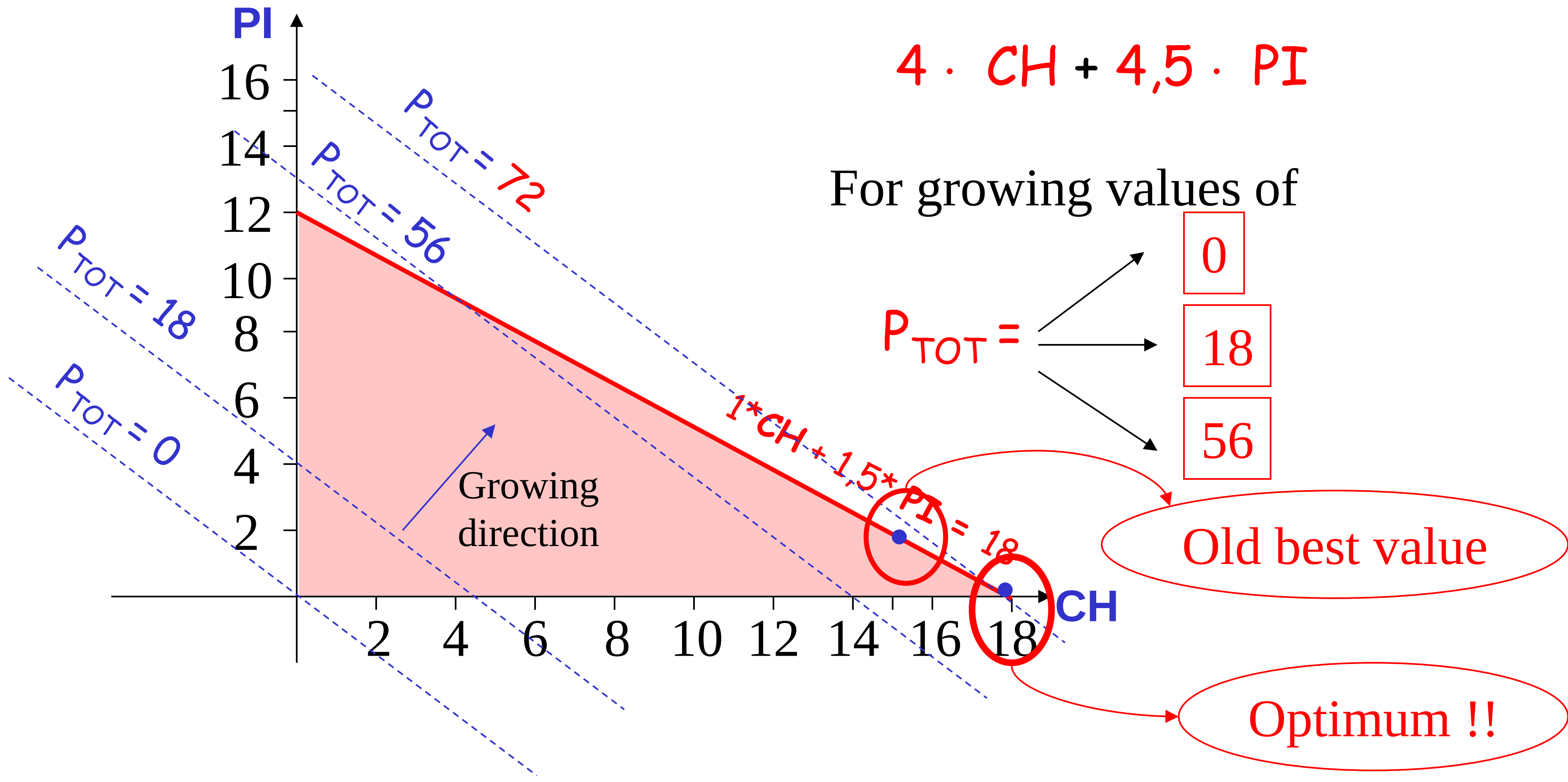
Actual best value

Geometric view of the problem

Draw the line of the profit P_{TOT}

$$4 \cdot CH + 4{,}5 \cdot PI$$

For growing values of



Another constraint: technology

To produce 1000 pliers or 1000 spanners is required 1 hour of usage of a plant

