Material for

Operation Research Laboratory

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A simple assignment problem

A company considers the problem of assigning 3 worker to 3 jobs. Each man must be assigned a job and each job must be filled. It is important to the company how to do assignment.

Suppose a value or benefit c_{ij} is known for each possible assignment pair man$i$-job$j$. The values of c_{ij} are in the followig table.

	job♯1	job♯2	job♯3
worker♯1	0,9	0,8	1
worker♯2	0,7	0,5	1
worker♯3	0,8	0,4	1

Find the assignment that gives the *maximum average gain.*

The mathematical model

– *Decision variables.*

$$x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ is assigned to job } j \\ 0 & \text{otherwise} \end{cases} \qquad i, j = 1, \dots, n$$

– *Objective function.* We want to maximize

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

– *Constraints.*

- At each job is assigned exactly one worker

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for all } j = 1, \dots, n$$

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The full model is

$$\begin{aligned}
\max \quad & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \\
& \sum_{i=1}^{n} x_{ij} = 1 \quad \text{for all } j = 1, \ldots, n \\
& \sum_{j=1}^{n} x_{ij} = 1 \quad \text{for all } i = 1, \ldots, n \\
& x_{ij} \in \{0, 1\} \quad i = 1, \ldots, n \;\; j = 1, \ldots, n
\end{aligned}$$

Actually the constraints $x_{ij} \in \{0, 1\}$ can be removed without changing the optimal solution, due to a special structure of the constraints (Total Unimodularity). Hence it ia a linear programming problem.