The plant can work for a maximum of 14 hours per day

PI = number of pliers

CH = number of spanners

H_{TOT} = total hours nedeed

$$H_{TOT} = 0.001 \cdot CH + 0.001 \cdot PI$$

Equation of the total hours

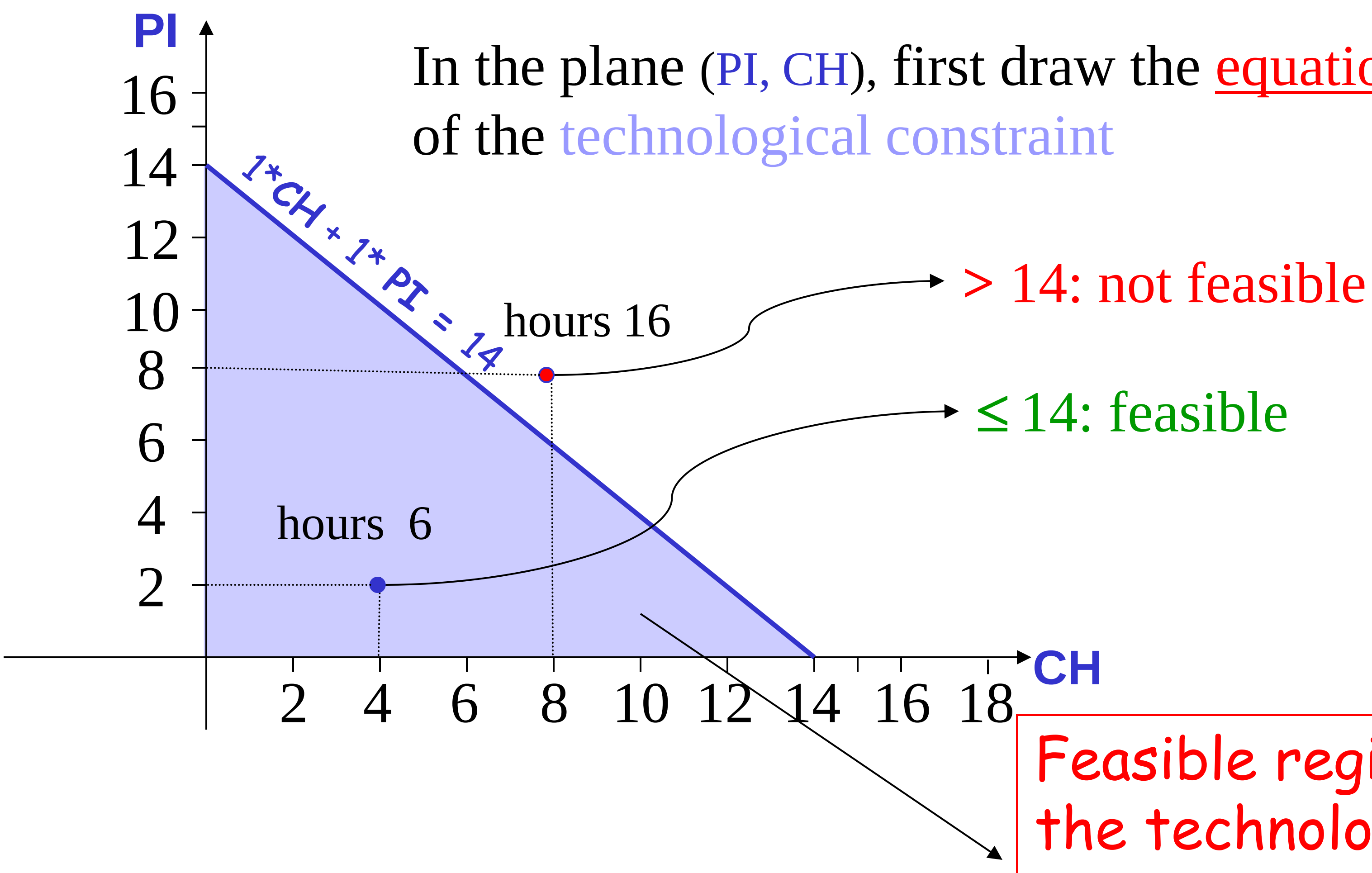
$$H_{TOT} = 0.001 \cdot CH + 0.001 \cdot PI \leq max\_hours=14$$

Technological constraint

Geometric view of the technological constraint

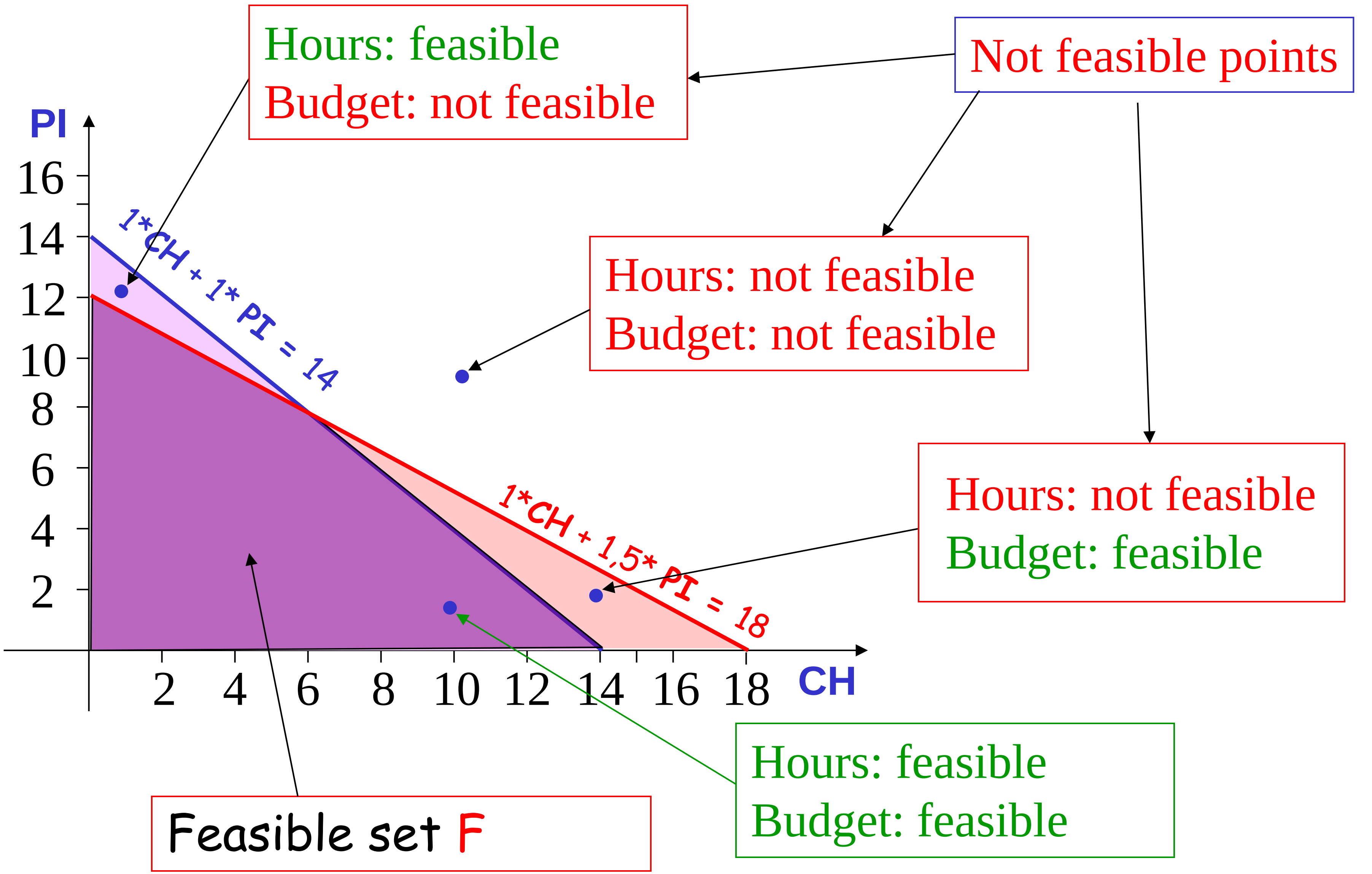
$0.001 \cdot CH + 0.001 \cdot PI \leq 14 \iff 1 \cdot CH + 1 \cdot PI \leq 14000$

In the plane (PI, CH), first draw the equation of the technological constraint

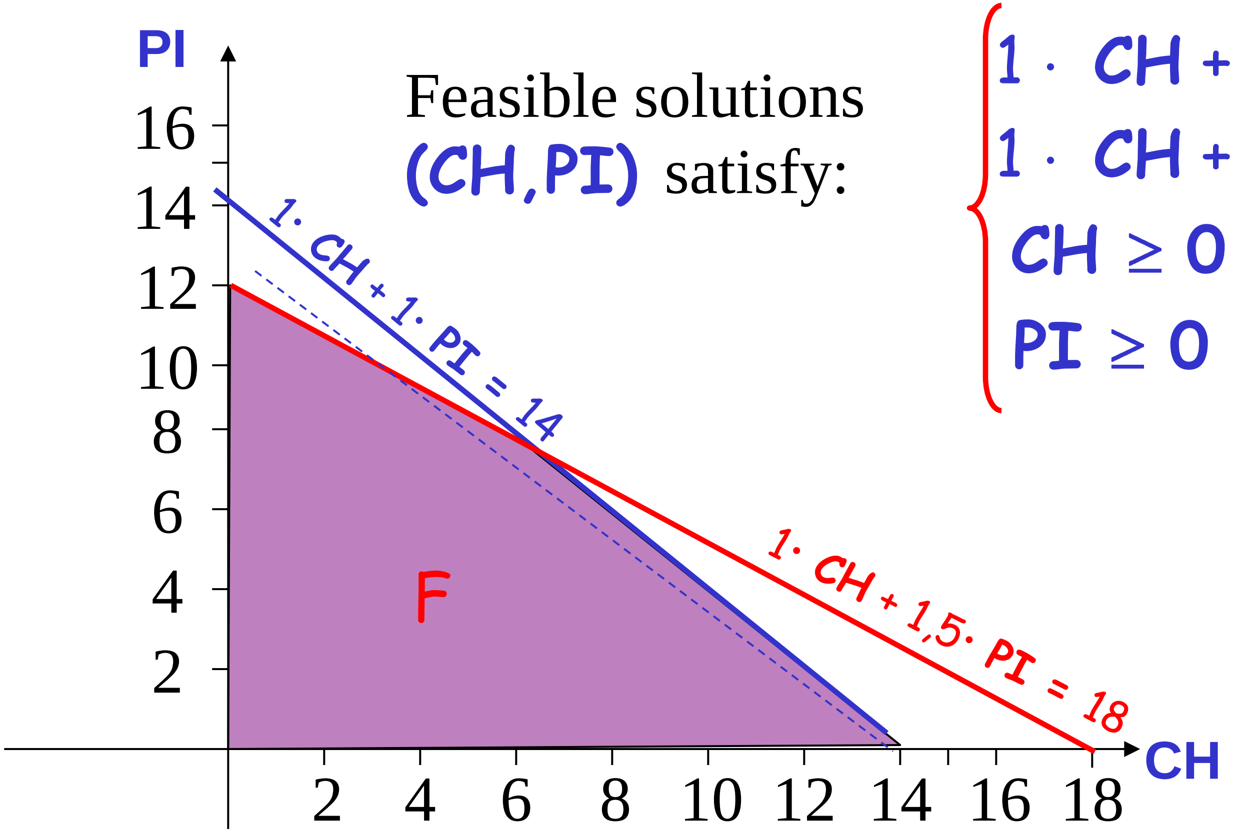


Feasible region of the technological constraint

Geometric view of both constraints



Final feasible region



Feasible solutions **(CH,PI)** satisfy:

$$\begin{cases} 1 \cdot CH + 1{,}5 \cdot PI \leq 18 & \text{budget} \\ 1 \cdot CH + 1 \cdot PI \leq 14 & \text{hours} \\ CH \geq 0 & \\ PI \geq 0 & \end{cases}$$

Non negativity

Which is the best value for **(CH,PI)*** among the feasible ones ?

The best one **(CH,PI)*** maximizes the profit $4 \cdot CH + 4{,}5 \cdot PI$

Linear Programming

CH, PI Real decision variables $CH, PI \in R$

Objective function

$$\max 4 \cdot CH + 4{,}5 \cdot PI \quad \text{profit}$$

constraints

$$1 \cdot CH + 1{,}5 \cdot PI \leq 18 \quad \text{budget}$$

$$1 \cdot CH + 1 \cdot PI \leq 14 \quad \text{hours}$$

$$CH \geq 0 \quad PI \geq 0 \quad \text{Non negativity}$$

$$CH, PI \in R$$

LINEAR PROGRAMMING (LP)

Max or Min of one linear objective function

Constraints are linear equalities (=) or inequalities (≥ = ≤)

Construction of a good model: general rules

Definition of the decision variables

CH, PI ∈ R ⟺ $x_1 , x_2 \in \mathbb{R}$

The name is not important

Definition of the objectives

max $4\,x_1 + 4{,}5\,x_2$ profit

Definition of the constraints

$x_1 + 1{,}5\,x_2 \leq 18$ budget

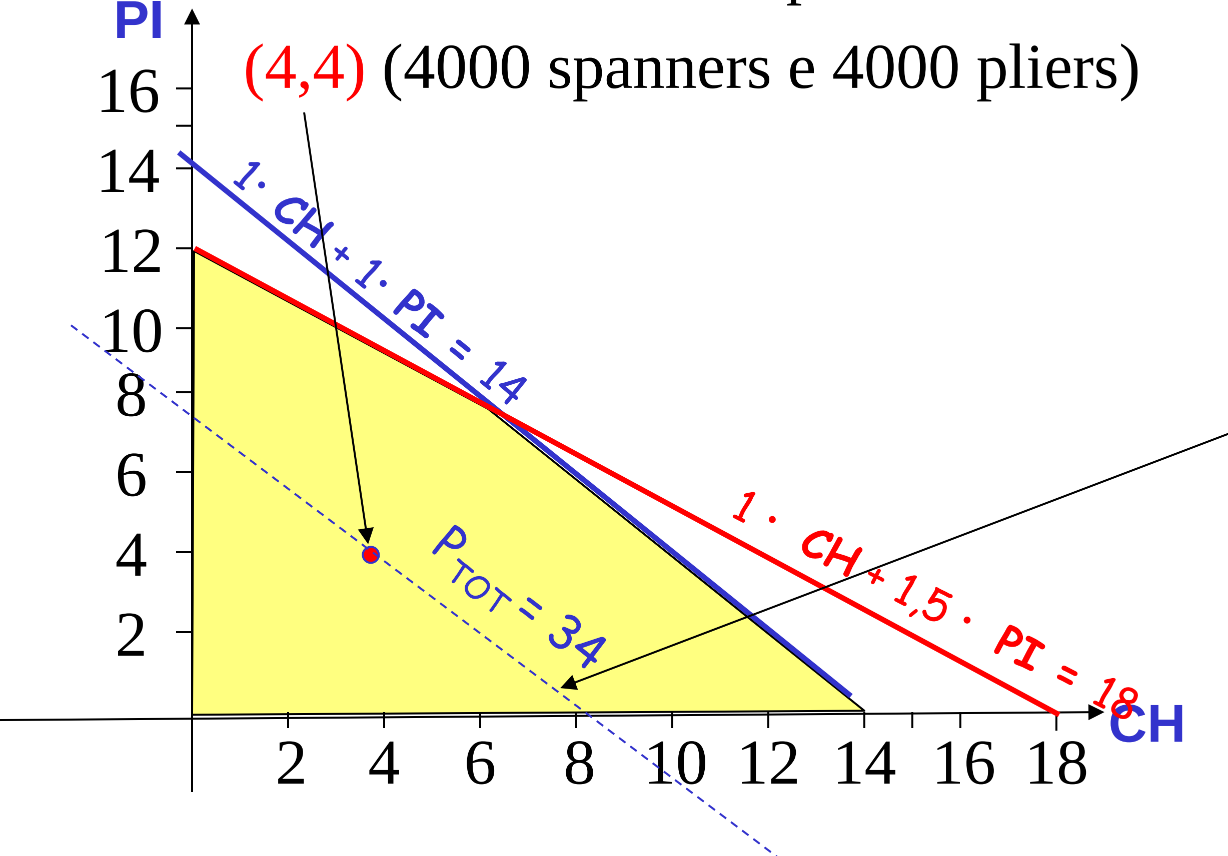
$x_1 + x_2 \leq 14$ hours

$x_1 , x_2 \geq 0$

Graphical solution for LP

Let choice a feasible point

(4,4) (4000 spanners e 4000 pliers)



The profit P_{TOT} is

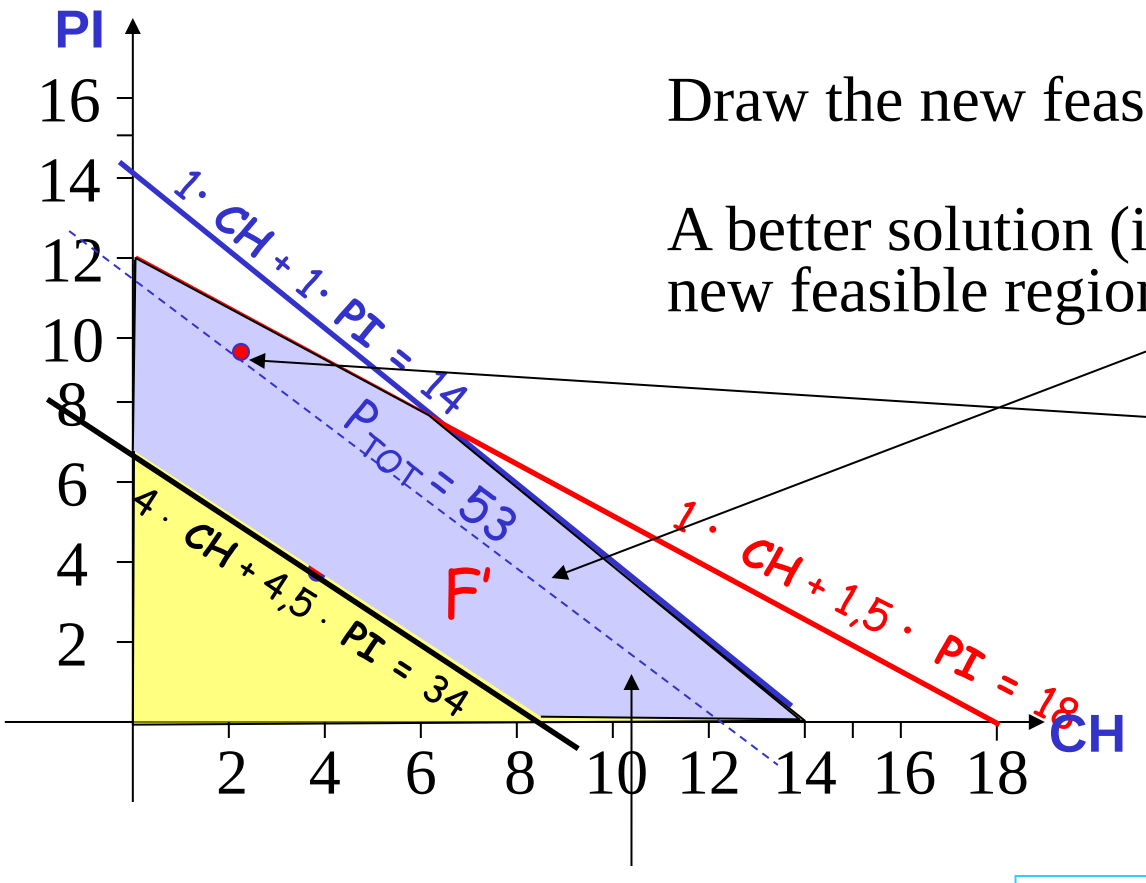
$4 \cdot CH + 4,5 \cdot PI =$

34 (34000 €)

Better solution must have a value of $P_{TOT} \geq 34$

The idea is looking for solutions that satisfy also the "new fictitious" constrait $4 \cdot CH + 4,5 \cdot PI \geq 34$.

Graphical solution for LP



Draw the new feasible region F'

A better solution (if any) must be in the new feasible region F'

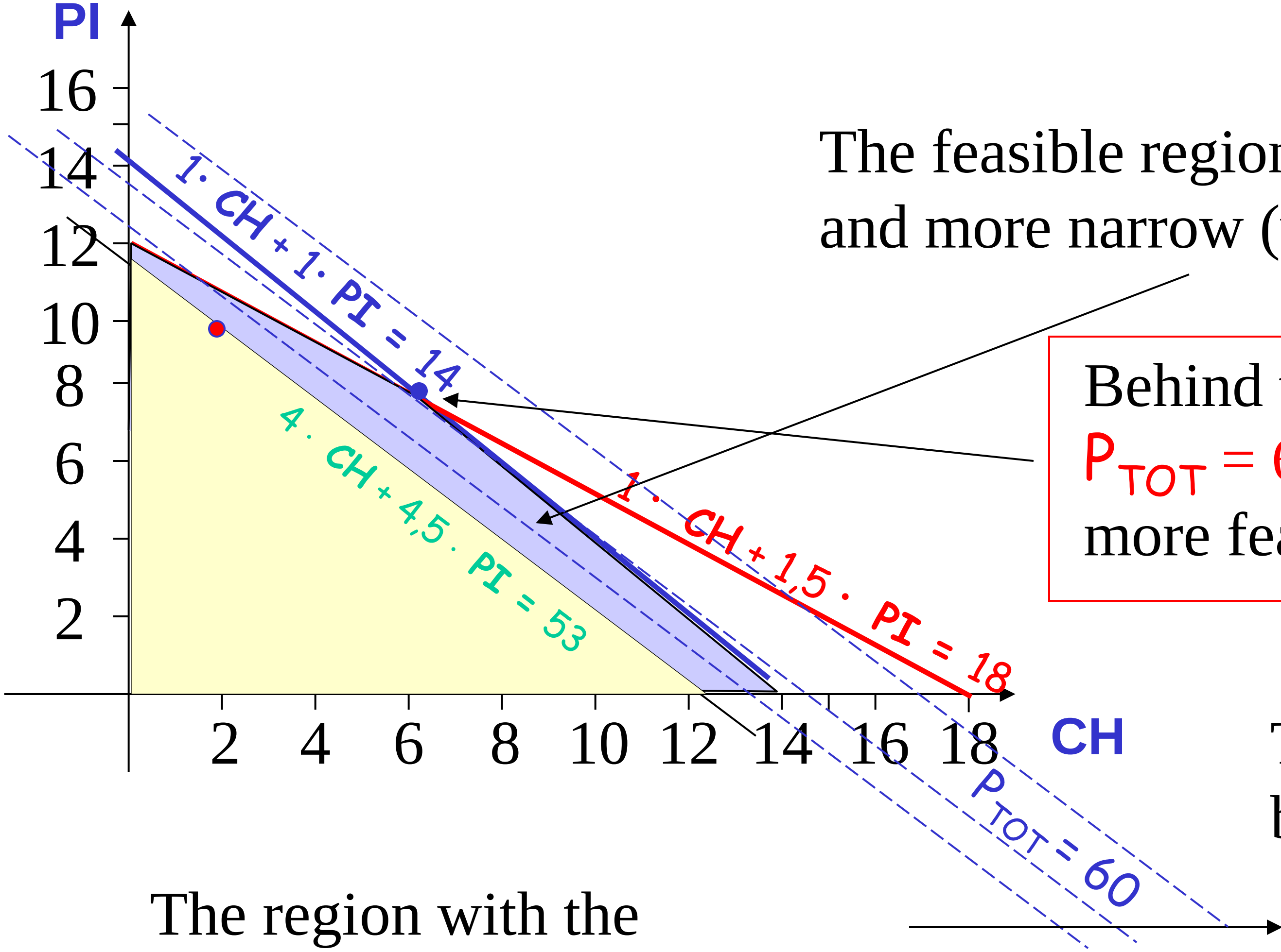
Let choice (2,10).

The profit P_{TOT} is now

$4 \cdot CH + 4{,}5 \cdot PI = 53$

(53000 €)

Graphical solution for LP



The feasible region F″ is more and more narrow (violet region)

Behind this point (6,8) with P_{TOT} = 60, there are non more feasible better points

The point (6,8) is the best feasible one

The region with the constraint $P_{TOT} > 60$ is empty !!!

Increasing values of P_{TOT}

Graphical solution of LP: summary

When the number of variables is two:

1. Draw the constraints and the feasible region
2. Choice a feasible point
3. Draw the line of the objective function passing for this point
4. Parallel move the line of the objective function in the direction of better values
5. The last feasible point "touched" by the line is the optimal solution

Solution of LP

Graphical solution can be applied only when the number of variables is two

Real problems has usually more than two variables

Computer must be used as a tool to tackle large quantities of data and arithmetic

Many standard software exist to solve LP problems of different level of complexity

We use Excel Solver (www.frontsys.com)

http://www.frontsys.com/